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MARSHALL AND THE MATHEMATIZATION OF ECONOMICS

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I. INTRODUCTION

Historians of economic thought have long debated about the methodological position adopted by Alfred Marshall (1842-1924) concerning the mathematization of economics. While Francis Y. Edgeworth and Marshall's pupils, most notably A. C. Pigou and J. M. Keynes, agreed that it was mainly through him that mathematical economics has since become respectable (see Pigou 1925, p. 66) and proclaimed him the founder of modern diagrammatic economics (ibid., p. 24), more recent commentators have criticized his failure to give proper credit to mathematics, and have even depicted him as an enemy of such an approach (see, for example, Schumpeter 1951, p. 97; Coase 1975; Brems 1975; Creedy 1986, p. 126; Schabas 1989; Huriot 1994, p. 17).

This paper demonstrates that these discrepant interpretations occur mainly because Marshall himself made contradictory statements on several occasions and over a number of years. Although he heatedly claimed that "I have never compromised on any doctrine of any kind" and found the suggestion that he tried "to compromise between" or "reconcile divergent schools of thought" to be trumpery (Pigou 1925, p. 418), his attitude toward the applicability of mathematics and geometry to economics was ambiguous and often far from transparent. Even more so, Marshall's own retrospective account can be taken as an accurate representation of his characteristic awkwardness and hesitancy. Thus in his letter of February 27, 1906, he confided to his favorite

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pupil, Arthur L. Bowley, that "I have very indistinct memories of what I used to think on the subject" of mathematical economics (ibid., p. 427).

II. MARSHALL'S VIEWS ON MATHEMATICAL ECONOMICS

Marshall's acquaintance with economics. There seems no doubt that Marshall's acquaintance with economics was very dependent on mathematics. To begin with, it was as a mathematician that he entered St. John's College, Cambridge, in 1861, and he was elected to a Fellowship at his college after obtaining the second wranglership in the Mathematical Tripos, in 1865. As he explained much later to Clement Colson

> I thought much more easily in mathematics at that time than in English.... At that time and for long after I knew very little of the realities of economic life. But I worked at what I regard as the central problem of distribution and exchange. Before 1871...I had worked out the whole skeleton of my present system in mathematics though not in English (Marshall 1933, p. 221).

He also wrote to John Bates Clark on July 2, 1900 that "my acquaintance with economics commenced with reading Mill, while I was still earning my living by teaching Mathematics at Cambridge; and translating his doctrines into differential equations as far as they would go" (Pigou 1925, p. 412). Eight years later, in another letter of March 24, 1908, Marshall told Clark that by about 1874,

> I had practically completed the whole of the substance of my Mathematical Appendix, the only important exception being the treatment of elasticity (Note III) and Edgeworth's contract curve Note XII bis.... I worked that out for the greater part while still teaching mathematics; and while still regarding myself as a mere pupil in the hands of great masters, especially Cournot, von Thünen and Ricardo; and while still extremely ignorant of economic realities (ibid., pp. 416-17).

That the young Marshall greatly admired A. A. Cournot and was particularly influenced by his celebrated *Recherches sur les principes mathématiques de la théorie des richesses* (1838) is supported by the preface to the first edition (1890) of his *Principles*, where Marshall recognized that Cournot's mathematical conception of continuity "affected, more than any other, the substance of the view expressed in the

present book" (Marshall 1961, *1*, p. x). Marshall's reverence for Cournot in this regard led him to write: "Cournot's genius must give a new mental activity to everyone who passes through his hands" (ibid., p. xi).

Marshall's acquaintance with Whewell. It is important to note that by the time Marshall started on economics the application of mathematics to economics in Britain was already in the air, championed by what James Henderson (1985) called "the Whewell group of mathematical economics," centered around the leading figure of William Whewell (1794-1866), Professor of Moral Philosophy at Cambridge and Master of Trinity College. Since, like Whewell before him, Marshall also began by translating David Ricardo and J. S. Mill into mathematics, some writers have found it rather remarkable that no reference to Whewell's papers on mathematical economics was ever made by Marshall (Macgregor 1942, p. 316; Hutchison 1953, pp. 64-5; Henderson 1973, p. 339; 1985, p. 422; Whitaker 1975, *1*, p. 45, n. 26; O'Brien 1990, p. 136, n. 28; Creedy 1990, p. 99; 1992, p. 32; see also Collard 1968, p. xviii; Cochrane 1975, p. 398, n. 7; Dimand 1988, p. 614). There is, however, direct evidence that Marshall was aware of Whewell's work at an early stage of his career. In fact, a comment by Marshall on Whewell's 1829 paper on Ricardo regarding p. 208 (not p. 108 as erroneously printed in Groenewegen 1990, p. 110, n. 11) is found opposite page 507 in Marshall's annotated copy of the sixth edition (1856) of Mill's *Principles of Political Economy* now preserved in the Cambridge University Library (class-mark: Marshall d. 61).

Marshall also referred to Whewell's practice of assuming linearity in Note XIII of the Mathematical Appendix to the *Principles* (Marshall 1961, *1*,p. 846). As we have seen, this note was written by 1874, and Whewell's reference goes back to the first edition of the *Principles* (Marshall 1961, *2*, p. 835). Unfortunately, we do not know how Marshall become acquainted with Whewell's work. If not earlier, he learned of him in 1872, while reviewing W. S. Jevons's *Theory of Political Economy*. In the preface to his book, Jevons himself quoted and much praised Whewell's early mathematical papers. Again, Marshall knew of Whewell's work through Léon Walras's references in his *Éléments d'économie politique pure*, sent to him by the author in 1889.[1] And in the first edition of the *Principles*, Marshall mentioned the list of mathematical economic writings appended to Jevons's second

1. See Marshall's letter to Léon Walras, September 19, 1889, reprinted in Jaffé 1965, *2*, letter 922, p. 355.

edition of his *Theory*, published in 1879, which includes Whewell's four mathematical papers and two by J. E. Tozer (ibid., pp. 247-48).

In 1891, Marshall was again aware of Whewell (and Tozer) because of the high tribute paid by the Italian Filippo Virgilii in his article entitled "L'applicazione della Matematica all'Economia Politica." An offprint of it with the author's dedication is preserved in the Marshall Library, Cambridge (Magazines & Pamphlets, Ref. 19-h). Finally, it should be noted that in the fourth edition (1898) of the *Principles*, Marshall (1961, *1*, p. 101; *2*, p. 247) cited the excellent bibliography of mathematical economics given by Irving Fisher as an appendix to Nathaniel T. Bacon's translation of Cournot's *Recherches*. It also includes Whewell's (and Tozer's) papers.

Marshall was also long aware of Whewell's great personality, for in his obituary eulogy of Henry Sidgwick he recognized that "as a freshman I learnt that I should *cap* Dr. Whewell and the Vice-Chancellor, but no one else outside my own College" (quoted in Pigou 1925, p. 317). Also, in 1892 Marshall recognized that one of the first books on economics that he came across, and which gave direction to much of his subsequent reading, was by Richard Jones, "edited and published by Whewell in 1859" (Marshall 1892, p. 510; Pigou 1925, p. 296). Moreover, as John K. Whitaker (1975, *2*, p. 264) perceptively remarked, a text in Marshall's copy of Whewell's edition of Jones's book, which Whitaker dates in the early 1870s, reveals respectful appreciation for Whewell when he recommends that "time may be spent not unprofitably in reading somewhat rapidly the prefatory notice [by Whewell]...of the book."[2]

Marshall and mathematical analysis. In his early and otherwise unsympathetic review of Jevons's *Theory* for the *Academy* in 1872, Marshall was eager to recognize the importance of mathematical reasoning in economics, though he already showed his preferences for geometry over algebra:

> We owe several valuable suggestions to the many investigations in which skilled mathematicians, English and continental, have applied their favourite method to the treatment of economical problems. But all that has been important in their reasonings and results has, with scarcely an exception, been capable of being described in ordinary language: while the language of

2. Of course, Marshall's acquaintance with Whewell does not mean he owed a great deal to him, particularly with respect to the concept of elasticity of demand, which is wholly Marshall's.

> diagrams, or, as Professor Fleeming Jenkin calls it, of graphic representation, could have expressed them as tersely and as clearly as that of the mathematics. The latter method, moreover,...is not intelligible to all readers. The book before us would be improved if the mathematics were omitted, but the diagrams retained (quoted in Pigou 1925, pp. 98-9).

Nine years later, in 1881, on the occasion of his review of Edgeworth's *Mathematical Physics*, he returned to the theme. What was true in the case of Jevons was nevertheless not entirely true in the case of his friend Edgeworth:

> This book shows clear signs of genius, and is a promise of great things to come. It is called "An Essay on the Application of Mathematics to the Moral Sciences." But the moral sciences are various and vast; and a goodly volume might be filled with a mere enumeration of the openings which they offer for the use of mathematical language and mathematical method....
>
> The discussion of this problem is introduced by an argument tending to show that that 'mathematical reasonings are possible without numerical *data*.' It is well put, but there is a certain air of unreality about all such arguments... The real question is not whether it is *possible*, but whether it is *profitable* to apply mathematical reasoning in the moral sciences. And this is a question which cannot be answered *a priori*; it can be answered only from the experience of those who make the attempt. When a man has cleared up his mind about a difficult economic question by mathematical reasoning, he generally finds it best to throw aside his mathematics and express what he has to say in language that is understanded [*sic*] of the people...
>
> It will be interesting to watch the development of his theory, and, in particular, to see how far he succeeds in preventing his mathematics from running away with him, and carrying him out of sight of the actual facts of economics....If, however, Mr Edgeworth can prevent his theories from becoming too abstract he may do great things by them (quoted in Whitaker 1975, 2, pp. 265-68; emphasis in original).

In the 1885 jubilee volume of the *Journal of the Royal Statistical Society* Marshall published an article on the graphic method of statistics, where he again proclaimed the advantages of the mathematical language in economics. He wrote: "Mathematical language enables us to express general statements as to quantity with the utmost brevity,

precision and force; and mathematical theory reasons on the basis of these statements" (Marshall 1885, p. 225). Later, in an 1898 article published in the *Economic Journal*, titled "Distribution and Exchange," Marshall confessed again that "in the far-off years, when I used to think naturally in the mathematical language, I jotted down a hundred and one mathematical versions of my central doctrine of distribution" (Marshall 1898, p. 56). Curiously, though devotees of the mathematical method frequently claim that the complexity of economic problems, rather than their simplicity, requires the aid of mathematics for solution, Marshall seems to have adopted exactly the opposite view. He went on to say that "the most helpful applications of mathematics to economics are those which are short and simple, which employ few symbols; and which aim at throwing a bright light on a small part of the great economic movement rather than at representing its endless complexities" (ibid., p. 39).

Significant too is Marshall's first letter to the editor Frederick Macmillan, advising him that "Mathematics cannot now be avoided in some branches of economics."[3] However, to Walras, who had tried to gain his support for mathematical economics, Marshall replied in 1889: "I have not myself retired from the conclusion that I think I communicated to you some time ago, viz that the right place for mathematics in a treatise on Economics is at the background."[4] Again, writing to Bowley in 1901, Marshall admitted the existence of economic questions not tractable in mathematical terms, and rejected disparagingly the application of such methods on the grounds of their complex interrelations:

> In my view every economic fact, whether or not it is of such a nature as to be expressed in numbers, stands in relation as cause and effect to many other facts: and since it *never* happens that all of them can be expressed in numbers, the application of exact mathematical methods to those which can is nearly always a waste of time, while in the large majority of cases it is positively misleading; and the world would have been further on its way forward if the work had never been done at all. It is chiefly when the mathematical method is used not for direct construction, but to train sound instinctive habits (like the practising of scales on the piano) that it seems to me generally helpful (quoted in Pigou 1925, p. 422).

3. Marshall to Macmillan, April 12, 1887, reprinted in Guillebaud 1965, p. 519.

4. Marshall to Walras, September 19, 1889, reprinted in Jaffé 1965, 2, p. 355, letter 922.

But perhaps even more important in explaining Marshall's reluctance to rely on mathematics was his belief that its application implied rigorous abstraction and "pure" economic theory, a word that Marshall much disliked. Thus, in his letter of August 28, 1902, he actively advised Edgeworth against applying mathematics and abstract theory to human affairs, a warning that Edgeworth undoubtedly did not follow:

> But I conceive no more calamitous notion than that abstract, or general, or 'theoretical' economics was economics 'proper.' It seems to me an essential but very small part of economics proper: and by itself sometimes even-well, not a very good occupation of time.
>
> The key-note of my *Plea* is that *the* work of the economist is 'to disentangle the interwoven effects of complex causes;' and that for this, general reasoning is essential, but a wide and thorough study of facts is equally essential, and that a combination of the two sides of the work is *alone* economics *proper*. Economic theory is, in my opinion, as mischievous an impostor when it claims to be economics *proper* as is mere crude unanalysed history (quoted in Pigou 1925, p. 437; emphasis in original).

Marshall's aversion to pure economic theory is well documented too in his correspondence with W. A. S. Hewins, the first Director of the London School of Economics. Having been asked his opinion on the subject, Marshall's letter in reply was quite definite:

> It seems strange to me to be asked my views as to the study of pure economic theory; as tho' that were a subject on wh[ich] I were fit to speak. For indeed I was never a partisan of it; and for more than a quarter of a century I have set my face away from it...
>
> The fact is I am the dull mean man, who holds Economics to be an organic whole, and has as little respect for pure theory (otherwise than as a branch of mathematics or the science of numbers), as for that crude collection and interpretation of facts without the aid of high analysis which sometimes claims to be a part of economic history.... I repeat I regard the use of mathematics on the way as a gain when convenient, but not as of the essence of the work (quoted in Coats 1967, pp. 410-11).

In his next letter to Hewins, dated May 29th, 1900, Marshall raised the following interesting methodological and semantic issue:

> I find that the subject wh[ich] you had described as economic *science* is officially called 'pure theory.' I knew that that had been assigned some place: but I am rather indifferent about it. Much of 'pure theory' seems to me to be elegant toying: I habitually describe my own pure theory of international trade as a 'toy.' I understand economic science to be the application of powerful analytical methods to unravelling the actions of economic and social causes, to assigning each its part, to tracing mutual interactions and modifications; and above all to lying bare the hidden *causas causantes* (ibid., p. 413; emphasis in original).

But doubtless Marshall's most acid misgivings for the use of mathematics in economics are found in his often-cited letter to Bowley, in February 1906:

> I never read mathematics now: in fact I have forgotten even how to integrate a good many things. But I know I had a growing feeling in the later years of my work at the subject that a good mathematical theorem dealing with economic hypotheses was very unlikely to be good economics: and I went more and more on the rules—(1) Use mathematics as a shorthand language, rather than as an engine of inquiry. (2) Keep to them till you have done. (3) Translate into English. (4) Then illustrate by examples that are important in real life. (5) Burn the mathematics. (6) If you can't succeed in 4, burn 3. This last I did often....
>
> Mathematics used in a Fellowship thesis by a man who is not a mathematician by nature—and I have come across a good deal of that—seems to me an unmixed evil. And I think you should do all you can to prevent people from using Mathematics in cases in which the English Language is as short as the Mathematical... (quoted in Pigou 1925, p. 427).

The mathematical reasoning in Marshall's Principles. Marshall's monumental *Principles of Economics* provides additional examples of his far from transparent attitude towards the role of mathematics and geometry in the study of economics. In the preface to the first edition, Marshall openly admitted the difficulty of getting a clear full view of the important notion of continuity "without the aid either of mathematical symbols or of diagrams." But on the same page he went on to qualify that

> the chief use of pure mathematics in economic questions seems

> to be in helping a person to write down quickly, shortly and exactly, some of his thoughts for his own use: and to make sure that he has enough, and only enough, premises for conclusions (i.e., that his equations are neither more nor less in number that his unknowns)[5]...yet it seems doubtful whether any one spends his time well in reading lengthy translations of economic doctrines into mathematics, that have not been made by himself (Marshall 1961, *1*, pp. x-xi).

In the text, Marshall first wondered "whether much has been gained by the use of complex mathematical formulae," but then he praised the great service rendered by the application of mathematical language and mathematical habits of thought because

> it has led people to refuse to consider a problem until they are quite sure what the problem is; and to insist on knowing what is, and what is not intended to be assumed before proceeding further. This has in its turn compelled a more careful analysis of all the leading conceptions of economics, and especially of demand (ibid., pp. 84-5).

It was also in the first edition of the *Principles* that Marshall included a passage, omitted in subsequent editions, praising the use of semi-mathematical language as having been "by far the more important" step that "had brought about a great change in the manner of economic thought" in his own generation (ibid., p. 101).

In analyzing the theory of distribution, Marshall not only admitted that the problem was so complex "that it is impossible to comprehend the whole in a single statement," but felt impelled to add: "But yet by aid of the terse, compact, precise language of Mathematics it is possible to lead up to a fairly unified general view" (ibid., p. 545). In appendix D, titled "Uses of Abstract Reasoning in Economics," Marshall first warned readers against the frequent use of mathematical formulae in economic studies, since they may lead to the illusory belief that there is room in economics for long chains of deductive reasoning, when in his own words "this is not the case except perhaps when a pure mathemati-

5. Incidentally, Note XXI (XX in the first edition) in the mathematical appendix to *Principles* confirms Marshall's failure to realize that such equality is not sufficient to ensure the existence of an equilibrium. He wrote: "Thus, however complex the problem may become, we can see that it is theoretically determinate, because the number of unknowns is always exactly equal to the number of equations which we obtain" (Marshall 1961, *1*, pp. 855-56).

cian uses economic hypotheses for the purpose of mathematical diversions." For such cases he bears no responsibility for the appropriateness of the material supplied by economic theory. Curiously, however, on the same page Marshall encouraged his students to study mathematics as a way of training their minds because

> a training in mathematics is helpful by giving command over a marvellously terse and exact language for expressing clearly some general relations and some short processes of economic reasoning; which can indeed be expressed in ordinary language, but not with equal sharpness of outline. And, what is of far greater importance, experience in handling physical problems by mathematical methods gives a grasp, that cannot be obtained equally well in any other way, of the mutual interaction of economic changes (ibid., p. 781).

In 1893, in another article in the *Economic Journal*, Marshall also admitted by implication that such concepts as "final utility," "marginal production," etc., could only have originated in a mathematical mind:

> These terms are used to enable ordinary readers to get the chief advantages which mathematicians derive from their training in the analysis of the laws of continuous growth. And after a little trial and error, at the hands of two generations of workers, they have reached a form which experience shows enables them to render great service to the student (Marshall 1893, pp. 81-82).

Finally, it is worthwhile to mention Marshall's disdain for Eugen von Böhm-Bawerk and for his lack of mathematical training. In his letter of December 19, 1904 to Knut Wicksell, Marshall proudly confessed to be a "trained mathematician" and accused to Böhm-Bawerk for having gone seriously astray in his criticisms on him simply because of his failure to comprehend the mathematical notes in *Principles*, where he might have found "a complete solution to his mystery" (quoted in Gårdlung 1958, pp. 342-43).

III. MARSHALL ON GEOMETRY

By the same token, what is true about Marshall's attitude towards mathematical analysis appears to be equally true for geometry. To begin with, his acknowledged elegance and facility with diagrams certainly contrasts with his extreme diffidence concerning publication. Marshall's reluctance to give diagrams any prominence is also known, the obvious example being their relegation to footnotes in the *Princi-*

ples. However, in a draft of a letter, dated 1878 by Whitaker (1975, *1*, p. 63), Marshall announced to the publisher Macmillan that he intended to get out three books before the end of 1881. The first of these would be "on the method of diagrams {distinct from but allied to the methods of analytical mathematics} applied to economic theory including Foreign trade curves."

We have already mentioned that in his review of Jevons' *Theory*, Marshall showed his predilection for geometry over algebra. More significantly, in his early manuscript privately printed and circulated by Sidgwick in 1879 titled "The Pure Theory of Foreign Trade," Marshall disparagingly described the usefulness of mathematical apparatus in the analysis of dynamic problems and his entire argument favoured diagrams exclusively: "The pure theory of economic science requires the aid of an apparatus which can grasp and handle the general quantitative relations on the assumption of which the theory is based. The most powerful engines for such a purpose are supplied by the various branches of mathematical calculus" (Marshall 1879, p. 5).

While granting the value of the mathematical method, Marshall went on to qualify that proposition strongly:

> But diagrams are of great service, wherever they are applicable, in interpreting to the eye the processes by which the methods of mathematical analysis obtain their results. It happens that with a few unimportant exceptions all the results which have been obtained by the application of mathematical methods to pure economic theory can be obtained independently by the method of diagrams.
>
> Diagrams present simultaneously to the eye the chief forces which are at work, laid out, as it were, in a map; and thereby suggest results to which attention has not been directed by the use of the methods of mathematical analysis. The method of diagrams can be freely used by every one who is capable of exact reasoning, even though he have no knowledge of Mathematics (ibid.).

There was still another reason leading Marshall to prefer diagrams: the inability for the mathematical analysis to discover the exact laws governing the shapes of curves for particular problems:

> For the mathematical functions introduced into the original differential equation could not, in the present condition of our knowledge, be chosen so as to represent even approximately the economic forces that actually operate in the world. And by

> integrating them we should move further away from, instead of approaching nearer to the actual facts of life. For this reason, among others, the method of diagrams seems to me to be generally speaking of greater use to the Economist, than the methods of mathematical analysis. For when using the former method we have continually before us those assumptions which are justified by economic facts, and no others. Whereas the use of mathematical analysis has been found to tempt men to expend their energy on the elaboration of minute and complex hypotheses, which have indeed some distant analogy to economic conditions, but which cannot properly be said to represent in any way economic laws (ibid., p. 25, n.).

Similarly, writing to Edgeworth in 1880, Marshall admitted on the one hand the existence of economical questions which he could not reduce to curves, and on the other his reluctance to quantify economic relationships:

> When tackling a new problem, I generally use analysis, because it is handier: and in the book which I am just going to begin to write I shall retain (in footnotes) a little mathematical analysis for questions which I can't reduce under the grasp of curves. But partly because curves require a special training, partly because they bear more obviously on the science of statistics —I intend never to use analysis when I can use geometry— my experience of the exact treatment of supply and demand in inference has been disappointing. The intricacies of the question are so numerous, the difficulties connected with the time element so great, that I have never got any curves relating to it which have satisfied me for many months after I first drew them (quoted in Mirowski 1990, p. 84).

In contrast, in his rather grudging letter of January 1912 in reply to Henry Moore, Marshall made it quite clear that he was not a partisan of modern econometrics, contrary to what J. A. Schumpeter (1954, p. 840) was tempted to believe:

> I will be frank. I have your book on Laws of Wages... intending to read it when opportunity come. It has not come; and I fear it never will come. For what dips I have made into the book made me believe that it proceeds on lines which I deliberately decided not to follow many years ago; even before mathematics had ceased to be a familiar language to me. My reasons for it are mainly two. (1) No important economic chain of

> events seems to [*sic*] likely to be associated with any one cause so predominantly that a study of the concomitant variation of the two can be made as well by mathematics, as by a comparison of a curve representing these two elements with a large number of the other curves representing other operative causes: the 'ceteris paribus' clause —though formally adequate seems to me impracticable. (2) Nearly a half of the whole operative economic causes have refused as yet to be tabulated statistically (quoted in Stigler 1965, pp. 352-53).

We learn from Marshall himself, however, that he did not praise geometry at all times. Thus, in his cited article of 1898 in the *Economic Journal*, he confessed that twenty years before, that is, in 1878, "I abandoned the use of curves for market problems because they were not really wanted.... Later on I found an even stronger objection of the same kind to the use of curves for wages problems; so I ceased to use them also" (Marshall 1898, p. 46). Similarly, in the 1902 letter to Edgeworth cited above, he wrote:

> You know I never apply curves or mathematics to market values. For I don't think they help much.... I think curves do naturally avoid the money difficulty: but I do not think they are essential for that line of argument. And I think they only get at the outer fringe of the outside of real problems of International Trade (in Pigou 1925, p. 435).

And in the preface to the first edition of the *Principles*, while asserting "the argument in the text is never dependent on them [diagrams]; and they may be omitted," Marshall observed, based in his own experience, that "they give a firmer grasp of many important principles than can be got without their aid; and that there are many problems of pure theory, which no one who has once learnt to use diagrams will willingly handle in any other way" (Marshall 1961, *1*, p. x). After all, we learn from Keynes that "Marshall himself always used them [diagrams] freely in his lectures" (quoted in Pigou 1925, p. 26).

IV. MARSHALL ON METHOD

Closely related to Marshall's view on mathematical analysis is his conception of method in economics. Ronald Coase (1975) has shown that Marshall did not only show little interest in the question of method in economics, but had also no definite ideas about it. Coase observed that in an early letter to John Neville Keynes, Marshall stated his gener-

al position: "I say of *methods* simply that economics has to use every method known to science" (ibid., p. 26). Much later, in the *Principles*, Marshall insisted that "there is not any one method of investigation which can properly be called the method of economics," and thereby encouraged economists to use every known scientific tool, either singly or in combination with others, adjusting to the particular circumstances (Marshall 1961, *1*, p. 29).

Significantly, for Marshall "induction and deduction are both needed for the scientific thought" and "rest on the same tendencies, the same beliefs, the same needs of our reason" (ibid., pp. 29, 773). It seems, however, that Marshall's own preferences favoured inductive methods, since he declared it "obvious that there is no room in economics for long series of deductive reasoning" (ibid., p. 781).[6] "Even in mechanics," he added, "long chains of deductive reasoning are directly applicable only to the occurrences of the laboratory" (ibid., p. 771). Though not comparable in their exactitude with physical or mechanical sciences, Marshall maintained that economics belonged in any case to the same group of sciences, and aspired alike to establish quantitative laws. Yet, he was convinced that economic reality was complex enough to prevent general propositions. "Economic causes," he said, "are intermingled with others in so many different ways, that exact scientific reasoning will seldom bring us very far on the way to the conclusion for which we are seeking" (ibid., p. 779). His little faith in abstract reasoning led him to say that "the function then of analysis and deduction in economics is not to forge a few long chains of reasoning, but to forge rightly many short chains and single connecting links" (ibid., p. 773). A final illustration of Marshall's methodology is that in an article in the *Economic Journal* he ventured to state that "the Mecca of the economist is economic biology rather than economic dynamics" (Marshall 1898, p. 43). He incorporated that epigram in the prefaces to all subsequent editions of the *Principles*.[7]

V. CONCLUDING REMARKS

What then can be said of Marshall's methodological position concerning the mathematization of economics? Evidently, he was not opposed

6. It is worthwhile to recall the following attached sentence, since deleted from the third edition (1895): "Such chains [of reasoning] might indeed afford interesting speculation in the closet" (Marshall 1961, *2*, p. 770).

7. See Marshall 1961, *1*, p. xiv; *2*, pp. 39, 47.

to mathematical methods, as many have noted. It is indeed true that he eschewed mathematics in the text of the *Principles*, and even relegated diagrams to footnotes, but as J. R. Hicks (1930, p. 215) remarked perceptively long ago, Marshall's work in the *Principles* "is presented in a non-mathematical form, but is mathematical in essence." Pigou, Marshall's successor at Cambridge, was also quick to realize, in his lecture in memory of Alfred Marshall in 1924, that in the text of the *Principles* the mathematical reasoning lies beneath the surface:

> A smooth platitudinous argument it seems at first: later on one discovers with a shock that the central part of it is a translation into ordinary language of a close mathematical argument, not perhaps to be grasped completely until it has been translated back again into the symbolic form in which it must first have been built up (quoted in Pigou 1925, p. 86).

We also learn from Pigou's experience that Marshall was against the use of mathematics in economics because

> he saw that excessive reliance on this instrument might lead us astray in pursuit of intellectual toys, imaginary problems not conforming to the conditions of real life: and, further, might distort our sense of proportion by causing us to neglect factors that could not easily be worked up in the mathematical machine (ibid., p. 84).

Later, in his book *Alfred Marshall and Current Thought*, Pigou (1953, p. 12) insisted that "so far as he [Marshall] was against mathematical elaborations in economics, it was only because he feared that realism might suffer. Convince him that any particular line of mathematical attack would indirectly help realism, and he would have been its enthusiastic friend." Keynes, Marshall's most distinguished pupil, has expressed a similar point of view, when he noticed in his famous obituary essay that

> Marshall's mathematical and diagrammatic exercises in Economic Theory were of such a character in their grasp, comprehensiveness and scientific accuracy and went so far beyond the 'bright ideas' of his predecessors, that we may justly claim him as the founder of modern diagrammatic economics (in Pigou 1925, p. 24).

Along similar lines, Edgeworth remarked that Marshall's writings are characterized by a phrase which Marshall himself acknowledged to be appropriate: "bearing under the garb of literature the armour of mathe-

matics" (ibid., p. 66). Finally, in his centenary paper on Marshall, Gerald F. Shove wrote:

> It may therefore be worth while to observe how naturally—one is tempted to say inevitably—the theoretical framework of the *Principles* grows out of an attempt to test, and fill the gaps in, Ricardian doctrines by the use of a mathematical apparatus—in other words, 'to translate them into differential equations' and 'make them more general' (Shove 1942, p. 296).

In light of this, what can be said of Marshall's failure to give proper credit to mathematical reasoning, when there is little doubt that through it the theoretical framework of the *Principles* was developed? For Keynes, the explanation is to be found in Marshall's fear that businessmen would shy away from his book (in Pigou 1925, p. 26). To Edgeworth, Marshall may have sidestepped mathematics to avoid contradicting his esteemed mentor Benjamin Jowett (ibid., p. 66).[8] More interesting is Jacob Viner's suspicion that it was Marshall's lurking puritan conception, the potential immorality of highly pleasurable activities, which led him to criticize the mathematical method. "Mathematics, and especially graphs," he said, "were Marshall's fleshpots, and if he frequently succumbed to their lure it was not without a struggle with his conscience" (Viner 1941, p. 231).

It is worthwhile to recall in this context Marshall's experience with mathematics early on. If we believe Keynes, repeating Mary Marshall, Marshall's tyrant father "hated the sight of a mathematical book," but Alfred read mathematics on the sly, "as he walked to and from the school." Keynes went on to add the following significant sentence: "Mathematics represented for Alfred emancipation, and he used to rejoice greatly that his father could not understand them" (in Pigou 1925, p. 3). Two other authors, Coase (1984) and Juan Urrutia (1983), agree that Marshall's extreme sensitivity to criticism, his agonizing over mistakes, his dislike of controversy, his evasiveness upon any hint of disagreement, in sum his complex personality, were, to a large extent, the result of his tyrannical father. Thus, although it is not easy to say exactly whether unconscious motivations also influenced Marshall's attitude to the use of mathematics in economics, it is clear, both in his

8. For Jowett, mathematics were not a method of discovery, but only a short language. In his letter of December 25, 1884, he even advised Marshall to relegate mathematical symbols to notes and appendices in his forthcoming "opus magnum" (quoted in Whitaker 1975, *1*, p. 28).

formal writings and in private correspondence, that he made contradictory and far from transparent pronouncements on the subject.

On the other hand, Marshall was a highly respected late-Victorian, and mathematics formed the core of Victorian liberal education (Becher 1971, Richards 1988). Significantly, in the Victorian context the emphasis was on applied mathematics rather than abstract principles. As Victorians become convinced that pure analysis was empty and artificial, treating all problem alike, they also came to believe that it actually solves little. Further, they advocated the subordination of pure mathematics to its applied counterpart on the grounds that mere analysis threatened the foundations of liberal education. It was also a characteristic of Victorians to nurture the conviction that analytical reasoning is bad for the intellect, while geometry would strengthens it. Good education, then, was geometry rather than analysis. From this perspective, we may conclude then that Marshall's methodological position on the role of mathematics and geometry in the study of economics was a product of his late-Victorian cultural upbringing and education.

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