PRODUCTION OF COMMODITIES BY MEANS OF COMMODITIES

PRELUDE TO A CRITIQUE OF ECONOMIC THEORY

PIERO SRAFFA

CAMBRIDGE UNIVERSITY PRESS

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PREFACE

Anyone accustomed to think in terms of the equilibrium of demand and supply may be inclined, on reading these pages, to suppose that the argument rests on a tacit assumption of constant returns in all industries. If such a supposition is found helpful, there is no harm in the reader's adopting it as a temporary working hypothesis. In fact, however, no such assumption is made. No changes in output and (at any rate in Parts I and II) no changes in the proportions in which different means of production are used by an industry are considered, so that no question arises as to the variation or constancy of returns. The investigation is concerned exclusively with such properties of an economic system as do not depend on changes in the scale of production or in the proportions of 'factors'.

This standpoint, which is that of the old classical economists from Adam Smith to Ricardo, has been submerged and forgotten since the advent of the 'marginal' method. The reason is obvious. The marginal approach requires attention to be focused on change, for without change either in the scale of an industry or in the 'proportions of the factors of production' there can be neither marginal product nor marginal cost. In a system in which, day after day, production continued unchanged in those respects, the marginal product of a factor (or alternatively the marginal cost of a product) would not merely be hard to find—it just would not be there to be found.

Caution is necessary, however, to avoid mistaking spurious 'margins' for the genuine article. Instances will be met in these pages which at first sight may seem indistinguishable from examples of marginal production; but the sure sign of their spuriousness is the absence of the requisite kind of change. The most familiar case is that of the product of the 'marginal land' in agriculture, when lands of different qualities are cultivated side by side: on this, one need only refer to P. H. Wicksteed, the purist of marginal theory, who condemns such a use of the term 'marginal' as a source of 'dire confusion'.[1]

The temptation to presuppose constant returns is not entirely fanciful. It was experienced by the author himself when he started on these studies many years ago—and it led him in 1925 into an attempt to argue that only the case of constant returns was generally consistent with the premises of economic theory. And what is more, when in 1928 Lord Keynes read a draft of the opening propositions of this paper, he recommended that, if constant returns were *not* to be assumed, an emphatic warning to that effect should be given.

These allusions give incidentally some indication of the disproportionate length of time over which so short a work has been in preparation. Whilst the central propositions had taken shape in the late 1920's, particular points, such as the Standard commodity, joint products and fixed capital, were worked out in the 'thirties and early 'forties. In the period since 1955, while these pages were being put together out of a mass of old notes, little was added, apart from filling gaps which had become apparent in the process (such as the adapting of the distinction between 'basics' and 'non-basics' to the case of joint products).

As was only natural during such a long period, others have from time to time independently taken up points of view which are similar to one or other of those adopted in this paper and have developed them further or in different directions from those pursued here. It is, however, a peculiar feature of the set of propositions now published that, although they do not enter into any discussion of the marginal theory of value and distribution, they have nevertheless been designed to serve as the basis for a critique of that theory. If the foundation holds, the critique may be attempted later, either by the writer or by someone younger and better equipped for the task.

My greatest debt is to Professor A. S. Besicovitch for invaluable mathematical help over many years. I am also indebted for similar

[1] 'Political Economy in the Light of Marginal Theory', in *Economic Journal*, xxiv (1914), pp. 18–20, reprinted as an appendix to his *Common Sense of Political Economy*, ed. Lionel Robbins (1933), pp. 790–2.

help at different periods to the late Mr Frank Ramsey and to Mr Alister Watson. It will be only too obvious that I have not always followed the expert advice that was given to me—particularly with regard to the notation adopted, which I have insisted on retaining (although admittedly open to objection in some respects) as being easy to follow for the non-mathematical reader.

P. S.

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PART I

SINGLE-PRODUCT INDUSTRIES AND CIRCULATING CAPITAL

CHAPTER I

PRODUCTION FOR SUBSISTENCE

1 Let us consider an extremely simple society which produces just enough to maintain itself. Commodities are produced by separate industries and are exchanged for one another at a market held after the harvest.

Suppose at first that only two commodities are produced, wheat and iron. Both are used, in part as sustenance for those who work, and for the rest as means of production—wheat as seed, and iron in the form of tools. Suppose that, all in all, and including the necessaries for the workers, 280 quarters of wheat and 12 tons of iron are used to produce 400 quarters of wheat; while 120 quarters of wheat and 8 tons of iron are used to produce 20 tons of iron. A year's operations can be tabulated as follows:

$$
\begin{aligned}
280 \text{ qr. wheat} + 12 \text{ t. iron} &\rightarrow 400 \text{ qr. wheat} \\
120 \text{ qr. wheat} + 8 \text{ t. iron} &\rightarrow 20 \text{ t. iron.}
\end{aligned}
$$

Nothing has been added by production to the possessions of society as a whole: 400 qr. of wheat and 20 t. of iron have been used up in the aggregate and the same quantities are produced. But each commodity, which initially was distributed between the industries according to their needs, is found at the end of the year to be entirely concentrated in the hands of its producer.

(We shall call these relations 'the methods of production and productive consumption', or, for short, *the methods of production.*)

There is a unique set of exchange-values which if adopted by the market restores the original distribution of the products and makes it possible for the process to be repeated; such values spring directly from the methods of production. In the particular example we have taken, the exchange-value required is 10 qr. of wheat for 1 t. of iron.

2 The same applies to three commodities, or indeed to any number. Adding as a third product pigs:

$$\begin{array}{l}
240 \text{ qr. wheat} + 12 \text{ t. iron} + 18 \text{ pigs} \rightarrow 450 \text{ qr. wheat} \\
90 \text{ qr. wheat} + 6 \text{ t. iron} + 12 \text{ pigs} \rightarrow 21 \text{ t. iron} \\
120 \text{ qr. wheat} + 3 \text{ t. iron} + 30 \text{ pigs} \rightarrow 60 \text{ pigs}
\end{array}$$

The exchange-values which ensure replacement all round are 10 qr. wheat = 1 t. iron = 2 pigs.

It may be noticed that, while in the two-industry system the amount of iron used in wheat-growing was necessarily of the same value as the amount of wheat used in iron-making, this, when there are three or more products, is no longer necessarily true of any pair of them. Thus in the last example there is no such equality and replacement can only be effected through triangular trade.

3 To restate the position in general terms, we have the commodities 'a', 'b', ..., 'k', each of which is produced by a separate industry.

We call A the quantity annually produced of 'a'; B the similar quantity of 'b'; and so on.

We also call A_a, B_a, ..., K_a the quantities of 'a', 'b', ..., 'k' annually used by the industry which produces A; and A_b, B_b, ..., K_b the corresponding quantities used for producing B; and so on.

All these represent known quantities. The unknowns to be determined are p_a, p_b, ..., p_k, respectively the values of units of the commodities 'a', 'b', ..., 'k' which if adopted restore the initial position.

The conditions of production now appear as follows:

$$\begin{array}{l}
A_a p_a + B_a p_b + \ldots + K_a p_k = A p_a \\
A_b p_a + B_b p_b + \ldots + K_b p_k = B p_b \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
A_k p_a + B_k p_b + \ldots + K_k p_k = K p_k
\end{array}$$

where, since the system is assumed to be in a self-replacing state, $A_a + A_b + \ldots + A_k = A$; $B_a + B_b + \ldots + B_k = B$; ...; and $K_a + K_b + \ldots + K_k = K$. That is to say, the sum of the first column is equal to the first line, that of the second column to the second line, and so on.

It is not necessary to suppose that every commodity enters *directly* into the production of every other; accordingly some of the quantities on the left-hand side, i.e. on the side of the means of production, may be zero.

One commodity is taken as standard of value and its price made equal to unity. This leaves $k-1$ unknowns. Since in the aggregate of the equations the same quantities occur on both sides, any one of the equations can be inferred from the sum of the others.[1] This leaves $k-1$ independent linear equations which uniquely determine the $k-1$ prices.

[1] This formulation presupposes the system's being in a self-replacing state; but every system of the type under consideration is capable of being brought to such a state merely by changing the proportions in which the individual equations enter it. (Systems which do so with a surplus are discussed in §4ff. Systems which are incapable of doing so under any proportions and show a deficit in the production of some commodities over their consumption even if none has a surplus do not represent viable economic systems and are not considered.)

CHAPTER II

PRODUCTION WITH A SURPLUS

4 If the economy produces more than the minimum necessary for replacement and there is a surplus to be distributed, the system becomes self-contradictory. In effect, if we add up all the equations, the right-hand side of the resulting sum-equation (or gross national product) will contain, besides all the quantities that are found on the left-hand side (or means of production and subsistence), some additional ones that are not. Reckoning as in § 3, there are now k independent equations with only $k-1$ unknowns.

The difficulty cannot be overcome by allotting the surplus *before* the prices are determined, as is done with the replacement of raw-materials, subsistence, etc. This is because the surplus (or profit) must be distributed in proportion to the means of production (or capital) advanced in each industry; and such a proportion between two aggregates of heterogeneous goods (in other words, the rate of profits) cannot be determined before we know the prices of the goods. On the other hand, we cannot defer the allotment of the surplus till after the prices are known, for, as we shall see, the prices cannot be determined before knowing the rate of profits. The result is that the distribution of the surplus must be determined through the same mechanism and at the same time as are the prices of commodities.

JB RM

Accordingly we add the rate of profits (which must be uniform for all industries) as an unknown which we call r and the system becomes

$$\begin{aligned}
(A_a p_a + B_a p_b + \ldots + K_a p_k)\,(1+r) &= A p_a \\
(A_b p_a + B_b p_b + \ldots + K_b p_k)\,(1+r) &= B p_b \\
\cdots\cdots\cdots\cdots\cdots\cdots \\
(A_k p_a + B_k p_b + \ldots + K_k p_k)\,(1+r) &= K p_k
\end{aligned}$$

where, since the system is assumed to be in a self-replacing state, $A_a + A_b \ldots + A_k \leqslant A; B_a + B_b + \ldots + B_k \leqslant B; \ldots; K_a + K_b + \ldots + K_k \leqslant K$; that is to say the quantity produced of each commodity is *at least* equal to the quantity of it which is used up in all branches of production together.

This system contains a number k of independent equations which determine the $k-1$ prices and the rate of profits.

5 As an example we may in the two-commodity case (§ 1) increase the output of wheat from 400 qr. to 575 qr. leaving all the other quantities unchanged. This gives a social surplus of 175 qr. of wheat and the resulting position is:

$$\begin{aligned} &280 \text{ qr. wheat} + 12 \text{ t. iron} \rightarrow 575 \text{ qr. wheat} \\ &120 \text{ qr. wheat} + 8 \text{ t. iron} \rightarrow 20 \text{ t. iron.} \end{aligned}$$

The exchange-ratio which enables the advances to be replaced and the profits to be distributed to both industries in proportion to their advances is 15 qr. of wheat for 1 t. of iron; and the corresponding rate of profits in each industry is 25 %.

(Let us, as an illustration, do the arithmetic for the iron industry. Of the 20 t. produced, 8 go to replace the iron used and 12 are sold, at the price of 15 qr. wheat per ton, thereby obtaining 180 qr. wheat: of these, 120 qr. go to replace the wheat used and 60 qr. are profit at the rate of 25 % on the 240 qr. wheat which is the aggregate value of the wheat and iron used as means of production and subsistence in the iron industry.) 120 + (8×15)

6 One effect of the emergence of a surplus must be noticed. Previously, all commodities ranked equally, each of them being found both among the products and among the means of production; as a result each, directly or indirectly, entered the production of all the others, and each played a part in the determination of prices. But now there is room for a new class of 'luxury' products which are not used, whether as instruments of production or as articles of subsistence, in the production of others. Basic

These products have no part in the determination of the system. Their role is purely passive. If an invention were to reduce by half the quantity of each of the means of production which are required to produce a unit of a 'luxury' commodity of this type, the commodity itself would be halved in price, but there would be no further consequences; the price-relations of the other products and the rate of profits would remain unaffected. But if such a change occurred in the production of a commodity of the opposite type, which *does* enter the means of production, all prices would be affected and the rate of profits would be changed. This can be seen if we eliminate from the system the equation representing the production of a 'luxury' good. Since by the same act we eliminate an unknown (the price of that good) which only appears in that equation, the remaining equations will still form a determinate system which will be satisfied by the solutions of the larger system. On the other hand, if we eliminated one of the other, non-luxury, equations, the number of unknowns would not thereby be diminished since the commodity in question appears among the means of production in the other equations and the system would become indeterminate.

What has just been said of the passive role of luxury goods can readily be extended to such 'luxuries' as are merely used in their own reproduction, either directly (e.g. racehorses) or indirectly (e.g. ostriches and ostrich-eggs) or merely for the production of other luxuries (e.g. raw silk).

The criterion is whether a commodity enters (no matter whether directly or indirectly) into the production of *all* commodities. Those that do we shall call *basic*, and those that do not, *non-basic* products.

We shall assume throughout that any system contains at least one basic product.

7 It is desirable at this stage to explain why the ratios which satisfy the conditions of production have been called 'values' or 'prices' rather than, as might be thought more appropriate, 'costs of production'.

The latter description would be adequate so far as *non*-basic products were concerned, since, as it follows from what we have seen in the preceding section, their exchange ratio is merely a reflection of what must be paid for means of production, labour and profits in order to produce them—there is no mutual dependence.

But for a basic product there is another aspect to be considered.

Its exchange-ratio depends as much on the *use* that is made of it in the production of other basic commodities as on the extent to which those commodities enter its own production. (One might be tempted, but it would be misleading, to say that 'it depends as much on the Demand side as on the Supply side'.)

In other words, the price of a non-basic product depends on the prices of its means of production, but these do not depend on it. Whereas in the case of a basic product the prices of its means of production depend on its own price no less than the latter depends on them.

A less one-sided description than cost of production seems therefore required. Such classical terms as 'necessary price', 'natural price' or 'price of production' would meet the case, but value and price have been preferred as being shorter and in the present context (which contains no reference to market prices) no more ambiguous.

It may be added that not only in this case but *in general* the use of the term 'cost of production' has been avoided in this work, as well as the term 'capital' in its quantitative connotation, at the cost of some tiresome circumlocution. This is because these terms have come to be inseparably linked with the supposition that they stand for quantities that can be measured independently of, and prior to, the determination of the prices of the products. (Witness the 'real costs' of Marshall and the 'quantity of capital' which is implied in the marginal productivity theory.) Since to achieve freedom from such presuppositions has been one of the aims of this work, avoidance of the terms seemed the only way of not prejudicing the issue.

8 We have up to this point regarded wages as consisting of the necessary subsistence of the workers and thus entering the system on the same footing as the fuel for the engines or the feed for the cattle. We must now take into account the other aspect of wages since, besides the ever-present element of subsistence, they may include a share of the surplus product. In view of this double character of the wage it would be appropriate, when we come to consider the division of the surplus between capitalists and workers, to separate the two component parts of the wage and regard only the 'surplus' part as variable; whereas the goods necessary for the subsistence of the

Wages

workers would continue to appear, with the fuel, etc., among the means of production.

We shall, nevertheless, refrain in this book from tampering with the traditional wage concept and shall follow the usual practice of treating the whole of the wage as variable.

The drawback of this course is that it involves relegating the necessaries of consumption to the limbo of non-basic products. This is due to their no longer appearing among the means of production on the left-hand side of the equations: so that an improvement in the methods of production of necessaries of life will no longer directly affect the rate of profits and the prices of other products. Necessaries however are essentially basic and if they are prevented from exerting their influence on prices and profits under that label, they must do so in devious ways (e.g. by setting a limit below which the wage cannot fall; a limit which would itself fall with any improvement in the methods of production of necessaries, carrying with it a rise in the rate of profits and a change in the prices of other products.)

In any case the discussion which follows can easily be adapted to the more appropriate, if unconventional, interpretation of the wage suggested above.

9 We shall also hereafter assume that the wage is paid *post factum* as a share of the annual product, thus abandoning the classical economists' idea of a wage 'advanced' from capital. We retain however the supposition of an annual cycle of production with an annual market.

10 The quantity of labour employed in each industry has now to be represented explicitly, taking the place of the corresponding quantities of subsistence. We suppose labour to be ~~uniform in quality~~ or, what amounts to the same thing, we assume any differences in quality ~~to have been, previously~~ reduced to equivalent differences in quantity so that each unit of labour receives the same wage.

We call $L_a, L_b, \ldots, L_k$ the annual quantities of labour respectively employed in the industries producing $A, B, \ldots, K$ and we define them as fractions of the total annual labour of society, which we take as unity, so that

$$L_a + L_b + \ldots + L_k = 1.$$

We call w the wage per unit of labour, which like prices will be expressed in terms of the chosen standard. (See further, on the choice of a standard, § 12.)

11 On this basis the equations take the form:

$$\begin{gathered}
(A_a p_a + B_a p_b + \ldots + K_a p_k)(1+r) + L_a w = A p_a \\
(A_b p_a + B_b p_b + \ldots + K_b p_k)(1+r) + L_b w = B p_b \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
(A_k p_a + B_k p_b + \ldots + K_k p_k)(1+r) + L_k w = K p_k
\end{gathered}$$

where, as in the earlier cases, the system is assumed to be in a self-replacing state, namely such that $A_a + A_b + \ldots + A_k \leqslant A$; $B_a + B_b + \ldots + B_k \leqslant B$; ...; $K_a + K_b + \ldots + K_k \leqslant K$.

12 The national income of a system in a self-replacing state consists of the set of commodities which are left over when from the gross national product we have removed item by item the articles which go to replace the means of production used up in all the industries. N-I

The value of this set of commodities, or 'composite commodity' as it may be called, which forms the national income, we make equal to unity. It thus becomes the standard in terms of which the wage and the k prices are expressed (taking the place of the arbitrarily chosen single commodity in terms of which $k-1$ prices, besides the wage, were expressed).

We have therefore the additional equation

$$[A-(A_a+A_b+\ldots+A_k)]p_a + [B-(B_a+B_b+\ldots+B_k)]p_b + \ldots \\
+ [K-(K_a+K_b+\ldots+K_k)]p_k = 1.$$

(It is impossible for the aggregate quantity of any commodity represented in this expression to be negative owing to the condition of self-replacement assumed in § 11.)

This gives $k+1$ equations as compared with $k+2$ variables (k prices, the wage w and the rate of profits r).

The result of adding the wage as one of the variables is that the number of these now exceeds the number of equations by one and the system can move with one degree of freedom; and if one of the variables is fixed the others will be fixed too.

CHAPTER III

PROPORTIONS OF LABOUR TO MEANS OF PRODUCTION

13 We proceed to give the wage (w) successive values ranging from 1 to 0: these now represent fractions of the national income (cp. §§ 10 and 12). The object is to observe the effect of changes in the wage on the rate of profits and on the prices of individual commodities, on the assumption that the methods of production remain unchanged.

14 When we make w equal to 1 the whole national income goes to wages and r is eliminated. We thus revert, in effect, to the system of linear equations from which we started, with the difference that the quantities of labour are now shown explicitly instead of being represented by quantities of necessaries for subsistence.

At this level of wages the relative values of commodities are in proportion to their labour cost, that is to say to the quantity of labour which directly and indirectly has gone to produce them.[1] At no other wage-level do values follow a simple rule.

15 Starting from the situation in which the whole of the national income goes to labour, we imagine wages to be reduced: a rate of profits will thereby arise.

The key to the movement of relative prices consequent upon a change in the wage lies in the inequality of the proportions in which labour and means of production are employed in the various industries. It is clear that if the proportion were the same in all industries no price-changes could ensue, however great was the diversity of the commodity-composition of the means of production in different industries. For in each industry an equal deduction from the wage would yield just as much as was required for paying the profits on

[1] See Appendix A, On Sub-systems.

its means of production at a uniform rate without need to disturb the existing prices.[1]

16 For the same reason it is impossible for prices to remain unchanged when there is inequality of 'proportions'. Suppose that prices *did* remain unchanged when the wage was reduced and a rate of profits emerged. Since in any one industry what was saved by the wage-reduction would depend on the number of men employed, while what was needed for paying profits at a uniform rate would depend on the aggregate value of the means of production used, industries with a sufficiently low proportion of labour to means of production would have a deficit, while industries with a sufficiently high proportion would have a surplus, on their payments for wages and profits. (Nothing is assumed at the moment as to what rate of profits corresponds to what wage reduction; all that is required at this stage is that there should be a uniform wage and a uniform rate of profits throughout the system.) NB

17 There would be a 'critical proportion' of labour to means of production which marked the watershed between 'deficit' and 'surplus' industries. An industry which employed that particular 'proportion' would show an even balance—the proceeds of the wage-reduction would provide exactly what was required for the payment of profits at the general rate. Whatever the precise value of that 'proportion' in any particular system, it can be said *a priori* that, in a system including two or more basic industries, the industry with the lowest proportion of labour to means of production would be a 'deficit' industry and the one with the highest proportion would be a 'surplus' industry. NB

[1] In these 'proportions' the means of production must be measured by their values, but since values may change with a change in the wage the question arises, which values? The answer is that, as regards establishing the equality or non-equality of the proportions (which is all that we are concerned with at the moment), all the possible sets of values give the same result. In effect, as we have seen, if the proportions of all the industries are equal, values, and therefore proportions, do not change with the wage; and from this it follows that if the proportions are unequal at the set of values corresponding to one wage they cannot be equal at any other, and so they are unequal at all values.

18 It follows that, with a wage-reduction, price-changes would be called for to redress the balance in each of the 'deficit' and in each of the 'surplus' industries.

To achieve this object it is first of all the price-ratio between each product and its means of production that one expects to come into play. Consider the situation of a 'deficit' industry when the wage is reduced. A rise in the price of the product relatively to the means of production would help to eliminate the deficit since it would release some of that share of the gross product of the industry which had been going to pay for the replacement of the now cheapened means of production; this would be added to the quantity available for distribution as wages or profits. The price rise by itself would thus result in an increase in the magnitude (and not merely in the value) of that part of the product of the industry which is available for distribution, despite the fact that the methods of production were unchanged.

A further effect of the rise in the price of the product relatively to the means of production would of course be to help a given quantity of product to go a longer way towards achieving the required rate of profit.

In the second place, and independently of this, the steeper the rise in the price of the product relative to labour, the smaller would be the quantity of it absorbed by the wage.

In a like way price-movements in an opposite direction could accomplish the disposal of the surplus which otherwise would appear in an industry using a high 'proportion' of labour to means of production.

19 It does not by any means follow, however, that the price of the product of an industry having a low proportion of labour to means of production (and therefore a potential deficit) would necessarily rise, with a wage-reduction, relative to its own means of production. On the contrary, it might quite possibly fall. The reason for this seeming contradiction is that the means of production of an industry are themselves the product of one or more industries ~~which may in~~ their turn employ a still lower proportion of labour to ~~means-of-production~~ (and the same may be the case with these latter means of production;

and so on); in that case the price of the product, although produced by a 'deficit' industry, might *fall* in terms of its means of production, and its deficiency would have to be made good through a particularly steep rise relative to labour.

The result is that as the wages fall the price of the product of a low-proportion (or 'deficit') industry may rise or it may fall, or it may even alternate in rising and falling, relative to its means of production; while the price of the product of a high-proportion (or 'surplus') industry may fall or it may rise, or it may alternate. What neither of such products can do, as we shall presently see (§§ 21–22), is to remain stable in price relative to its means of production throughout any range, whether long or short, of the wage-variation.

20 To conclude this preliminary survey of the subject it may be pointed out that these considerations dominate not only the price-relation of a product to its means of production but equally its relations to any other product. As a result, the relative price-movements of two products come to depend, not only on the 'proportions' of labour to means of production by which they are respectively produced, but also on the 'proportions' by which those means have themselves been produced, and also on the 'proportions' by which the means of production of those means of production have been produced, and so on. The result is that the relative price of two products may move, with the fall of wages, in the opposite direction to what we might have expected on the basis of their respective 'proportions'; besides, the prices of their respective means of production may move in such a way as to reverse the order of the two products as to higher and lower proportions; and further complications arise, which will be considered subsequently.

However complex the pattern of the price-variations arising from a change in distribution, their net result, and their complete justification, remains the simple one of redressing the balance in each industry. They fully achieve that object, but it could not be achieved with anything less.

21 We now revert to the 'critical' proportion which has been mentioned before (§ 17) as constituting the borderline between 'deficit' industries and 'surplus' industries. Suppose that there was an industry which employed labour and means of production in that precise proportion, so that with a wage-reduction, and on the basis of the initial prices, it would show an exact balance of wages and profits. Suppose further that the means of production which it used, taken as an aggregate, were themselves produced by labour and means of production in that proportion; and suppose finally that the same proportion applied to the production of the aggregate means of production by which those means of production were produced, and similarly to the successive layers of means of production involved, however far we traced them back.

The commodity produced by such an industry would be under no necessity, arising from the conditions of production of the industry itself, either to rise or to fall in value relative to any other commodity when wages rose or fell; for, as we have seen, a necessity of this sort can originate only from a potential deficit or surplus and an industry operating under the conditions described would *ipso facto* be in balance. A commodity of this type would in any case be incapable of changing in value relative to the aggregate of its own means of production since the recurrence of the same 'proportion' would apply equally to them.

Two separate conditions have been assumed to obtain this result, namely (1) that the 'balancing' proportion is used, and (2) that one and the same proportion *recurs* in all the successive layers of the industry's aggregate means of production without limit. We shall, however, find that the first condition is necessarily implied in the second for, as will presently appear (§22), within any one system complete 'recurrence' is only possible with the balancing proportion. So that there is in effect only one condition, that of 'recurrence'.

22 In trying to identify the 'balancing' proportion it is convenient to replace the hybrid 'proportion' of the quantity of labour to the value of the means of production which we have been using up to this point, with one of the corresponding 'pure' ratios between homogeneous quantities. There are two such corresponding ratios, namely the quantity-ratio of direct to indirect labour employed, and the value-ratio of net ~~product to means of production~~.[1] We shall adopt the latter here.

While the rate of profits is uniform in all industries, and depends only on the wage, the value-ratio of the net product to the means of production is in general different for each industry and mainly depends on its particular circumstances of production.

There is however an exception to this. When we make the wage equal to zero and the whole of the net product goes to profits, in each industry the value-ratio of net product to means of production necessarily comes to coincide with the general rate of profits. However different from one another they may have been at other wage-levels, at this level the 'value-ratios' of all industries are equal.

It follows that the only 'value-ratio' which can be invariant to changes in the wage, and therefore is capable of being 'recurrent' in the sense defined in §21, is the one that is equal to the rate of profits which corresponds to zero wage. And *that* is the 'balancing' ratio.

We shall call *Maximum rate of profits* the rate of profits as it would be if the whole of the national income went to profits. And we shall denote by a single letter, R, the two coincident ratios, namely the Maximum rate of profits and the 'balancing' ratio of net product to means of production.

[1] In general (i.e. for all the industries that do *not* use the 'balancing' proportion) these two ratios will coincide only when the value-ratio is calculated at the values for $w = 1$.

CHAPTER IV

THE STANDARD COMMODITY

23 The necessity of having to express the price of one commodity in terms of another which is arbitrarily chosen as standard, complicates the study of the price-movements which accompany a change in distribution. It is impossible to tell of any particular price-fluctuation whether it arises from the peculiarities of the commodity which is being measured or from those of the measuring standard. The relevant peculiarities, as we have just seen, can only consist in the inequality in the proportions of labour to means of production in the successive 'layers' into which a commodity and the aggregate of its means of production can be analysed; for it is such an inequality that makes it necessary for the commodity to change in value relative to its means of production as the wage changes.

The 'balanced' commodity which we have just considered (§ 21) would present no peculiarities of this type, since the same proportion would be found in all its 'layers'. It is true that, as wages fell, such a commodity would be no less susceptible than any other to rise or fall in price relative to other individual commodities; but we should know for certain that any such fluctuation would originate exclusively in the peculiarities of production of the commodity which was being compared with it, and not in its own. If we could discover such a commodity we should therefore be in possession of a standard capable of isolating the price-movements of any other product so that they could be observed as in a vacuum.

24 It is not likely that an individual commodity could be found which possessed even approximately the necessary requisites. A mixture of commodities, however, or a 'composite commodity', would do equally well; it might do even better, since it could be 'blended' to suit our requirements, modifying its composition so as to smooth out a price-bulge at one wage-level or to fill in a depression at another level.

We should, however, not get very far with the attempt to concoct such a mixture before realising that the perfect composite commodity of this type, in which the requirements are fulfilled to the letter, is one which consists of the same commodities (combined in the same proportions) as does the aggregate of its own means of production—in other words, such that both product and means of production are quantities of the self-same composite commodity.

The question is, can such a commodity be constructed?

25 The problem is one that concerns industries rather than commodities and is best approached from that angle.

Suppose we segregate from the actual economic system such fractions of the individual basic industries as will together form a complete miniature system endowed with the property that the various commodities are represented among its aggregate means of production *in the same proportions* as they are among its products.

As an example let us assume that the actual system from which we start includes only basic industries and that these produce respectively iron, coal and wheat in the following way:

	90 t. iron +	120 t. coal +	60 qr. wheat +	$\frac{3}{16}$ labour	→ 180 t. iron
	50 t. iron +	125 t. coal +	150 qr. wheat +	$\frac{5}{16}$ labour	→ 450 t. coal
	40 t. iron +	40 t. coal +	200 qr. wheat +	$\frac{8}{16}$ labour	→ 480 qr. wheat
Totals	180	285	410	1	

where, since iron happens to be produced in a quantity just sufficient for replacement (180 t.), the national income includes only coal and wheat and consists of 165 t. of the former and 70 qr. of the latter.

To obtain from this a reduced-scale system in the required proportions we must take, with the whole of the iron industry, $\frac{3}{5}$ of the coal industry and $\frac{3}{4}$ of the wheat-growing one. The resulting system is:

	90 t. iron +	120 t. coal +	60 qr. wheat +	$\frac{3}{16}$ labour	→ 180 t. iron
	30 t. iron +	75 t. coal +	90 qr. wheat +	$\frac{3}{16}$ labour	→ 270 t. coal
	30 t. iron +	30 t. coal +	150 qr. wheat +	$\frac{6}{16}$ labour	→ 360 qr. wheat
Totals	150	225	300	$\frac{12}{16}$	

The proportions in which the three commodities are produced in the new system (180:270:360) are equal to those in which they enter its aggregate means of production (150:225:300). The composite commodity sought for is accordingly made up in the proportions

$\frac{150}{180} : \frac{225}{270} : \frac{300}{360}$

$\frac{5}{6} \quad \frac{5}{6} \quad \frac{5}{6}$

1 t. iron: $1\frac{1}{2}$ t. coal: 2 qr. wheat.

26 We shall call a mixture of this type the *Standard composite commodity* or, for short, the *Standard commodity*; and the set of equations (or of industries), taken in the proportions that produce the Standard commodity, the *Standard system.*

It can be said that in any actual economic system there is embedded a miniature Standard system which can be brought to light by chipping off the unwanted parts. (This applies as much to a system which is not in a self-replacing state as to one which is.)

We shall as a rule find it convenient to take as unit of the Standard commodity the quantity of it that would form the net product of a Standard system employing the whole annual labour of the actual system. (For such a unit to form the net product in the above example, each industry must be increased by $\frac{1}{3}$, the aggregate labour employed being thereby raised from $\frac{12}{16}$ to $\frac{16}{16}$; as a result the unit would consist of 40 t. iron, 60 t. coal and 80 qr. wheat.) Such a unit we shall call the *Standard net product* or *Standard national income.*

27 The fact that in the Standard system the various commodities are produced in the same proportions as they enter the aggregate means of production implies that the rate by which the quantity produced exceeds the quantity used up in production is the same for each of them. In the above example the rate for each commodity is 20 %, as can be seen if the figures are so rearranged that the aggregate quantity of each commodity entering the means of production is set against the quantity of it that is produced:

$$(90+30+30)\left(1+\tfrac{20}{100}\right) = 180 \text{ t. iron}$$
$$(120+75+30)\left(1+\tfrac{20}{100}\right) = 270 \text{ t. coal}$$
$$(60+90+150)\left(1+\tfrac{20}{100}\right) = 360 \text{ qr. wheat.}$$

28 The rate that applies to the individual commodities is naturally also the rate by which the total product of the Standard system exceeds its aggregate means of production, or the ratio of the net product to the means of production of the system. This ratio we shall call the *Standard ratio*.

The possibility of speaking of a ratio between two collections of miscellaneous commodities without need of reducing them to the common measure of price arises of course from the circumstance that both collections are made up in the same proportions—from their being in fact quantities of the same composite commodity.

The result would therefore not be affected by multiplying the individual component commodities by their prices. The ratio of the values of the two aggregates would inevitably be always equal to the ratio of the quantities of their several components. Nor, once the commodities had been multiplied by their prices, would the ratio be disturbed if those individual prices were to vary in all sorts of divergent ways.

Thus in the Standard system the ratio of the net product to the means of production would remain the same whatever variations occurred in the division of the net product between wages and profits and whatever the consequent price changes.

29 What has just been said of the ratio of the net product to the means of production in the Standard system applies equally if we replace the net product by any fraction of it: the ratio of such fraction to the means of production will remain unaffected by any variation of prices.

Now suppose the Standard net product to be divided between wages and profits, taking care that the share of each consists always, as the whole does, of Standard commodity: the resulting rate of profits would be in the same proportion to the Standard ratio of the system as the share allotted to profits was to the whole of the net product. In the example given above, where the Standard ratio was 20 %, if $\frac{3}{4}$ of the Standard national income went to wages and $\frac{1}{4}$ to profits, the rate of profits would be 5 %; if half went to each, it would be 10 %; and if the whole went to profits the rate of profits would reach its maximum level of 20 % and coincide with the Standard ratio.

The rate of profits in the Standard system thus appears as a ratio between quantities of commodities irrespective of their prices.

30 To restate the position in general terms, as far as the Standard system is concerned, we may say that, if R is the Standard ratio or Maximum rate of profits and w the proportion of the net product that goes to wages, the rate of profits is

$$r = R(1-w).$$

Thus as the wage is gradually reduced from 1 to 0 the rate of profits increases in direct proportion to the total deduction made from the wage. The relation can be represented graphically by a straight line as shown in Fig. 1.

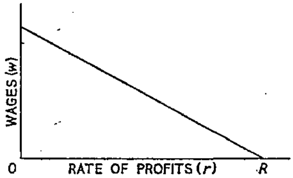

FIG. 1. Relation between wages (as a proportion of the Standard net product) and the rate of profits.

31 Such a relation is of interest only if it can be shown that its application is not limited to the imaginary Standard system but is capable of being extended to the actual economic system of observation.

This turns on whether the decisive role which the Standard commodity plays in this connection lies in its being the constituent material of the national income and of the means of production (which is peculiar to the Standard system), or in its supplying the medium in which wages are estimated. For the latter is a function which the appropriate Standard commodity can fulfil in any case, whether the system is in the Standard proportions or not.

Now it is true that appearances are against the second alternative. In the Standard system the circumstance of the wage being paid in Standard commodity seems to draw its special significance from the fact that the residue left over for profits will itself be a quantity of Standard commodity and therefore similar in composition to the means of production: the result is that the rate of profits, being the ratio of these two homogeneous quantities, can be *seen* to rise in direct proportion to any reduction made in the wage. There would therefore appear to be no reason to expect that in the actual system, when the equivalent of the same quantity of Standard commodity has been paid for wages, the *value* of what is left over for profits should stand in the same ratio to the *value* of the means of production as the corresponding *quantities* do in the Standard system.

But the actual system consists of the same basic equations as the Standard system, only in different proportions; so that, once the wage is given, the rate of profits is determined for both systems regardless of the proportions of the equations in either of them. Particular proportions, such as the Standard ones, may give transparency to a system and render visible what was hidden, but they cannot alter its mathematical properties.

The straight-line relation between the wage and the rate of profits will therefore hold in all cases, provided only that the wage is expressed in terms of the Standard product. The same rate of profits, which in the Standard system is obtained as a ratio between *quantities* of commodities, will in the actual system result from the ratio of aggregate *values*.

32 Reverting to our example, if in the actual system (as outlined in § 25 ff., with $R = 20\%$) the wage is fixed in terms of the Standard net product, to $w = \frac{3}{4}$ there will correspond $r = 5\%$. But while the share of wages will be equal in value to $\frac{3}{4}$ of the Standard national income, it does not follow that the share of profits will be equivalent to the remaining $\frac{1}{4}$ of the Standard income. The share of profits will consist of whatever is left of the *actual* national income after deducting from it the equivalent of $\frac{3}{4}$ of the *Standard* national income for wages: and prices must be such as to make the value of what goes to profits equal to 5 % of the value of the actual means of production of society.

33 To restate it in general terms, the problem of constructing a Standard commodity amounts to finding a set of k suitable multipliers, which may be called $q_a, q_b, \ldots, q_k$, to be applied respectively to the production-equations of commodities 'a', 'b', ..., 'k'.

The multipliers must be such that the resulting quantities of the various commodities will bear the same proportions to one another on the right-hand sides of the equations (as products) as they do on the aggregate of the left-hand sides (as means of production).

This, as we have seen, implies that the percentage by which the output of a commodity exceeds the quantity of it entering the aggregate means of production is equal for all commodities. This percentage we have called the Standard ratio and we have denoted it by the letter R.

Such a condition is expressed by a system of equations which contains the same constants (representing quantities of commodities) as the production equations, but arranged in a different order (the rows of one system corresponding to the columns of the other). This system of equations, which we shall refer to as the q-system, is as follows:

$$\begin{aligned}
(A_a q_a + A_b q_b + \ldots + A_k q_k)\,(1+R) &= A q_a \\
(B_a q_a + B_b q_b + \ldots + B_k q_k)\,(1+R) &= B q_b \\
&\cdots \\
(K_a q_a + K_b q_b + \ldots + K_k q_k)\,(1+R) &= K q_k.
\end{aligned}$$

To complete the system it is necessary to define the unit in which the multipliers are to be expressed; and since we wish the quantity of labour employed in the Standard system to be the same as in the actual system (§ 26), we define the unit by an additional equation which embodies that condition, namely:

$$L_a q_a + L_b q_b + \ldots + L_k q_k = 1.$$

We have thus a number $k+1$ of equations which determine the k multipliers and R.

34 By solving this system of equations we obtain a set of numbers for the multipliers (we may call these numbers $q'_a, q'_b, \ldots, q'_k$). We apply these to the equations of the production system (§ 11) and thus transform it into a Standard system as follows:

$$\begin{aligned}
q'_a[(A_a p_a + B_a p_b + \ldots + K_a p_k)\,(1+r) + L_a w] &= q'_a A p_a \\
q'_b[(A_b p_a + B_b p_b + \ldots + K_b p_k)\,(1+r) + L_b w] &= q'_b B p_b \\
&\cdots \\
q'_k[(A_k p_a + B_k p_b + \ldots + K_k p_k)\,(1+r) + L_k w] &= q'_k K p_k
\end{aligned}$$

From this we derive the Standard national income which henceforward we shall adopt as unit of wages and prices in the original system of production. The unit equation of § 12 is therefore replaced by the following where the q''s stand for known numbers while the p's are variables:

$$[q'_a A - (q'_a A_a + q'_b A_b + \dots + q'_k A_k)]p_a + [q'_b B - (q'_a B_a + q'_b B_b + \dots + q'_k B_k)]p_b + \dots + [q'_k K - (q'_a K_a + q'_b K_b + \dots + q'_k K_k)]p_k = 1.$$

This composite commodity is the Standard of wages and prices that we have been seeking for (§ 23).

35 It is evidently impossible for those non-basic products which are *completely* excluded from the role of means of production to satisfy these conditions and find a place in the Standard system. The multiplier appropriate to their equations can therefore only be zero.

The same, if slightly less obviously, is true of those other non-basics which, while not entering the means of production of commodities in general, yet are used in the production of one or more non-basics, which may include themselves (e.g. special raw materials for luxury goods; and luxury animals or plants).

In so far as a commodity of this kind entered only the production of a non-basic product of the type previously considered, it would clearly follow the latter's fate and have zero for multiplier.

And in so far as it entered its own production, the ratio of its quantity as product to its quantity as means of production would be exclusively determined by its own production-equation and would therefore in general be unrelated to R and consequently be incompatible with the Standard system. The multiplier appropriate to it would therefore also be zero.[1]

We may in consequence simplify the discussion by assuming that all non-basic equations are eliminated at the outset so that only basic industries come under consideration.

It is to be noted that the absence of the non-basic industries from the Standard system does not prevent the latter from being equivalent in its effects to the original system since, as we have seen (§6), their presence or absence makes no difference to the determination of prices and of the rate of profits.

[1] Strictly speaking the multiplier would be zero for every possible value of R except the one that was equal to the ratio of the quantity of that non-basic in the net product to its quantity in the means of production. This is a freak case of the type referred to in Appendix B: at that particular value of R all prices would be zero in terms of the non-basic in question.

CHAPTER V

UNIQUENESS OF THE STANDARD SYSTEM

36 In the following five sections it is sought to prove that there always is a way, and never more than one way, of transforming a given economic system into a Standard system: in other words, that there always is one, and only one, set of multipliers which, if applied to the several equations or industries composing the system, will have the effect of rearranging them in such proportions that the commodity-composition of the aggregate means of production and that of the aggregate product are identical.

37 That any actual economic system of the type we have been considering can always be transformed into a Standard system may be shown by an imaginary experiment.

(The experiment involves two types of alternating steps. One type consists in changing the proportions of the industries; the other in reducing in the same ratio the quantities produced by all industries, while leaving unchanged the quantities used as means of production.)

We start by adjusting the proportions of the industries of the system in such a way that of each basic commodity a larger quantity is produced than is strictly necessary for replacement.

Let us next imagine gradually to reduce by means of successive small proportionate cuts the product of all the industries, without interfering with the quantities of labour and means of production that they employ.

As soon as the cuts reduce the production of any one commodity to the minimum level required for replacement, we readjust the proportions of the industries so that there should again be a surplus of each product (while keeping constant the quantity of labour employed in the aggregate). This is always feasible so long as there is a surplus of some commodities and a deficit of none.

We continue with such an alternation of proportionate cuts with the re-establishment of a surplus for each product until we reach the point where the products have been reduced to such an extent that all-round replacement is just possible without leaving anything as surplus product.

Since to reach this position the products of all the industries have been cut in the same proportion we are now able to restore the original conditions of production by increasing the quantity produced in each industry by a uniform rate; we do not, on the other hand, disturb the proportions to which the industries have been brought. The uniform rate which restores the original conditions of production is R and the proportions attained by the industries are the proportions of the Standard system.

38 We now consider the question whether the Standard system into which a given system of industries can be transformed is unique or whether there may be alternative ways of rearrangement which satisfy the conditions.

The equations of the q-system (§33) are reducible to an equation of the kth degree in R and therefore there may be as many as k values of R (each with its corresponding set of values of the q's) which satisfy them. To show that only one of these sets represents a possible way of rearranging the industries into a Standard system it is sufficient to prove that there cannot be more than one value of R to which there corresponds an all-positive set of values of the q's.

39 As a preliminary to doing so, we must show that, just as there always is a possible set of multipliers (§37), so there is at all values of the wage including zero a set of prices which satisfy the condition of replacement of the means of production with uniform profits: that is to say, there always is a set of *positive* values of the p's.

We start from the level of $w = 1$ where, since prices are equal to labour cost (§14), the values of the p's must necessarily be all positive. If the value of w is moved continuously from 1 to 0, the values of the p's will also move continuously, so that any p to become negative must go through zero. However, while wages and profits are positive, the price of no commodity can become zero until the price of at least one of the other commodities entering its means of production has become negative. Thus, since no p can become negative before any other, none can become negative at all.[1]

40 As a second, and final, preliminary it is convenient for purposes of comparison to rewrite here the production equations as they appear when wages are made equal to zero. The labour terms, having to be multiplied by 0, may be omitted altogether and instead of r we write R which stands for the Maximum rate of profit. We can take the price of any one of the commodities as unity.

The production system thus becomes

$$
\begin{aligned}
(A_a p_a + B_a p_b + \ldots + K_a p_k)\,(1+R) &= A p_a \\
(A_b p_a + B_b p_b + \ldots + K_b p_k)\,(1+R) &= B p_b \\
\ldots \quad \ldots \quad \ldots \quad & \ldots \\
(A_k p_a + B_k p_b + \ldots + K_k p_k)\,(1+R) &= K p_k.
\end{aligned}
$$

41 We can at last proceed to show that there can be no more than one set of positive multipliers. Let R' be a possible value of R to which there correspond *positive* prices $p'_a, p'_b, \ldots, p'_k$ and *positive* multipliers $q'_a, q'_b, \ldots, q'_k$. Let R'' be another possible value of R to which there correspond prices $p''_a, p''_b, \ldots, p''_k$ and multipliers $q''_a, q''_b, \ldots, q''_k$. We must prove that it is impossible for the q'''s to be all positive.

Putting in the production equations (as re-written for $w = 0$ in the preceding section) R' for R and $p'_a, p'_b, \ldots, p'_k$ for $p_a, p_b, \ldots, p_k$ and multiplying them respectively by $q''_a, q''_b, \ldots, q''_k$ we obtain the system

$$
\begin{aligned}
q''_a(A_a p'_a + B_a p'_b + \ldots + K_a p'_k)\,(1+R') &= q''_a A p'_a \\
q''_b(A_b p'_a + B_b p'_b + \ldots + K_b p'_k)\,(1+R') &= q''_b B p'_b \\
\ldots \quad \ldots \quad \ldots \quad & \ldots \\
q''_k(A_k p'_a + B_k p'_b + \ldots + K_k p'_k)\,(1+R') &= q''_k K p'_k
\end{aligned}
$$

and adding these up we have

$$
\begin{aligned}
&[q''_a(A_a p'_a + B_a p'_b + \ldots + K_a p'_k) + q''_b(A_b p'_a + B_b p'_b + \ldots + K_b p'_k) + \ldots \\
&\quad + q''_k(A_k p'_a + B_k p'_b + \ldots + K_k p'_k)]\,(1+R') = q''_a A p'_a + q''_b B p'_b + \ldots + q''_k K p'_k \quad (1)
\end{aligned}
$$

[1] For the proof to be complete it is necessary to show in addition that the p's representing prices of basic products cannot become negative through becoming infinite—unlike the p's of non-basics which can do so. This is shown in the Note on Self-reproducing Non-basics (Appendix B).

Now, putting in the q-equations (as given in §30) R'' for R and $q''_a, q''_b, \ldots, q''_k$ for $q_a, q_b, \ldots, q_k$, and multiplying them respectively by $p'_a, p'_b, \ldots, p'_k$ we obtain

$$\begin{aligned} p'_a(A_a q''_a + A_b q''_b + \ldots + A_k q''_k)\ (1+R'') &= p'_a A q''_a \\ p'_b(B_a q''_a + B_b q''_b + \ldots + B_k q''_k)\ (1+R'') &= p'_b B q''_b \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ p'_k(K_a q''_a + K_b q''_b + \ldots + K_k q''_k)\ (1+R'') &= p'_k K q''_k \end{aligned}$$

and adding these up we have

$$\begin{aligned} [p'_a(A_a q''_a + A_b q''_b + \ldots + A_k q''_k) + p'_b(B_a q''_a + B_b q''_b + \ldots + B_b q''_k) + \ldots \\ + p'_k(K_a q''_a + K_b q''_b + \ldots + K_k q''_k)]\ (1+R'') = p'_a A q''_a + p'_b B q''_b + \ldots + p'_k K q''_k \quad (2) \end{aligned}$$

The terms of sum-equation (1) are identical with those of sum-equation (2) (although grouped in a different way), with the exception that R' and R'' are distinct numbers. Therefore, for the equations to be true, both sides of both equations must be equal to zero: which, since all the p''s are positive, denotes that some of the q'''s must be negative.

This proves that, *if* there is a set of positive values for the p's there can be no more than one set of positive values for the q's.[1]

We had previously seen (in §37) that there always is a set of positive q's and (in §39) that there always is a set of positive p's. We can therefore conclude that there always is one, and only one, value of R to which there corresponds a set of positive multipliers (q's) which will transform a given economic system into a Standard system.

42 It can be shown, as an immediate consequence of the above, that the value of R to which correspond all-positive prices (and which we shall go on calling R') is the *lowest* of the k possible values of R.

In effect, suppose this not to be true; then there exists a value of R lower than R' which we shall call R''. As an example, make $R' = 15\%$ and $R'' = 10\%$.

[1] A similar argument, only putting in the p'''s and the q''s instead of the p''s and the q'''s, proves that, if there is a set of positive values for the q's, there can be no more than one set of positive values for the p's.

To ascertain whether this is possible, we revert to the system with w and r (§11). We assign as wage a quantity of the Standard commodity, which, as we know, corresponds to R'. Thus we replace the labour terms ($L_a w$, $L_b w$, etc.) with proportionate quantities of the Standard commodity, such that their total is a fraction

$$1-\frac{R''}{R'}$$

(in the example which we have chosen, $\frac{1}{3}$) of the Standard national income. At the same time we take as standard of prices an arbitrarily chosen basic commodity 'a' and make its value equal to unity.

Consider now two sets of solutions of the resulting system. One corresponds to R', giving

$$r = R'(1-\tfrac{1}{3}) = 10\,\%$$

and *all-positive prices* (since, being positive at $r = R'$, they will remain so at all values of r down to 0; cf. §39).

The second set of solutions corresponds to R''. We know from the last section that at the prices corresponding to R'' the value of the Standard commodity, which is formed in the proportions that correspond to R', is 0, so that the wage vanishes and

$$r = R'' = 10\,\%.$$

This implies, as indeed was said in the last section, that among the prices corresponding to R'' *some must be negative and others positive.*

The two sets of solutions thus give the same value (10 %) for r, but two different sets of prices.

This, however, is impossible, for to any one value of r there can correspond only one set of prices; in effect, when r is replaced by a known number such as 10 % the equations form a linear system and for the remaining unknowns[1] there is a unique set of solutions.

Thus R', the value of R to which correspond all-positive prices,

[1] In these conditions, one of the equations is implicit in the others (see §3, last paragraph) and the number $(k-1)$ of independent equations is equal to the number of the remaining unknowns.

cannot be higher, and therefore must be lower, than any other value R'' to which correspond some positive and some negative prices.[1]

43 The Standard system is a purely auxiliary construction. It should therefore be possible to present the essential elements of the mechanism under consideration without having recourse to it.

We know that, if we make the Standard net product equal to unity, so that the wage is measured in terms of it, a relation of proportionality is established between a deduction from the wage and the corresponding addition to the rate of profits, in accordance with the expression

$$r = R'(1-w),$$

where R' is the ratio of the Standard net product to its means of production which results from the q-equations.

This proposition is reversible, and if we make it a condition of the economic system that w and r should obey the proportionality rule in question, the wage and commodity-prices are then *ipso facto* expressed in Standard net product, without need of defining its composition, since with no other unit can the proportionality rule be fulfilled.

To do this we have only to substitute for the equation (p. 25) which makes the Standard net product equal to unity, the above relation linking w and r with R'. And to find R', namely the value of R to which correspond positive multipliers and positive prices, we need not have recourse to the q-equations; we can find it as the Maximum rate of profits from the production equations, by making $w = 0$.

[1] It may be noted that the straight-line relation represented by

$$r = R(1-w)$$

would continue to hold good if the wage were to be measured in any of the other Standard commodities which correspond to the possible values of R higher than R' (if it is possible to conceive of Standard commodities which include negative components; a point to which we revert in ch. VIII). The prices of the various Standard commodities, relative to each other, would with the change of r move in such a way that although the wage, at any given value of r, would represent different proportions of the respective Standard national incomes, yet these different fractions of different Standard incomes would all be of equal value. When r was made equal to R' the wage in terms of any one of the other Standard commodities would consist of a non-zero quantity of such Standard commodity but the value of the latter would be zero if expressed in terms of the Standard commodity formed by means of all-positive multipliers and which corresponds to R'.

The above condition is sufficient to ensure that the wage and commodity-prices are expressed in terms of the Standard net product. And it is curious that we should thus be enabled to use a standard without knowing what it consists of.

There is available however a more tangible measure for prices of commodities which makes it possible to displace the Standard net product even from this attenuated function. This measure, as we shall presently see, is 'the quantity of labour that can be purchased by the Standard net product'. In effect, as soon as we have fixed the rate of profits, and without need of knowing the prices of commodities, a parity is established between the Standard net product and a quantity of labour which depends only on the rate of profits; and the resulting prices of commodities can be indifferently regarded as being expressed either in the Standard net product or in the quantity of labour which at the given level of the rate of profits is known to be equivalent to it. This quantity of labour will vary inversely with the Standard wage (w) and directly with the rate of profits. If the annual labour of the system is taken as unit, this equivalent quantity of labour, derived from the above relation, is

$$\frac{1}{w} = \frac{R'}{R'-r}.$$

Thus all the properties of 'an invariable standard of value', as described in §23, are found in a variable quantity of labour, which, however, varies according to a simple rule which is independent of prices: this unit of measurement increases in magnitude with the fall of the wage, that is to say with the rise of the rate of profits, so that, from being equal to the annual labour of the system when the rate of profits is zero, it increases without limit as the rate of profits approaches its maximum value R'.

The last remaining use of the Standard net product is as the medium in terms of which the wage is expressed—and in this case there seems to be no way of replacing it. If we wish to eliminate it altogether, we must cease to regard w as an expression for the wage and treat it instead as a pure number which helps to define the quantity of labour which at the given rate of profits constitutes the unit of prices: then, the prices of commodities being expressed in terms of such quantity of labour, we can find its wage in terms of any commodity by taking the reciprocal of the price of that commodity.

44 The last steps of the preceding argument have led us to reverse the practice, followed from the outset, of treating the wage rather than the rate of profits as the independent variable or 'given' quantity.

The choice of the wage as the independent variable in the preliminary stages was due to its being there regarded as consisting of specified necessaries determined by physiological or social conditions which are independent of prices or the rate of profits. But as soon as the possibility of variations in the division of the product is admitted, this consideration loses much of its force. And when the wage is to be regarded as 'given' in terms of a more or less abstract standard, and does not acquire a definite meaning until the prices of commodities are determined, the position is reversed. The rate of profits, as a ratio, has a significance which is independent of any prices, and can well be 'given' before the prices are fixed. It is accordingly susceptible of being determined from outside the system of production, in particular by the level of the money rates of interest.

In the following sections the rate of profits will therefore be treated as the independent variable.

CHAPTER VI

REDUCTION TO DATED QUANTITIES OF LABOUR

45 In this chapter prices are considered from their cost-of-production aspect, and the way in which they 'resolve themselves' into wages and profits is examined. Had it not been for the necessity of following one line of argument at a time, the subject would have been introduced earlier in the discussion. Indeed, although not properly introduced, it has been anticipated in allusions to the quantity of labour which 'directly and indirectly' enters a product.

46 We shall call 'Reduction to dated quantities of labour' (or 'Reduction' for short) an operation by which in the equation of a commodity the different means of production used are replaced with a series of quantities of labour, each with its appropriate 'date'.

Take the equation which represents the production of commodity 'a' (and where the wage and prices are expressed in terms of the Standard commodity):

$$(A_a p_a + B_a p_b + \ldots + K_a p_k)(1+r) + L_a w = A p_a.$$

We begin by replacing the commodities forming the means of production of A with *their own* means of production and quantities of labour; that is to say, we replace them with the commodities and labour which, as appears from their own respective equations, must be employed to produce those means of production; and they, having been expended a year earlier (§ 9), will be multiplied by a profit factor at a compound rate for the appropriate period, namely the means of production by $(1+r)^2$ and the labour by $(1+r)$. (It may be noted that A_a, the quantity of commodity 'a' itself which is used in the production of A, is to be treated like any other means of production, that is to say, replaced by its own means of production and labour.)

We next proceed to replace *these latter* means of production with their own means of production and labour, and to these will be applied a profit factor for one more year, or, to the means of production $(1+r)^3$ and to the labour $(1+r)^2$.

We can carry this operation on as far as we like and if next to the direct labour L_a we place the successive aggregate quantities of labour which we collect at each step and which we shall call respectively $L_{a_1}, L_{a_2}, \ldots, L_{a_n}, \ldots$, we shall obtain the 'reduction equation' for the product in the form of an infinite series

$$L_a w + L_{a_1} w(1+r) + \ldots + L_{a_n} w(1+r)^n + \ldots = Ap_a.$$

How far the reduction need be pushed in order to obtain a given degree of approximation depends on the level of the rate of profits: the nearer the latter is to its maximum, the further must the reduction be carried. Beside the labour terms there will always be a 'commodity residue' consisting of minute fractions of every basic product; but it is always possible, by carrying the reduction sufficiently far, to render the residue so small as to have, at any prefixed rate of profits short of R, a negligible effect on price. It is only at $r = R$ that the residue becomes all-important as the sole determinant of the price of the product.

47 As the rate of profits rises, the value of each of the labour terms is pulled in opposite directions by the rate of profits and by the wage, and it moves up or down as the one or the other prevails. The relative weight of these two factors varies of course at different levels of distribution; and, besides, it varies differently in the case of terms of different 'date', as we shall presently see.

We have seen (§30) that, if the wage is expressed in terms of the Standard net product, when the rate of profits (r) changes, the wage (w) moves as

$$w = 1 - \frac{r}{R}$$

where R is the maximum rate of profits.

Substituting this expression for the wage in each term of the reduction-equation the general form of any nth labour term becomes

$$L_{a_n}\left(1 - \frac{r}{R}\right)(1+r)^n.$$

Consider now the values assumed by this expression as r moves from 0 to its maximum R.

At $r = 0$ the value of a labour term depends exclusively on its size, irrespective of date.

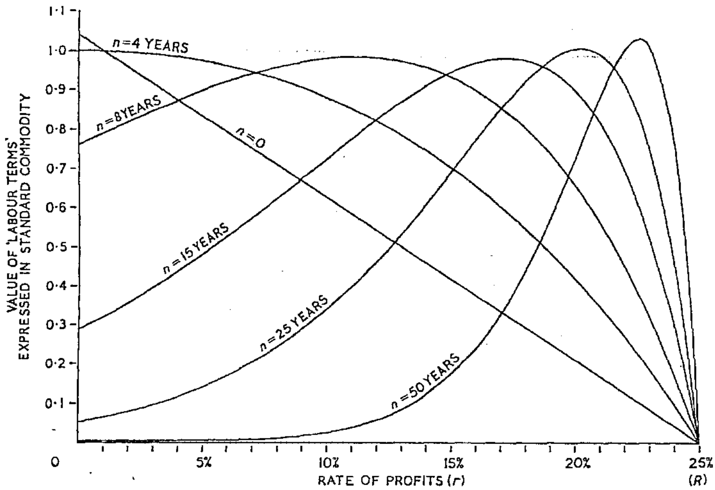

FIG. 2. Variation in value of 'Reduction terms' of different periods $[L_n w(1+r)^n]$ relative to the Standard commodity as the rate of profits varies between zero and R (assumed to be 25%).

The quantities of labour (L_n) in the various 'terms', which have been chosen so as to keep the curves within the page, are as follows: $L_0 = 1{\cdot}04$; $L_4 = 1$; $L_8 = 0{\cdot}76$; $L_{15} = 0{\cdot}29$; $L_{25} = 0{\cdot}0525$; $L_{50} = 0{\cdot}0004$.

With the rise of the rate of profits, terms divide into two groups: those that correspond to labour done in a more recent past, which begin at once to fall in value and fall steadily throughout; and those representing labour more remote in time, which at first rise and then, as each of them reaches its maximum value, turn and begin the downward movement. In the end, at $r = R$, the wage vanishes and with it vanishes the value of each labour term.

This is best shown by a selection of curves, representing terms of widely different dates (n) and different quantities of labour, such as is given in Fig. 2. In this example R is supposed to be 25%.

It is as if the rate of profits, in its movement from 0 to R, generated a wave along the row of labour terms the crest of which was formed by successive terms, as one after the other they reached their maximum value. At any value of the rate of profits the term which reaches its maximum has the 'date'

$$n = \frac{1+r}{R-r}.$$

And, conversely, the rate of profits at which any term of date n is at its maximum is

$$r = R-\frac{1}{n}.$$

Accordingly, all the terms for which $n \leqslant \frac{1}{R}$ have their maximum at $r = 0$ and thus form the group of 'recent dates' mentioned above as falling in value throughout the increase of r.

48 The labour terms can be regarded as the constituent elements of the price of a commodity, the combination of which in various proportions may, with the variation of the rate of profits, give rise to complicated patterns of price-movement with several ups and downs.

The simplest case is that of the 'balanced commodity' (cf. §21) or of its equivalent, the Standard commodity taken as an aggregate: its Reduction would result in a perfectly regular series, the quantity of labour in any term being equal to $(1+R)$ times the quantity in the term immediately preceding it in date.

As an example of the more complicated type we may suppose two products which differ in three of their labour terms (chosen from those represented in Fig. 2), while being identical in all the others. One of them, 'a', has an excess of 20 units of labour applied 8 years before, whereas the excess of the other, 'b', consists of 19 units employed in the current year and 1 unit bestowed 25 years earlier. (They are thus not unlike the familiar instances, respectively, of the wine aged in the cellar and of the old oak made into a chest.) The difference between their Standard prices at various rates of profits, namely

$$p_a-p_b = 20w(1+r)^8-\{19w+w(1+r)^{25}\}$$

is represented in Fig. 3 on the following page.

The price of 'old wine' rises relatively to the 'oak chest' as the rate of profits moves from 0 to 9 %, then it falls between 9 % and 22 %, to rise again from 22 % to 25 %.

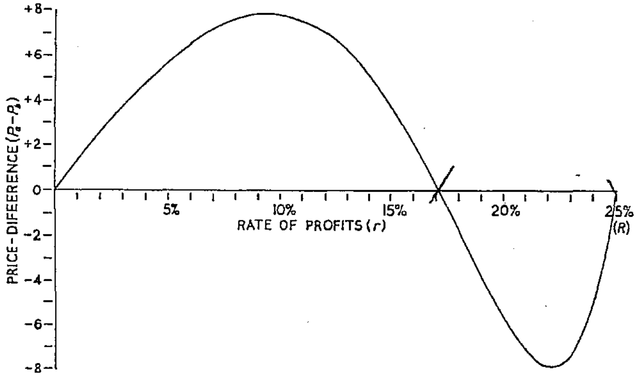

FIG. 3. Difference, at various rates of profits, between the prices of two commodities which are produced by equal quantities of labour equally distributed over time, with the exception that:

(1) a unit of commodity 'a' requires in addition 20 units of labour to be performed 8 years before its production is completed;

(2) a unit of commodity 'b' requires in addition 1 unit of labour 25 years before its production is completed and 19 units in the last year.

The equation of the curve is

$$p_a - p_b = 20w(1+r)^8 - \{19w + w(1+r)^{25}\},$$

where

$$w = 1 - \frac{r}{25\%}.$$

(The reduction to dated labour terms has some bearing on the attempts that have been made to find in the 'period of production' an independent measure of the quantity of capital which could be used, without arguing in a circle, for the determination of prices and of the shares in distribution. But the case just considered seems conclusive in showing the impossibility of aggregating the 'periods' belonging to the several quantities of labour into a single magnitude which could be regarded as representing the quantity of capital. The reversals in the direction of the movement of relative prices, in the face of unchanged methods of production, cannot be reconciled with *any* notion of capital as a measurable quantity independent of distribution and prices.)

49 There is however a restriction to the movement of the price of any product: if as a result of a rise in the rate of profits the price falls, its rate of fall cannot exceed the rate of fall of the wage. Thus, if we

draw two lines which show how the price of a product 'a' and the wage, both expressed in terms of the Standard commodity, vary with the rise of the rate of profits, the price line cannot cut the wage line more than once, and then only in one direction, such that the price, from being lower, becomes higher than the wage with the rise of the rate of profits.

This can be readily seen, whether we look at the Reduction series or at the original production equation of 'a'. Consider the former. The only variables, beside the price of 'a', are the wage and the rate

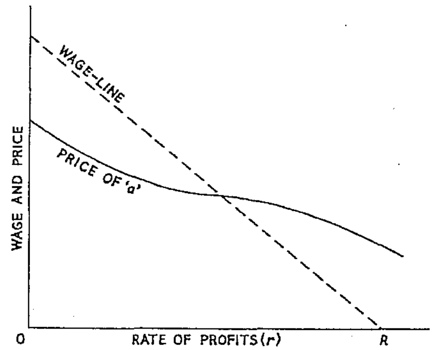

FIG. 4. Not more than one intersection is possible (in a system of single-product industries).

of profits, which rises with the fall of the wage, so that the combined effect of the two can never be a fall in the price more than in proportion to that of the wage.

If we turn to the production equation of commodity 'a', the prices of the means of production might upset the proposition if they were themselves capable of falling at a greater rate. But to see that this is impossible it is sufficient to turn our attention to the product whose rate of fall exceeds that of all the others: this product, since it cannot have means of production which are capable of falling at a greater rate than it does, must itself fall less than the wage.

The conclusion is not affected if instead of the Standard commodity, we take, as measure of wages and prices, any arbitrarily chosen product, since what we are concerned with is the price-relation between labour and the given product, a relation which is independent of the medium adopted.

It follows that if the wage is cut in terms of *any* commodity (no matter whether it is one that will consequently rise or fall relatively to the Standard) the rate of profits will rise; and vice versa for an increase of the wage.

It also follows that if the wage is cut in terms of one commodity, it is thereby cut in terms of all; and similarly for an increase. The direction of change is the same in relation to all commodities, however different may be the extent.

PART II

MULTIPLE-PRODUCT INDUSTRIES AND FIXED CAPITAL

CHAPTER VII

JOINT PRODUCTION[1]

50 In Part I it has been assumed that each commodity was produced by a separate industry. We shall now suppose two of the commodities to be jointly produced by a single industry (or rather by a single process, as it will be more appropriate to call it in the present context). The conditions would no longer be sufficient to determine the prices. There would be more prices to be ascertained than there are processes, and therefore equations, to determine them.

In these circumstances there will be room for a second, parallel process which will produce the two commodities by a different method and, as we shall suppose at first, in different proportions. Such a parallel process will not only be possible—it will be necessary if the number of processes is to be brought to equality with the number of commodities so that the prices may be determined. We shall therefore go a step further and assume that in such cases a second process or industry does in fact exist.[2]

This may seem an unreasonable assumption to make, implying as it appears to do that in every case there will be available a second method of production, distinct from the first and yet neither more nor less productive, so as to be capable of being employed side by side with it. But no such condition as to equal productiveness is implied, nor would it have a definite meaning before the prices were determined; and, with different proportions of products, a set of prices can generally be found at which two different methods are equally profitable.

[1] The next three chapters on Joint Production are in the main a preliminary to the discussion of Fixed Capital and Land in chs. x and xi. Readers who find them too abstract may like to move on to chs. x and xi and to refer back when necessary.

[2] Incidentally, considering that the proportions in which the two commodities are produced by any one method will in general be different from those in which they are required for use, the existence of two methods of producing them in different proportions will be necessary for obtaining the required proportion of the two products through an appropriate combination of the two methods.

Therefore, *any* other method of producing the two commodities will be compatible with the first, subject only to the general requirement of the resulting equations being mutually independent and having at least one system of real solutions: which rules out, for example, proportionality of *both* products and means of production in the two processes. However (and this is the only economic restriction), while the equations may be formally satisfied by negative solutions for the unknowns, only those methods of production are practicable which, in the conditions actually prevailing (i.e. at the given wage or at the given rate of profits) do not involve other than positive prices.

The same result as to the determination of prices which is obtained from the two commodities being jointly *produced* in different proportions by the two methods could be achieved (even though they were produced in the same proportions) through their being *used* as means of production in different proportions in various processes.

It could be achieved even if the two commodities were jointly produced by only *one* process, provided that they were *used* as means of production to produce a third commodity by two distinct processes; and, more generally, provided that the number of independent processes in the system was equal to the number of commodities produced.

(The assumption previously made of the existence of 'a second process' can now be replaced by the more general assumption that the number of processes should be equal to the number of commodities.)

51 The possibility of an industry having more than one product makes it necessary to reconstruct to some extent the equations devised for the case of exclusively single-product industries. In order to do so in a perfectly general way we shall, instead of regarding joint products as the exception, assume them to be universal and to apply to all processes and all products.

We consider a system of k distinct processes each of which turns out, in various proportions, the same k products.

This does not rule out the possibility of some of the products having a zero coefficient (that is to say, not being produced) in some of the processes: just as it has been admitted throughout that it is not necessary for each of the basic products to be used *directly* as means of production by all the industries.

The system of single-product industries is thus subsumed as an extreme case in which each of the products, while having a positive coefficient in one of the processes, has a zero coefficient in all the others.

An industry or production-process is consequently characterised, no longer by the commodity which it produces, but by the proportions in which it uses and the proportions in which it produces, the various commodities.

Accordingly, in the present chapter, processes will be distinguished (instead of, as formerly, by their products 'a', 'b', ..., 'k') by arbitrarily assigned numbers 1, 2, ..., k.

Thus, $A_1, B_1, \ldots, K_1$ will denote the quantities of the various goods 'a', 'b', ..., 'k' which are used as means of production in the first process; $A_2, B_2, \ldots, K_2$, those used in the second; ..., and $A_k, B_k, \ldots, K_k$, those used in the last process.

The quantities of the various goods *produced* by each process, on the other hand, to distinguish them from the means of production, will have their suffix enclosed in parentheses: $A_{(1)}, B_{(1)}, \ldots, K_{(1)}$ being the products of the first process; $A_{(2)}, B_{(2)}, \ldots, K_{(2)}$ the products of the second; ..., and $A_{(k)}, B_{(k)}, \ldots, K_{(k)}$ the products of the last process.

Using for the rest the same notation as in the case of single-product industries, the joint-production equations present themselves as follows:

$$
\begin{aligned}
(A_1p_a+B_1p_b+\ldots+K_1p_k)\ (1+r)+L_1w &= A_{(1)}p_a+B_{(1)}p_b+\ldots+K_{(1)}p_k \\
(A_2p_a+B_2p_b+\ldots+K_2p_k)\ (1+r)+L_2w &= A_{(2)}p_a+B_{(2)}p_b+\ldots+K_{(2)}p_k \\
\cdots\cdots\cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots\cdots\cdots \\
(A_kp_a+B_kp_b+\ldots+K_kp_k)\ (1+r)+L_kw &= A_{(k)}p_a+B_{(k)}p_b+\ldots+K_{(k)}p_k
\end{aligned}
$$

52 We can also construct the Standard system in the same way as was done in the case of exclusively single-product industries (§33); namely by finding a set of multipliers which, applied to the k production-equations, will result in the quantity of each commodity in the aggregate means of production of the system bearing to the quantity of the same commodity in the aggregate product a ratio which is equal for all commodities.

Before proceeding to do so, however, it is necessary to remove certain difficulties which stand in the way. These arise from the greater complexity of the interrelations, which results on the one hand in the creeping in of negative quantities and on the other in the disappearance of the one-one relation between products and industries.

CHAPTER VIII

THE STANDARD SYSTEM WITH JOINT PRODUCTS

53 As soon as we consider in detail the construction of a Standard system with joint products, it becomes obvious that some of the multipliers may have to be negative.

Take for example the case of two products jointly produced by each of two different methods. The possibility of varying the extent to which one or the other method is employed ensures a certain range of variation in the proportions in which the two goods may be produced in the aggregate. But this range finds its limits in the proportions in which the two goods are produced respectively by each of the two methods, so that the limits are reached as soon as one or the other method is exclusively employed.

Now suppose that in all cases in which two joint products 'a' and 'b' are used as means of production, the proportion in which 'a' is *employed* relatively to 'b' is invariably higher than the highest of the proportions in which it is *produced*. In such circumstances we can say from the outset that some process must enter the Standard system with a negative multiplier: but whether such a multiplier will have to be applied to the low producer or to a high user of commodity 'a' cannot be determined *a priori*—it can only be discovered by the solution of the system.

54 The most fertile ground for negative multipliers, however, is among non-basic products. (The latter need redefining in the new circumstances, but it may be said in advance that the main class, namely products which are altogether excluded from the means of production, will still be reckoned as non-basic; cf. § 60.)

Consider the case of two commodities (jointly produced in different proportions by two processes) of which one is to be included in the Standard product while the other, not entering the means of production of any industry, must be excluded from the Standard product. This will be effected by giving a negative multiplier to the process that produces relatively more of the second commodity and a positive one to the other process: the two multipliers being so proportioned that when the two equations are added up the two quantities produced of the non-basic exactly cancel out, while a positive balance of its companion product is retained as a component of the Standard commodity.

55 Once negative multipliers have been admitted for some processes, others, which with regard to negative multipliers shine with a reflected light, are liable to appear. Thus, if a raw material is directly used in only one process, and that happens to be one that receives a negative multiplier, the industry which produces the raw material in question will itself have to follow suit and enter the Standard system with a negative multiplier.

56 The outcome of this, since no meaning can be attached to the 'negative industries' which such multipliers entail, is that it becomes impossible to visualise the Standard system as a conceivable rearrangement of the actual processes. We must therefore in the case of joint-products be content with the system of abstract equations, transformed by appropriate multipliers, without trying to think of it as having a bodily existence.

The *raison d'être* of the Standard system, however, is to provide a Standard commodity. And in the case of the latter there is fortunately no insuperable difficulty in conceiving as real the negative quantities that are liable to occur among its components. These can be interpreted, by analogy with the accounting concept, as liabilities or debts, while the positive components will be regarded as assets.

Thus a Standard commodity which includes both positive and negative quantities can be adopted as money of account without too great a stretch of the imagination provided that the unit is conceived as representing, like a share in a company, a fraction of each asset and of each liability, the latter in the shape of an obligation to deliver without payment certain quantities of particular commodities.

57 There is another difficulty arising from the complexity of the joint-products system that must be considered before we can go on to construct the Standard commodity.

The criterion previously adopted for distinguishing between basic and non-basic products (namely whether they do, or do not, enter directly or indirectly the means of production of all commodities) now fails, since, each commodity being produced by several industries, it would be uncertain whether a product which entered the means of production of only one of the industries producing a given commodity should or should not be regarded as entering directly the means of production of that commodity.[1] And the uncertainty would naturally extend to the question whether it did or did not enter 'indirectly' the production of commodities into which the latter entered as means of production.

58 Taking advantage of the circumstance that all the three distinct types of non-basics which are met in the single-products system find their equivalents in the case of multiple-product industries, we shall begin by defining for the latter case the three types of non-basics, each as the extension of the corresponding single-product type (cf. § 35).

(1) Products which do not enter the means of production of *any* of the industries. This type can be immediately extended to the multiple-products system without need of adaptation.

(2) Products each of which enters only *its own* means of production. The equivalent of this type in the multiple system is a commodity which enters the means of production of each of the processes by which it is itself produced, and of no others—but enters them to such an extent that the ratio of its quantity among the means of production to its quantity among the products is exactly the same in each of the processes concerned.

(3) Products which only enter the means of production of an interconnected group of non-basics; in other words, products which,

[1] The trouble, however, lies deeper and as we shall presently see there would be uncertainty even if the commodity entered directly the means of production of *all* the processes in the system. Cf. below, §59.

as a group, behave in the same way as a non-basic of the second type does individually.

In order to define in the multiple system of k processes the type which corresponds to this third case we shall (supposing the inter-connected group to consist of three products, 'a', 'b' and 'c') arrange the quantities in which these commodities enter any one process, as means of production, and as products, in a row: we shall thus obtain k rows ordered in 2×3 columns as follows:[1]

$$\begin{array}{cccccc} A_1 & B_1 & C_1 & A_{(1)} & B_{(1)} & C_{(1)} \\ A_2 & B_2 & C_2 & A_{(2)} & B_{(2)} & C_{(2)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ A_k & B_k & C_k & A_{(k)} & B_{(k)} & C_{(k)} \end{array}$$

The condition for the three products being non-basic is that not more than three of the rows should be independent, the other rows being obtainable from combinations of those three rows formed after giving them suitable multipliers. (See for the general definition § 60.)

59 The third type may give rise to curiously intricate patterns. An example will indicate the possibilities in this direction.

Suppose that, in a system of four processes and four products, two commodities, 'b' and 'c', are jointly produced by *one* process, and are produced by no other; but while 'b' does not enter the means of production of *any* process, 'c' enters the means of *all* the four processes. Supposing the process that produces 'b' and 'c' to be represented by the equation

$$(A_1 p_a + C_1 p_c + K_1 p_k)\ (1+r) + L_1 w = A_{(1)} p_a + B_{(1)} p_b + C_{(1)} p_c + K_{(1)} p_k$$

the 'rows' for the two commodities will be

$$\begin{array}{cccc} \cdot & C_1 & B_{(1)} & C_{(1)} \\ \cdot & C_2 & \cdot & \cdot \\ \cdot & C_3 & \cdot & \cdot \\ \cdot & C_4 & \cdot & \cdot \end{array}$$

Only the first row and any one of the three others are independent, the remaining two rows being linear transformations of the latter. So that both 'b' and 'c' are non-basic.

[1] Some of the quantities may of course be zero.

If we look at the matter from the standpoint of constructing the Standard system, while it is obvious that 'b' is incapable of entering the Standard commodity, 'c' appears at first sight to be a suitable component of it. However, since 'b' occurs only in one process, the only way to eliminate 'b' is to omit that process altogether (i.e. to give it a zero multiplier). But that process was also the exclusive producer of 'c', so that 'c' now survives only on the side of the means of production and therefore becomes incapable of entering the Standard commodity. So 'c' must itself be eliminated, which is to be done by subtracting one of the remaining equations from each of the others, after giving it in each case an appropriate multiplier which will result in cancelling out every quantity of 'c'.

60 The formal definition just given is not nearly so satisfactory from the economic standpoint as the intuitive criterion of 'entering, or not entering, the means of production of all commodities' which it supersedes. It has, however, the advantage of greater generality.

It is clear, to begin with, that the first two types of non-basics can be absorbed, as particular cases, in the third.

The definition covers, besides, the three types of the single-products system. (It is indeed quite general and, as the example of §59 suggests, it includes also a final type of non-basic, which is to be introduced subsequently, namely commodities which enter the means of production but are not produced—a type of which land is the outstanding example.)

We can therefore give this general formulation of the distinction between basic and non-basic goods:

In a system of k productive processes and k commodities (no matter whether produced singly or jointly) we say that a commodity or more generally a group of n linked commodities (where n must be smaller than k and may be equal to 1) are *non-basic* if of the k rows (formed by the $2n$ quantities in which they appear in each process) not more than n rows are independent, the others being linear combinations of these.[1]

[1] In the language of algebra, the matrix of k rows and $2n$ columns is of rank less than, or equal to, n.

All commodities which do not satisfy this condition are *basic*. (Note that, as has been stated in §6, every system is assumed to include at least one basic product.)

61 It follows directly from this that we can, by linear transformations, entirely eliminate the non-basic commodities from the system, both on the side of the means of production and on that of the products. That is to say, we can find a set of multipliers (some positive and some negative) which applied to the original k equations make it possible to combine them into a smaller number of equations (equal in number to the basic products) in each of which any quantity of a non-basic is cancelled by an equal quantity of opposite sign, so that only basics are included in quantities different from zero.

This operation achieves the same result as is obtained in the single-products system by the much simpler method of crossing out the equations of the industries which produce non-basics (§35). In both cases the effect is to simplify the subsequent steps in the argument.

62 If the number of basic products is j, the system thus obtained will consist of j equations: these may be described as the Basic equations.

Supposing the j basic commodities to be 'a', 'b', ..., 'j' we shall denote the net quantities in which they appear in a Basic equation by barred letters $\bar{A}$, $\bar{B}$, ..., $\bar{J}$ to distinguish them from the quantities in the original processes. The Basic equations will accordingly be as follows:

$$
\begin{aligned}
(\bar{A}_1 p_a+\bar{B}_1 p_b+\ldots+\bar{J}_1 p_j)\ (1+r)+\bar{L}_1 w &= A_{(1)} p_a+\bar{B}_{(1)} p_b+\ldots+\bar{J}_{(1)} p_j \\
(\bar{A}_2 p_a+\bar{B}_2 p_b+\ldots+\bar{J}_2 p_j)\ (1+r)+\bar{L}_2 w &= \bar{A}_{(2)} p_a+\bar{B}_{(2)} p_b+\ldots+\bar{J}_{(2)} p_j \\
\cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots \\
(\bar{A}_j p_a+\bar{B}_j p_b+\ldots+\bar{J}_j p_j)\ (1+r)+\bar{L}_j w &= \bar{A}_{(j)} p_a+\bar{B}_{(j)} p_b+\ldots+\bar{J}_{(j)} p_j
\end{aligned}
$$

This system is equivalent to the original one inasmuch as the values which it determines for R and the prices will necessarily be also solutions of that system.

It differs however from the original system, not only for excluding non-basics, but in two other respects. In the first place a Basic equation does not in general represent a productive process—it is merely the result of combining the equations of a number of processes. In the second place it may contain negative quantities as well as positive ones.

63 The Basic equations are designed for the construction of the Standard product.[1] The multipliers $q_1, q_2, \ldots, q_j$ which applied to the j Basic equations give the Standard system are determined by the following equations:

$$
\begin{aligned}
(\bar{A}_1 q_1 + \bar{A}_2 q_2 + \ldots + \bar{A}_j q_j)\ (1+R) &= \bar{A}_{(1)} q_1 + \bar{A}_{(2)} q_2 + \ldots + \bar{A}_{(j)} q_j \\
(\bar{B}_1 q_1 + \bar{B}_2 q_2 + \ldots + \bar{B}_j q_j)\ (1+R) &= \bar{B}_{(1)} q_1 + \bar{B}_{(2)} q_2 + \ldots + \bar{B}_{(j)} q_j \\
\cdots\cdots & \cdots\cdots \\
(\bar{J}_1 q_1 + \bar{J}_2 q_2 + \ldots + \bar{J}_j q_j)\ (1+R) &= \bar{J}_{(1)} q_1 + \bar{J}_{(2)} q_2 + \ldots + \bar{J}_{(j)} q_j
\end{aligned}
$$

The equations give an equation for R of the jth degree, so that there may be up to j possible values of R and corresponding sets of values of the q's; and each set will represent a Standard commodity of different composition.

64 In deciding which, among the j possible sets of values, is the one relevant to the economic system, we can no longer rely on there being, as the obvious choice, a value of R to which corresponds an all-positive Standard commodity; for in a system of joint production all may include negative quantities among their components.

If, however, we reconsider the matter from the standpoint of the single-products system, we shall find that while an all-positive Standard appeals to commonsense, its superiority is due at least as much to its being at the same time (as was shown in §42) the one that corresponds to the lowest possible value of R. And we shall see that the possession of this last property is by itself sufficient to make the Standard net product that is endowed with it (no matter whether all-positive or otherwise) the one eligible for adoption as unit of wages and prices.

In effect suppose that, R' being the lowest possible value of R, we adopted as unit the Standard product corresponding to another value,

[1] It would have been possible to construct the Standard product directly from the original equations, and the final result would of course have been the same. Why it has seemed simpler to go through the intermediate stage of the Basic equations is explained in Appendix C.

say R'', larger than R'. As the wage w measured in this Standard was gradually reduced from 1 it would, before reaching 0, arrive at a level w' such that

$$R''(1-w') = R'$$

when the rate of profits would be equal to R'.

If, at such level of w, we reckon on the basis of R', the wage must be zero, since the rate of profits is at its maximum; while on the basis of R'' the wage must be positive since the rate of profits is below its maximum. The reconciliation is effected through the wage w' being a positive quantity of a composite commodity the exchange value of which is zero. This is because (as was shown in § 41) the exchange value of a Standard commodity the composition of which corresponds to one solution of R (in our case R'') at the prices that correspond to another solution of R (in our case R') is zero.

This implies that, in these circumstances, the prices of all commodities would, in terms of the chosen Standard, be infinite. Such a result is economically meaningless. This anomaly, however, can be avoided if we adopt as unit the Standard net product that corresponds to the lowest of the values of R. This is the only Standard product in terms of which, at all the levels of the wage from 1 to 0 (and so at all the levels of the rate of profits from 0 to its maximum), it is possible for the prices of commodities to be finite.

65 The distinction between basics and non-basics has become so abstract in the Multiple-products system (whether because of the way in which it is defined or of the way in which it is applied in the construction of a Standard commodity) that it may be wondered whether it has retained any economic content at all.

From the start, however, the chief economic implication of the distinction was that basics have an essential part in the determination of prices and the rate of profits, while non-basics have none. And this we shall find to be still true under the new definition.

In the Single-products system this meant that, if an improvement took place in the method of production of a basic commodity, the result would necessarily be a change in the rate of profits and in the prices of all commodities; while a similar improvement in the case of a non-basic would affect only its particular price.

This cannot be extended directly to a system of multiple products, where both basics and non-basics may be produced by the same process. We can however find an equivalent in a tax (or subsidy) on the production of a particular commodity. Such a tax is best conceived as a tithe, which can be defined independently of prices and has the same effect as would have a fall in the output of the commodity in question all other things (namely the quantities of its means of production and of its companion products) remaining unchanged.

A tax on a basic product then will affect all prices and cause a fall in the rate of profits that corresponds to a given wage, while if imposed on a non-basic it will have no effect beyond the price of the taxed commodity and those of such other non-basics as may be linked with it.[1] This is obvious if we consider that the transformed system of Basic equations, which by itself determines the rate of profits and the prices of basic products, cannot be affected by changes in the quantity or price of non-basics which are not part of the system.

[1] The effect which the tax has on the price of a non-basic will vary with the type of non-basic. If it does not enter any of the means of production, its price will rise by the amount of the tax. If it enters its own means of production, its price will change to the extent required to maintain the original ratio of the value of the aggregate product of the process (after deduction of the wage and of the tax) to the value of its aggregate means of production. If it belongs to a group of interconnected non-basics, the prices of all or some of the components of the group will change so as to maintain that ratio. (In the example of §59 if the production of commodity 'c' were taxed, the price of 'c' itself would be unaffected and the brunt would be borne by the price of 'b' which would have to rise to the necessary extent.)

CHAPTER IX

OTHER EFFECTS OF JOINT PRODUCTION

66 It remains now to see to what extent the other conclusions reached in the case of single-product industries are applicable to joint-products.

One of those clearly needing verification is the rule that, when the rate of profits is zero, the relative value of commodities is proportional to the quantity of labour which, directly and indirectly, has gone to produce them (§ 14). For in the case of joint-products there is no obvious criterion for apportioning the labour among individual products, and indeed it seems doubtful whether it makes any sense to speak of a *separate* quantity of labour as having gone to produce one of a number of *jointly* produced commodities. We certainly get no help from the 'Reduction' approach, that is to say from looking upon the quantity of labour as being ascertained by tracing back the successive units of labour bestowed at various times on the product; for this method seems totally inapplicable to the case of joint-products. (The question is further referred to in §68.)

However, with the system of single-product industries we had an alternative if less intuitive line of approach in the method of 'Sub-systems' (see Appendix A) by which it was possible to determine for each of the commodities composing the net product the share of the aggregate labour which could be regarded as directly or indirectly entering its production. Now this method, with appropriate adaptation, is capable of extension to a system of joint-products, so that the conclusion about the quantity of labour 'contained' in a commodity and its proportionality to value at zero profits can also, without any straining of the ordinary meaning of words, be extended to commodities jointly produced.

Take first the case of two commodities which are jointly produced by each of two processes in different proportions; but instead of looking separately at the two processes and their products, let us consider the system as a whole and suppose that quantities of both commodities are included in the net product of the system. We shall further assume that the system is in a self-replacing state and that, whenever the net product is changed, the self-replacing state is immediately restored by means of suitable adjustments in the proportions of the processes composing it.

It may be noted as a preliminary that it is possible to change, within certain limits, the proportions in which the two commodities are produced if we alter the relative sizes of the two processes by each of which (although in different proportions) they are jointly produced.

Now, if we wish to increase by a given amount the quantity in which a commodity enters the net product of the system, while leaving all the other components of the net product unchanged, we normally must increase the total labour employed by society. It is, therefore, natural to conclude that the quantity by which labour has to be increased for this purpose goes in its entirety, whether directly or indirectly, to produce the additional quantity of the commodity in question. The commodity added will, at the price corresponding to a zero rate of profits, obviously be equal in value to the additional quantity of labour.

This conclusion seems no less cogent for a commodity which is jointly produced with another, than for one which is produced separately. Nor is the conclusion affected by the circumstance that it will in general be necessary, in order to maintain the self-replacing state, to change the quantities of the means of production used in the system, since any additional labour needed to produce the latter is included as indirect labour in the quantity that produces the addition to the net product.[1]

[1] The adjustments envisaged include, since joint-products are present, the contraction of some of the processes, and thus we might fall again into the awkwardness of 'negative industries'. This, however, can in general be avoided, provided that the initial increase of the commodity in question is supposed to be sufficiently small, and that the net product of the system is assumed to comprise at the start sufficiently large quantities of all the products, so that any necessary contraction can be absorbed by existing processes, without the need of any of them having to receive a negative coefficient.

67 A similar reasoning can be applied to the case of two commodities ('a' and 'b') which are jointly produced by only one process, but are used as means of production, in different relative quantities, by two processes each of which produces singly the same commodity 'c'.

While in this case we cannot change the proportions in which the two commodities appear in the output of the industry producing them, nevertheless we can, by altering the relative size of the two processes using them, vary the relative quantities in which they are used as means for producing a given quantity of 'c'. In this way we can vary the relative quantities in which the two enter the means of production of the system and this by itself (since the relative quantities in which the two enter the gross product are fixed) alters the relative quantities in which they respectively enter the *net* social product.

It is thus possible, as in the previous case, by an addition to the total labour, to arrive at a new self-replacing state, in which a quantity of one of the two joint products, say 'a', is added to the net product, while all the other components of the latter remain unchanged. And we can accordingly conclude that the addition to labour is the quantity which directly and indirectly is required to produce the additional amount of commodity 'a'.

68 As noted above, while the method just outlined is an extension of the approach by sub-systems, there is no equivalent in the case of joint-products to the alternative method, namely Reduction to a series of dated labour terms. In effect it is of the essence of such a Reduction that each commodity should be produced separately and by only one industry, and the whole operation consists in tracing back the successive stages of a single-track productive process.

To re-create with joint products the conditions necessary for such an operation we should have to give a negative coefficient to one of the two joint-production equations and a positive one to the other, so as to eliminate one of the products while retaining the other in isolation. Consequently some of the terms in the Reduction would represent negative quantities of labour, for which no reasonable interpretation could be suggested. What is worse, since the series would contain both positive and negative terms, the 'commodity residue' instead of decreasing towards zero at the successive stages of approximation, might show steady or even widening fluctuations, so that the series would not converge, that is to say, its sum would not tend to a finite limit. (An example of this type will be found in § 79.)

The Reduction could not even be attempted if the two products were jointly produced by a single process, or by two processes in the same proportions, since the apportioning of the value and of the quantities of labour between the two products would depend entirely on the way in which the products were *used* as means of production for other commodities.

69 Another statement that calls for reconsideration at this stage is the proposition that, if the prices of all commodities are positive at any *one* level of the wage between 1 and 0, no price can become negative as a result of the variation of the wage within those limits (§ 39). It may be said at once, however, that this proposition is not capable of extension to the case of joint products. The grounds on which it rested in the case of a system of single-product industries was that the price of a commodity could only become negative if the price of some other commodity (which was used as one of its means of production) had become negative first—so that no commodity could ever be the first to do so. But in the case of joint products there is a way round and the price of one of them might become negative, provided the balance was restored by a rise in the price of its companion product sufficient to maintain the aggregate value of the two products above that of their means of production by the requisite margin.

70 This conclusion is not in itself very startling. All that it implies is that, although in actual fact all prices were positive, a change in the wage might create a situation the logic of which required some of the prices to turn negative: and this being unacceptable, those among the methods of production that gave rise to such a result would be discarded to make room for others which in the new situation were consistent with positive prices.

But when the above conclusion is related to what we have previously seen concerning the quantity of labour entering a commodity, the combined effect of the two is indeed such as to require some explanation. For what is involved is not merely that, e.g., in the remote contingency of the rate of profits falling to zero, the price of such a commodity would, if other things remained equal, have to become negative; but that we are driven to the conclusion that in the actual situation, with profits at the perfectly normal rate of, say, 6 %, that commodity is in fact being produced by a *negative* quantity of labour.

This looks at first as if it were a freak result of abstraction-mongering that can have no correspondence in reality. But if we apply to it the test employed for the general case in §66, and under the conditions there described we suppose that the quantity of such a commodity entering the net product of the system is *increased* (the other components being kept unchanged), we shall find that as a result the aggregate quantity of labour employed by society has indeed been *diminished*.

Nevertheless, since the change in production is carried out while the ruling rate of profits is, as in the above example, at 6 % and the system of prices is the one appropriate to that rate, nothing abnormal will be noticeable: in effect the diminution in the expense for labour will be more than balanced by an increased charge for profits, so that the addition to net output will entail a positive addition to cost of production.

What happens is that in order to effect the required change in the net product, one of the two joint-production processes must be expanded while the other is contracted; and in the case under consideration the expansion of the former process employs (either directly or through such other processes as it carries in its train to ensure full replacement) a quantity of labour which is smaller, but means of production which at the prices appropriate to the given rate of profits are of greater value, and therefore attract a heavier charge for profits, than does (under a similar proviso) the contraction of the latter process.

It seems unnecessary to show in detail that what has been said in this section concerning negative quantities of labour can be extended (on the same lines as was done for positive quantities in §67) to the case in which two commodities are jointly produced by only one process, but are used as means of production by two distinct processes both producing a third commodity.

71 There is one further proposition about prices which needs reconsideration in the case of joint products.

We have seen (§49) that with single-product industries when the wage falls in terms of the Standard commodity no product can fall in price, in the same standard, at a higher rate than does the wage. This conclusion was based on the consideration that, were a product able to do so, it must be owing to one of its means of production falling in price at a still higher rate; and since this could not apply to the product that fell at the highest rate of all, that product itself could not fall at a higher rate than the wage.

With one of a group of joint products, however, there is the alternative possibility that the other commodities jointly produced with it should *rise* in price (or suffer only a moderate fall) with the fall of the wage so as to make up, in the aggregate product of the industry, for any excessive fall of the first commodity's price. To such a rise there is no limit and therefore there is none to the rate at which one of the several joint products may *fall* in price.

But as soon as it is admitted that the price of one, out of two or more joint products, can fall at a higher rate than does the wage, it follows that even a singly produced commodity can do so, provided that it employs, as one of its means of production, and to a sufficient extent, the joint product so falling.

72 The possibility that the price of a product may fall faster than the wage has some notable consequences. The first is that the rule that the fall of the wage in *any* standard involves a rise in the rate of profits must now admit of an exception.

Suppose that a 10 per cent fall in the Standard wage entails (at a certain level) a larger proportionate fall, say 11 per cent, in the price, also measured in Standard product, of commodity 'a'. This means that labour has risen in value by about 1 per cent relative to commodity 'a'. If therefore we were to express the wage in terms of commodity 'a', a *fall* in such a wage over the same range would involve a *rise* in the Standard wage and consequently a *fall* in the rate of profits.

We can thus no longer speak of a rise or of a fall in the wage unless we specify the standard, for what is a rise in one standard may be a fall in another.

For the same reasons it becomes possible for the wage-line and the price-line of commodity 'a' to intersect more than once as the rate of profits varies.

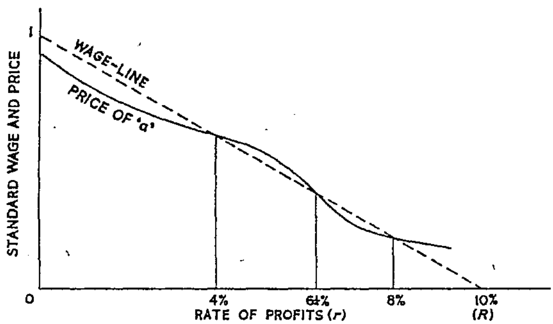

FIG. 5. Several intersections are possible in a system of multiple-product industries.

As a result, to any one level of the wage in terms of commodity 'a' there may correspond several alternative rates of profits. (In Fig. 5 the several points of intersection represent equality in value between a unit of labour and a unit of commodity 'a', i.e. the same wage in terms of 'a'; but of course they represent different levels of the wage in terms of the Standard commodity.) On the other hand, as in the case of the single-products system, to any one level of the rate of profits there can only correspond one wage, whatever the standard in which the wage is expressed.

CHAPTER X

FIXED CAPITAL

73 The interest of Joint Products does not lie so much in the familiar examples of wool and mutton, or wheat and straw, as in its being the genus of which Fixed Capital is the leading species. And it is mainly as an introduction to the subject of fixed capital that the preceding chapters devoted to the intricacies of joint products find their place.

We shall regard durable instruments of production as part of the annual intake of a process, on the same footing as such means of production (e.g. raw materials) as are entirely used up in the course of a year; while what is left of them at the end of the year will be treated as a portion of the annual joint product of the industry, of which the more conspicuous part consists of the marketable commodity that is the primary object of the process.

For example, a knitting-machine enters the means of production at the beginning of the year, along with the yarn, the fuel, etc., with which it is employed; and at the end of the year the partly worn-out, older machine which emerges from the process will be regarded as a joint product with the year's output of stockings.

74 This point of view implies that the same machine, at different ages, should be treated as so many different products, each with its own price. In order to determine these prices, an equal number of additional equations (and therefore of processes) is required.

Accordingly, an industry which employs a durable instrument must be regarded as being subdivided into as many separate processes as are the years of the total life of the instrument in question. Each of these processes is distinguished by the fact that it uses an instrument of a different age; and each of them 'produces', jointly with a quantity of a marketable commodity, an instrument a year older than the one which it uses—with the exception of the process using the expiring instrument in its last year, which produces singly the marketable commodity (or, at most, in addition, the residual scrap if it has any value).[1]

These processes need not be separate in ownership or in operation, and will indeed often be run side by side in the same shed; all that is necessary is that the amounts of means of production and labour employed by each should be separately ascertainable by the use of measures of quantity, without need of knowing the values—so that an independent production equation can be set up for each.[2]

Nor is it necessary that the instruments belonging to successive age-groups should actually be marketed for their prices to be effective; since even though these are only book-values, they are the basis for correctly allocating the profits and making allowance for depreciation in the case of each age-group: 'correctly' in the sense of just fulfilling the original condition of making possible the replacement of the means of production and the payment of a uniform rate of profits. This can be seen if we compare the results of the method here proposed with the usual way of calculating the depreciation and interest on a fixed capital asset.

75 The 'usual' method just referred to is as follows.

Supposing a machine 'm' to work with constant efficiency throughout its life, the annual charge to be paid for interest and depreciation in respect of it must be constant, if the price of all units of the product is to be uniform. This annual charge will be equal to a fixed annuity, the present value of which, calculated on the basis of the general rate of profits r, is equal to the original price of the machine. If that price

[1] If the scrap (metal, timber, etc.) is interchangeable in use with some other material already accounted for, it simply assumes the price of the latter without need of an additional process; if it is not completely interchangeable (e.g. scrap iron as compared with pig iron), then there will be room for two processes, producing the same commodity (e.g. steel), but differing in the proportions in which they use the two types of material.

[2] This does not rule out the possibility of there being overheads which cannot be allotted without going through the process of valuation. Where such exist, they merely represent another case of joint production superimposed on the case under consideration and like all such cases they both require and allow of a sufficient number of processes to determine the allocation of the joint costs.

is p_{m_0} and the life of the machine is n years, the annuity, as one can find from any handbook of commercial arithmetic, is

$$p_{m_0}\frac{r(1+r)^n}{(1+r)^n-1}$$

which is therefore the annual charge on the machine.

76 On the other hand, the method here proposed is based on the equations for the separate processes which correspond to the successive ages of the machine. The quantity of machines of a given type that are required to produce annually $G_{(g)}$ (a quantity of a commodity) will be denoted by M_0 when they are new, by M_1 when they are one-year old, etc., and by M_{n-1} when they enter their last year of usefulness; their respective prices, or book-values, per unit will be denoted by $p_{m_0}, p_{m_1}, \ldots, p_{m(n-1)}$. Under the condition assumed above of constant efficiency throughout the life of the machine, the equations representing the production of a commodity 'g' by the employment of the machine 'm', using for the rest the same notation as in §51, will be

$$\begin{aligned}
(M_0 p_{m_0} + A_g p_a + \ldots + K_g p_k)\,(1+r) + L_g w &= G_{(g)} p_g + M_1 p_{m_1} \\
(M_1 p_{m_1} + A_g p_a + \ldots + K_g p_k)\,(1+r) + L_g w &= G_{(g)} p_g + M_2 p_{m_2} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \cdots\cdots\cdots \\
(M_{n-1} p_{m(n-1)} + A_g p_a + \ldots + K_g p_k)\,(1+r) + L_g w &= G_{(g)} p_g.
\end{aligned}$$

The quantities of means of production, of labour and of the main product are equal in the several processes in accordance with the assumption of constant efficiency during the life of the machine. This circumstance makes it possible for the whole group to be combined into a single expression. If we multiply the n equations respectively by $(1+r)^{n-1}$, $(1+r)^{n-2}$, ..., $(1+r)$, 1 and add them, the machines of intermediate ages (above zero and under n years) which appear on both sides cancel out and we obtain

$$M_0 p_{m_0}(1+r)^n + \{(A_g p_a + \ldots + K_g p_k)\,(1+r) + L_g w\}\,\frac{(1+r)^n-1}{r} = G_{(g)} p_g \frac{(1+r)^n-1}{r}$$

Dividing both sides by $\dfrac{(1+r)^n-1}{r}$ we have

$$M_0 p_{m_0} \frac{r(1+r)^n}{(1+r)^n-1} + (A_g p_a + \ldots + K_g p_k)\,(1+r) + L_g w = G_{(g)} p_g$$

where the first term represents the annual charge for the machine and is identical with the expression that we obtained above (§75) by the annuity approach.

77 While the two methods give the same result in the extremely simplified case of constant efficiency to which both can be applied, the advantage of the joint-production-equations method is that it is not restricted to that case but has general validity. It will give the 'correct' answer in every case, no matter how complex, over the life of a durable instrument of production, may be the pattern of falling productivity or increasing maintenance and repairs. It will, besides, make due allowance for any variation in the prices of the different materials and services required.

In every case the price at any given age of a durable instrument of production or fixed capital asset, as it results from the equations, represents its correct book-value after depreciation. The difference between the values of the asset at two consecutive ages gives the allowance to be made for depreciation for that year. And this latter amount (for example, $M_1 p_{m_1} - M_2 p_{m_2}$) added to the profit at the general rate on the value of the asset at the beginning of the year ($M_1 p_{m_1} r$) gives the annual charge for that year. This charge will in general not be constant but changing, and probably falling, with the ageing of the instrument or asset.

78 The depreciation of a machine, however, is not determined exclusively by its employment in one particular industry, as the above might seem to imply.

The same type of machine (e.g. a lorry) may be used in several industries and it may be subject to greater wear and tear when employed in one than in the other and have a shorter life; or, even if the total life is the same, its efficiency may fall at different rates from year to year or require more repairs.

Since the price of the new machine is the same for all industries it can continue to be denoted by p_m. But in successive years it may have a different book-value according to the use to which it is put. The new uses will be represented by additional equations and the new book-values by additional symbols. Thus we may call $M_{g_1}p_{mg_1}$, $M_{g_2}p_{mg_2}$, etc. the machines at successive ages multiplied by their respective book-values in the 'g' industry; $M_{h_1}p_{mh_1}$, $M_{h_2}p_{mh_2}$, etc. those in the 'h' industry, and so on.

If in all the industries the machine had the same working life and constant efficiency, the book-values for each age would be equal in all of them, since the annual charges would all be equal to the annuity described in §75.

79 We now turn to inquire to what extent the complications that arise with Joint Products in general apply to the particular case of Fixed Capital. First as regards 'reduction'.

The equations for fixed capital make it easy to see how an attempt to effect the 'reduction' of a durable instrument to a series of dated quantities of labour will in general fail. To take the simplest case, suppose that a machine has a life of two years and its efficiency is constant. The equations would be

$$(M_0p_{m_0}+A_gp_a+\dots+K_gp_k)\ (1+r)+L_gw = G_{(g)}p_g+M_1p_{m_1}$$
$$(M_1p_{m_1}+A_gp_a+\dots+K_gp_k)\ (1+r)+L_gw = G_{(g)}p_g.$$

Now the first step towards the 'reduction' of the one-year-old machines M_1 to a series of labour terms is to subtract the second equation from the first so as to isolate M_1, leaving it as the sole product on the right-hand side. As a result of this there appears a similar quantity M_1 among the means of production; it has, however, a negative sign and its price is multiplied by $(1+r)$.

This is by itself sufficient to show that we are engaged in a blind alley: for when we come to the 'reduction' of the negative term containing M_1, there will appear among its residual means of production a positive M_1; and so, with successive steps, M_1 will constantly reappear, alternately positive and negative, and in each case multiplied by a higher power of $(1+r)$. This will make it impossible on the one hand for the residual aggregate of commodities to tend to vanishing-point and on the other for the sum of labour terms to tend to a limit. (This conclusion, based on the assumption of constant efficiency, holds *a fortiori* when the product of a machine diminishes with age; but it would cease to be true, and the 'reduction' to dated labour terms, some positive and some negative, would become possible if the annual product were to *increase* with age.)

80 Now consider how the value of a machine varies with its age. (For the sake of simplicity we take as in the previous example a machine of constant efficiency.) If we assume the rate of profits to be zero, the value of such a machine will fall by equal steps of $1/n$th of the original value for each of the n years of its life.

Since in this case of zero profits the original value represents the quantity of labour that has been required to produce the machine, it is natural to extend this notion to the subsequent years and say that its value at any given age represents the quantity of labour which it 'embodies', that is to say the quantity which has gone to produce it, minus such quantities as year by year have passed into its product. (What is more, this can be verified by the method described in §§66–7 and Appendix A, as is done in the following section.)

81 Suppose that a tractor requires, directly and indirectly, 4 units of labour to produce it and has a life of four years with constant efficiency: what is being suggested is that at the end of the first year's operation it will 'contain' only 3 units of labour, at the end of the second 2, etc., and at the end of the fourth, when it is ready to be scrapped, none.

To prove this we shall compare two systems which differ in their net products. We start with a self-replacing system the annual net product of which consists of, say, 1000 tons of wheat. It employs 20 tractors equally divided between the four age-groups of 0, 1, 2, 3 years; these require for replacement the production of 5 new tractors annually.

Next we introduce a second self-replacing system, similar to the preceding one in every respect, except that its net annual product includes some tractors which are half-way through their life. Thus, besides 1000 tons of wheat, the *net* product of this system will include 2 *two-year-old tractors*. We have to show that the second system must employ an extra 4 units of labour, i.e. the quantity said to be 'contained' in two tractors of that age.

Such a system to be self-replacing must, first of all, have among its means of production 2 additional one-year-old tractors and 2 additional new ones: these require for replacement 2 new tractors annually.

Since there are now at work an extra 4 tractors, whereas the quantity of wheat in the net product must remain unchanged, the former team of tractors must be reduced from 20 to 16, if the total number (20) is to remain as before: these 16 being still equally spread over the four ages, and requiring 4 (instead of 5) new tractors annually for replacement. Thus, although there are, as before, only 20 tractors at work, the output of 'new' (i.e. zero-age) tractors must be raised from 5 to 6 (i.e. 2+4) with the consequent employment by the system of 4 extra units of labour. No 'new' tractors are added to the net product (since all the 6 are required for the replacement of means of production) and the net product of the 4 units of labour is 2 two-year-old tractors.

82 If the rate of profits is zero, the criterion of equal depreciation-quotas for equal efficiency in successive years ensures equal prices for identical units of product, no matter what the age of the machines by which they are produced. But as soon as the rate of profits rises above zero, equal depreciation quotas would entail different charges (the 'charge' consisting of depreciation plus profit) on machines of different ages, since at any given rate of profits less would be payable for profits on the older and partly written-down machines; and therefore equal depreciation would be inconsistent with equal prices for all units of the product.

The equality of price can therefore only be maintained if the annual depreciation quotas are increased on the older machines relatively to the newer ones, so as to restore the equality of the charge at different ages. Thus, if we look at any one machine of a given age, the year's depreciation quota for it will change with the rise in the rate of profits. The sum, however, of the annual depreciation quotas over the whole lifetime of a machine must under all circumstances be constant, since it must be equal to its original price. The quotas for the later years must therefore rise by exactly as much as those for the earlier years fall.

Each depreciation quota is naturally equal to the difference between the values of the durable instrument in two consecutive years of its life. As a result the value of the instrument, instead of falling with age by equal annual steps, will as soon as a rate of profits emerges fall by steps which are bigger the higher is the age: and the higher the rate of profits the more will the steepness of the downward steps increase with age.

83 We now turn from the standpoint of the life-progress of a single machine to the standpoint of a complete range of n similar machines, each being one year older than the preceding one, and thus forming a group such as we might find in a self-replacing system. The requirement that the life-sum of the depreciation quotas should be constant and independent of the rate of profits is now embodied in the fact that under all circumstances such a group is maintained simply by bringing in a new machine every year.

But the redistribution over the various ages of this constant life-sum has the remarkable effect that with any rise in the rate of profits the value of the group as a whole *rises* relative to the original value of a new machine. This is the necessary result of the fact just noticed that with increasing age the value of a durable instrument falls by equal steps in successive years if the rate of profits is zero, but if the rate of profits is greater than zero the downward steps increase in size with age.

To see how this comes about, let us consider the position of an instrument which has reached any given age t out of a total life of n years. The sum of the steps by which its value has fallen during the first t years of its life is smaller if $r > 0$ than if $r = 0$; so that the sum of the steps by which it will fall to nothing in the remainder of its life, which is of course equal to its value at the present moment, is larger if $r > 0$ than if $r = 0$. By a similar reasoning it can be further seen that its value will not only be higher when $r > 0$, but will continue to rise with any increase of r.

There is, however, a limit to the rise in value of such an instrument even if the rate of profits were to rise without limit, and the limit to

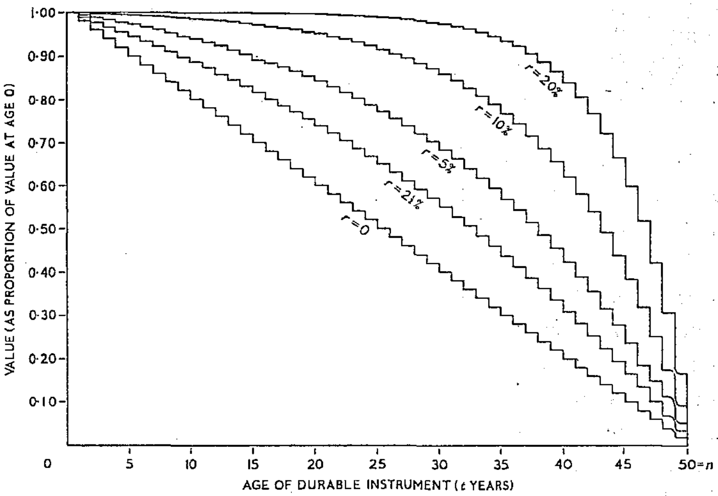

FIG. 6. Book value of a durable instrument at various rates of profit.

(The instrument is supposed to have a life of 50 years at constant efficiency.) Each stepped curve shows how, at a given rate of profits, the value of the instrument falls as its age increases. The area enclosed between each curve and the axes is proportional to the value of a set of 50 instruments of even age-distribution. Taking the value of a new instrument as unity, their aggregate value, which is 25 at $r = 0$, rises to 29·5 at $r = 2\frac{1}{2}$%, to 34 at 5%, to 39·5 at 10% and to 44 at 20%: it can of course never exceed 50.

which it tends is the value of a new instrument. If the total life of an instrument is n years, and its value when new is 1, at the age of t years its value is

$$\frac{(1+r)^n-(1+r)^t}{(1+r)^n-1}$$

and the range of variation of its value with the variation of r lies between $(n-t)/n$ and 1.

In the diagram above (Fig. 6), the ordinates represent the value, at each age, of a durable instrument, having a total life of 50 years, at various assumed levels of the rate of profits (r): and the area intercepted between each stepped curve and the axes represents the aggregate value of a complete set (or self-replacing group) of instruments of all ages. The value of such a set increases from $n/2$ to a maximum of n as the rate of profits increases from zero without limit.

This variation in the price of ageing machinery cannot be explained from the cost-of-production side. It arises exclusively from the necessity of maintaining, when the rate of profits changes, the equality in price of all units of the product whatever the differences in age of the instruments by which they are respectively produced.

Although the interest of this type of price-variation is chiefly from the standpoint of capital theory, its effect in the case of long-lived fixed-capital assets such as buildings can be appreciable.

Thus, when a number of plants are to be constructed in succession over a period of years the annual depreciation quotas of the first units put into operation are available for financing the construction of the subsequent units, and the early quotas will be larger the lower is the rate of profits: as a result, given the cost of building a plant, the total net investment required will be larger the higher is the rate of profits. In the example assumed in Fig. 6 the investment is proportional to the area between the relevant curve and the axes—an area which grows with the rise in the rate of profits.

84 In contrast with its intractability as regards 'reduction', fixed capital fits easily into the Standard system. What simplifies the matter is the circumstance that durable instruments as such do not necessarily involve negative multipliers.

Durable instruments, if basics, will have to be represented in the Standard commodity by specimens of the various ages in their due proportions. For example, consider a machine that has a life of three years, and suppose the Standard ratio to be 10%. The three processes employing machines of 0, 1 and 2 years of age will receive such multipliers as to result in the machines' entering the aggregate of the means of production of the three processes in the proportions of 100 two-year-old machines, 110 one-year-old ones and 121 new ones: hence, at the end of the year, the number of each age-group found in the product will exceed by 10 % the number of the same age that had been included in the means of production at the beginning of the year.

The similarity between the several processes which employ a durable instrument in its successive stages of wear will generally make it possible for the Standard system to be constructed by means of exclusively positive multipliers. As a result, a system which contained no other element of joint production besides what is implied in the presence of fixed capital would in general have an all-positive Standard commodity, thus reproducing in this respect the simplicity of the system of single-product industries.

CHAPTER XI

LAND

85 Natural resources which are used in production, such as land and mineral deposits, and which being in short supply enable their owners to obtain a rent, can be said to occupy among means of production a position equivalent to that of 'non-basics' among products. Being employed in production, but not themselves produced, they are the converse of commodities which, although produced, are not used in production. They are, in fact, already included under the wider definition of non-basics given in §60.

The similarity of rent-earning natural resources with non-basic products shows itself at once in the impossibility of their being counted among the components of the Standard product, since they appear on one side only of the production process. And as for the other property of non-basics with regard to taxation, it is hardly necessary to dwell on the doctrine that 'taxes on rent fall wholly on landlords' and thus cannot affect the prices of commodities or the rate of profits—a conclusion which could be proved in the present setting by merely repeating the argument used in the case of non-basic products (§65).

86 If n different qualities of land are in use, they will give rise to an equal number of different methods of producing corn (supposing, at first, corn to be the only agricultural product). There will therefore be n production-equations, to which must be added the condition that one of the lands pays no rent;[1] and to these equations there will correspond an equal number of variables representing the rents of the n qualities of land and the price of corn.

Only the process that produces corn on the no-rent land can enter into the composition of the Standard system, since the no-rent land

[1] By this token only can it be identified as the least productive land in use (cf. p. 75).

itself is eliminated from the equation, along with all other 'free' natural resources which, although necessary to production, are not reckoned among the means of production.

In setting up the production-equations, the C's will represent quantities of corn, $\Lambda_1, \Lambda_2, \ldots, \Lambda_n$ the different lands and $\rho_1, \rho_2, \ldots, \rho_n$ the respective rents; among these quantities, the ρ's are the unknowns. (Note that the suffixes are arbitrary and do *not* represent the order of fertility, which is not defined independently of the rents; that order, as well as the magnitude of the rents themselves, may vary with the variation of r and w.) The equations which, as part of the general system, represent the production of corn are as follows:

$$
\begin{aligned}
(A_{c_1}p_a + \ldots + C_{c_1}p_c + \ldots + K_{c_1}p_k)(1+r) + L_{c_1}w + \Lambda_1\rho_1 &= C_{(1)}p_c \\
(A_{c_2}p_a + \ldots + C_{c_2}p_c + \ldots + K_{c_2}p_k)(1+r) + L_{c_2}w + \Lambda_2\rho_2 &= C_{(2)}p_c \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\
(A_{cn}p_a + \ldots + C_{cn}p_c + \ldots + K_{cn}p_k)(1+r) + L_{cn}w + \Lambda_n\rho_n &= C_{(n)}p_c
\end{aligned}
$$

and the condition that one of the rents should be zero can be written

$$\rho_1\rho_2\ldots\rho_n = 0$$

the relevant solution being always the one in which the ρ's are $\geqslant 0$.

87 If land is all of the same quality and is in short supply, this by itself makes it possible for two different processes or methods of cultivation to be used consistently side by side on similar lands determining a uniform rent per acre. While any two methods would in these circumstances be formally consistent, they must satisfy the economic condition of not giving rise to a negative rent: which implies that the method that produces more corn per acre should show a higher cost per unit of product, the cost being calculated at the ruling levels of the rate of profits, wages and prices.

The production of corn would thus be represented in the general system by two equations with the two corresponding variables of the rent of land and the price of corn.

Both equations would enter the Standard system, although with coefficients of opposite signs and of such values as would in the aggregate eliminate the land from the means of production of that system.

88 While the case of lands of different qualities will be readily recognised as the outcome of a process of 'extensive' diminishing returns, it may be less obvious that a similar connection exists between the employment of two methods of producing corn on land of a single quality and a process of 'intensive' diminishing returns.

From this standpoint the existence side by side of two methods can be regarded as a phase in the course of a progressive increase of production on the land. The increase takes place through the gradual extension of the method that produces more corn at a higher unit cost, at the expense of the method that produces less. As soon as the former method has extended to the whole area, the rent rises to the point where a third method which produces still more corn at a still higher cost can be introduced to take the place of the method that has just been superseded.[1] Thus the stage is set for a new phase of increase in production through the gradual extension of the third method at the expense of the intermediate one. In this way the output may increase continuously, although the methods of production are changed spasmodically.

While the scarcity of land thus provides the background from which rent arises, the only evidence of this scarcity to be found in the process of production is the duality of methods: if there were no scarcity, only one method, the cheapest, would be used on the land and there could be no rent.

89 More complex cases can generally be reduced to combinations of the two that have been considered. The main type of complication arises from the multiplicity of agricultural products.

Thus, suppose that in the first case land of one quality was so exceptionally well-suited for one particular crop, that such a crop was grown on the whole of that land and on no other land; under these circumstances there would be room for two different methods of producing the crop in question on that land, and its rent would be determined independently of that of the other lands, becoming in effect an instance of the second case.

[1] The change in methods of production, if it concerns a basic product, involves of course a change of Standard system; see below, ch. XII.

Or consider the more general case in which each of several qualities of land can be used for several alternative crops, although none of the crops is grown on land of all the qualities; while on the other hand none of the lands is sufficiently specialised to have its rent determined independently of the others. What is required in any case is that the number of separate processes should be equal to the number of qualities of land plus the number of products concerned; and, moreover, that the links or overlaps between the various products and the various lands on which they are grown should be sufficient for the determination of the rents and of the prices. The type of link required may be sufficiently indicated by the consideration that the above condition would be satisfied if the links were such as to make possible the construction of a Standard commodity from which were excluded all the lands as well as any non-basics among the products.

In the case of a single quality of land, the multiplicity of agricultural products would not give rise to any complications. It may however be noted that only for *one* of the crops would two separate methods of production be compatible; for the rest, the number of processes would have to be equal to the number of products.

90 We must now turn back to reconsider, in the light of the discussion of rent, a distinction made in an earlier chapter.

We have just seen that, where rent arises from the use of a single quality of land, negative coefficients will be involved in the construction of the Standard system (although this will not necessarily happen in the case of 'differential' rent from lands of unequal fertility) with the consequent possibility of negative quantities among the components of the Standard commodity. Now this possibility of negative components is the characteristic feature of what we have called the 'multiple-products system' and is also the chief cause of its limited usefulness as a conception in contrast with the system of 'single-product industries'. It is therefore disconcerting to see it appear in a case where each of the processes produces a single commodity.

The fact is that the introduction of means of production which are not themselves produced, by rendering possible a multiplicity of processes producing the same commodity even though each process has no more than one product, has disrupted our distinction between the two types of system, making its reconstruction necessary.

To effect such reconstruction we must in the first place re-define a 'system' as a set of industries, or methods of production, equal in number, not as formerly to the different products, but to the different things that are produced and/or used as means of production. Besides, the properties which we had attributed to the system of 'single-product industries' must be transferred to a system in which each commodity is produced by *not more than one* method; and the properties of the system of 'multiple-product' industries must be transferred to a system in which at least one commodity is produced by *more than one* method, even though all industries were single-product industries. (This need not affect what was said in the previous chapters, since the two distinctions coincide up to the appearance of means of production which are not themselves produced.)

91 Machines of an obsolete type which are still in use are similar to land in so far as they are employed as means of production, although not currently produced. The quasi-rent (if we may apply Marshall's term in a more restricted sense than he gave it) which is received for those fixed capital items which, having been in active use in the past, have now been superseded but are worth employing for what they can get, is determined precisely in the same way as the rent of land. And like land such obsolescent instruments have the properties of non-basics and are excluded from the composition of the Standard commodity.

PART III

SWITCH IN METHODS OF PRODUCTION

CHAPTER XII

SWITCH IN METHODS OF PRODUCTION

92 We have been assuming that in a system of single-product industries only one way of producing each commodity is available, with the result that changes in distribution can have no effect on the methods of production employed.

Suppose now that, for the production of one of the commodities, two alternative methods are known. And, to take the simpler case first, suppose that the commodity in question is a *non-basic* product.

At any given level of the general rate of profits,[1] the method that produces at a lower price is of course the most profitable of the two for a producer who builds a new plant.

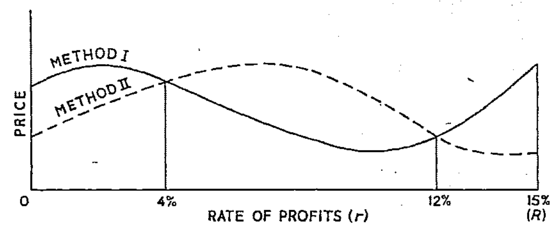

FIG. 7

The two curves in Fig. 7 show how the price of the commodity as produced by the two alternative methods varies with the rate of profits (the price, or cost of production, being expressed in terms of an arbitrarily chosen standard). The points of intersection where the prices are equal correspond to the switching from one to the other method as the rate of profits changes. There may be one or more such intersections within the range of possible rates of profit, by analogy with what we have seen in the case of two distinct commodities (§48);

[1] The rate of profits is taken as the independent variable in this connection; but the argument would not be affected if the wage, expressed in any given commodity or composite commodity, were taken instead.

if on the other hand there is no intersection, one of the methods is unprofitable in all circumstances and may be disregarded.

93 If the product is a basic one, the problem is complicated by the circumstance that each of the two alternative methods of producing it implies a distinct economic system, with a distinct Maximum rate of profit. As a result we seem to lack a common ground on which the comparison between the two methods can be carried out: since, according as one or the other method is used, we are in one or the other economic system, and to any given rate of profits there will correspond, in each system, a different wage, even though in the same standard, and a different set of relative prices; as a consequence a comparison of the prices by the two methods becomes meaningless since its result appears to depend on which commodity is chosen as standard of prices.

Two different methods of producing the same basic commodity can only co-exist at the points of intersection (that is to say at those rates of profits at which the prices of production by the two methods are equal), since the two economic systems (which are respectively characterised by the two methods, but are alike in every other respect) will at such points necessarily have also the same commodity-wage[1] and the same system of relative prices.

This co-existence is possible because with k basic equations (representing k methods of production) and $k+1$ unknowns (representing $k-1$ prices, the wage w and the rate of profits r) there is room for one more basic equation (or method of production) even though it does not bring with it an additional product and an additional price. With $k+1$ methods of production, however, it is no longer possible to vary at will the rate of profits, its level being now fully determined. At any other level of the rate of profits the two methods are incompatible, and the two distinct systems to which they belong have no point of contact.

Yet, if the two methods are to present themselves as alternatives, a comparison must be possible within the same system even at rates

[1] It may be noticed that, while the commodity-wage is the same at such points, yet it will be equivalent to different *proportions* of the respective Standard net products of the two systems: since to each of the two systems there will correspond a different value of R.

of profits at which the two methods are incompatible. This can be accomplished if we assume for a moment that the products of the two methods are two distinct commodities which, however, have such properties that, while for all possible *basic* uses they can be regarded as identical and are completely interchangeable, there are other, *non-basic* uses, some of which require the one, and some the other, of the two products, without possibility of interchange. The result is that for all basic uses the choice between the two methods will be exclusively grounded on cheapness; and at the same time the special non-basic uses will ensure that both methods are always employed to some extent, whatever the system.

Suppose that the commodity in question is copper and that it can be produced by two methods which we shall call I and II and which characterise respectively systems I and II with different Maximum rates of profits R_{I} and R_{II}. The products of the two methods (copper I and copper II) are, for basic uses, the same commodity produced in different ways. We can therefore assume either that we are in system I and regard copper II as non-basic, or that we are in system II with copper I as non-basic (and vice versa for basic).

The two assumptions will give different results, for in general to any given rate of profits, say 5 %, there will correspond in each of the two systems a different wage and a different set of relative prices; and according as one or the other assumption is made the cost-ratio between copper I and copper II will be different.

It can however be shown that, while the *extent* of the cheapness of one method of production relatively to the other will vary according as the comparison is carried out in system I or in system II, the *order* of the two methods as to cheapness must be the same in the two systems. In effect, as we shall see (§ 94), it is always the method whose product (say, 'copper II') is basic in the system which has the higher value of R that, in the upper reaches of the rate of profits,[1] is the cheapest in both systems. As the rate of profits is reduced, any change in the order of cheapness must apply equally to the two systems, since it involves going through a point of intersection and such points are common to both.

[1] I.e. above the rate corresponding to the highest point of intersection.

94 We have seen that as the rate of profits rises there may be several intersections between the prices at which the two methods produce, with as many switchings backwards and forwards from one method to the other and consequently from one system to the other.

In view of this possibility we cannot (contrary to what one might have expected) say in general that, of two alternative methods of production, the one that corresponds to a Standard system with a higher ratio of product to means of production (i.e. with a larger R) will be the most profitable when the rate of profits is comparatively high, and the least profitable when it is comparatively low.

There is, however, one statement of general validity that can be made in this connection. But for this purpose it is convenient to transfer our attention from the two methods of producing the commodity in question to the two corresponding economic systems.

From such a standpoint it is evident that at rates of profits which are intermediate between R_{I} and R_{II} (where R_{II} is larger than R_{I}) there can be no points of intersection, since over that range, while the wage (w) of system II would continue positive, in system I w would assume zero or negative values. (That is to say, over that range copper II would be, not merely the most profitable, but the only possible one as a basic.)

Since in the higher ranges of the rate of profits (i.e. between R_{I} and R_{II}) the method which corresponds to the higher Standard ratio of product to means of production is the only possible one for the basic product, it follows that *if* the two methods have a single point of intersection, the only possible switch as the rate of profits rises is from a lower to a higher Standard ratio of product to means of production (i.e. from a lower to a higher value of R).

The position can be illustrated by a diagram (Fig. 8) which exhibits the relation between the rate of profits and the wage in each of two systems (I and II) which, while being similar in all other respects, differ in so far as one uses method I while the other uses method II for producing one of the basic products.

The two lines show, for the respective systems, how the wage falls while the rate of profits rises from zero to its maximum value (which is $R_{\mathrm{I}} = 15\,\%$ for the first system and $R_{\mathrm{II}} = 16\,\%$ for the second).

A common standard being necessary for a comparison, the wage of *both* systems is expressed in terms of the Standard commodity of system II.[1] As a result the relation is represented by a straight line for system II and by a curve for system I. (This would of course be reversed if the Standard commodity of system I were adopted as common standard.) The point of intersection, at $r = 10\%$, is where the two alternative methods of production are equally profitable; beyond that point, with a further rise of the rate of profits, it becomes profitable to switch from method I to method II.

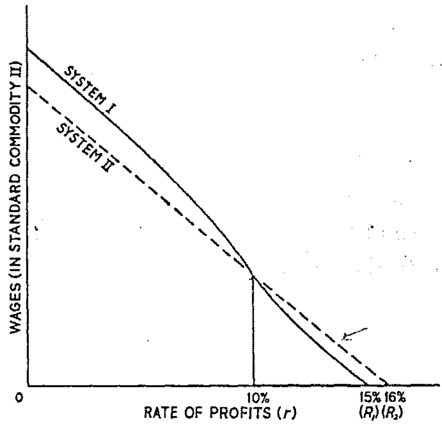

FIG. 8

95 We can now extend the supposition of an alternative method for producing one commodity and suppose that there are many such alternatives with at least as many distinct points of intersection; and not only for one of the products, but for each of them. So that as the rate of profits rises there will be a rapid succession of switches in the methods of production of one or other of the commodities.

Throughout such a series of changes, although the value of R may move alternately up and down, to each rise in the rate of profits there will invariably correspond (with systems of single-product

[1] It may be noticed that, although the composition of the Standard commodity in system I will in general be quite different from that of system II, yet all the commodities entering the latter can be produced in system I, even though some of them may appear in this system merely as non-basic products.

industries) a fall in the wage measured in terms of any commodity. This is because changes in the rate of profits and in the wage always take place *within* one system, so that the movements of the two are bound to be in opposite directions; whereas the switch from one method to the other (and consequently from one system to the other) entails no change in either the rate of profits or the wage—on the contrary, it becomes possible at a *point* of intersection between the old and the new systems, and therefore at *given* levels of the wage and of the rate of profits.

96 With single-product industries, each process or method of production is identified by the commodity which it produces, so that when an additional, $(k+1)$th, method is introduced there is no doubt as to which of the pre-existing methods it is an alternative to. When, however, each process or method produces several commodities, and each commodity is produced by several methods, this criterion fails. And the problem arises of how to identify among the pre-existing methods the one to which the new method is an alternative.

We first define the equivalent, for the case of multiple-product industries, of the rate of profits at which the intersection between the two price-curves of the single-product industries takes place: such equivalent is that rate of profits at which *each* of the k commodities is produced, whether by the new method or by the old ones, at the same price.

Our problem is to spot the method that will be superseded when the rate of profits rises beyond that point. In doing this we proceed in a somewhat roundabout way. We begin by turning our attention away from the individual methods of joint-production and concentrate upon the possible systems which are respectively defined by the *absence* of one of the methods from among their components. With $k+1$ methods (or processes) we can form k different systems of k processes, all of the systems including the new method and each of them omitting in turn one of the k old methods.

Suppose now that the rate of profits is raised by a very small fraction above that point. For all the k systems the resulting wage will be lower than before:[1] but it will be different for each of the systems (although expressed in the same standard). Consider the system which at the newly given rate of profits allows of the highest wage: if we regard the wage, instead of the rate of profits, as being given, we shall find that this system will also be the most profitable one since, given *any* of those wages, it will allow the payment of a higher rate of profits than does any other system. Now this system is distinguished by the *absence* from among its constituents of one particular method of production, which is present in all the other systems. That particular method is thus shown to be the least profitable to employ in the new circumstances, and is therefore the one that will be superseded by the new method.

[1] We assume here (and it is essential for the conclusion) that no commodity's price behaves in the peculiar way described in §§71–2.

APPENDIX A

ON 'SUB-SYSTEMS'[1]

Consider a system of industries (each producing a different commodity) which is in a self-replacing state.

The commodities forming the gross product (i.e. all quantities on the right-hand side of the equations in §11) can be unambiguously distinguished as those which go to replace the means of production and those which together form the net product of the system.

Such a system can be subdivided into as many parts as there are commodities in its net product, in such a way that each part forms a smaller self-replacing system the net product of which consists of only one kind of commodity. These parts we shall call 'sub-systems'.

This involves subdividing each of the industries of the original system (namely, the means of production, the labour and the product of each) into parts of such size as will ensure self-replacement for each sub-system.

Although only a fraction of the labour of a sub-system is employed in the industry which directly produces the commodity forming the net product, yet, since all other industries merely provide replacements for the means of production used up, the whole of the labour employed can be regarded as directly or indirectly going to produce that commodity.

Thus in the sub-system we see at a glance, as an aggregate, the same quantity of labour that we obtain as the sum of a series of terms when we trace back the successive stages of the production of the commodity (ch. VI).

At each level of the wage and of the rate of profits, the commodity forming the net product of a sub-system is equal in value to the wages of the labour employed plus the profits on the means of production. And when the wage absorbs the whole net product, the commodity is equal in value to the labour that directly or indirectly has been required to produce it.

[1] Cf. §14.

means of prod + w's + profits

NP_A = w's + profits

= L.w

APPENDIX B

NOTE ON SELF-REPRODUCING NON-BASICS[1]

Consider a commodity which enters to an unusually large extent into the production of itself. It may be imagined to be some crop such as a species of beans or of corn the wastage on which is so great that for every 100 units sown no more than 110 are reaped. It is clear that this would not admit of a rate of profits higher than, or indeed, since other means of production must be used as well, as high as, 10 %.

If the product in question is a basic one there is no problem; it simply means that the Maximum rate of profits of the system will have to be less than 10 %.

If however it is a non-basic product, complications arise. The way in which a non-basic is produced has, as we have seen, no influence on the general rate of profits, so that there would be nothing to prevent the Maximum rate of the system being higher than 10 %: and yet the product in question is incompatible with a rate as high as 10 %. This contradictory situation finds its outlet in the behaviour of the price of the product (we shall call it 'beans') when the wage is reduced. As the rate of profits in its rise approached 10 % the price of the beans would have to increase without limit since of the 10 units left over after replacing the seed more and more would be required for the profits on the seed itself, while the quantity which remained available for paying for the replacement of the other materials used, plus their profits, would approach vanishing-point.

That point would be reached at 10 %, when replacement of the other materials would be possible only if they were to be had for nothing, i.e. if the relative price of the beans was infinite.

When the rate of profits was above 10 % the conditions could be satisfied only if the particular p which represented the price of beans assumed a negative value. (The resulting situation could be visualised as a sort of fairyland in which, the product being insufficient even to replace the beans used up and pay in full the profit on them, a quantity of these would have to be 'bought' for the purpose and, as 'negative price', goods

[1] Cf. footnote to §39.

sufficient to replace the other means of production, with profits, would have to be received in addition.)

A simplified version of the 'beans' example is shown in the diagram, where the Maximum rate of profits is assumed to be 15 % and the price is expressed in Standard commodity. The price curve consists of *both* branches of a rectangular hyperbola which has for asymptotes the rate-of-profits axis and the parallel to the price-axis that passes through the 10 % point.

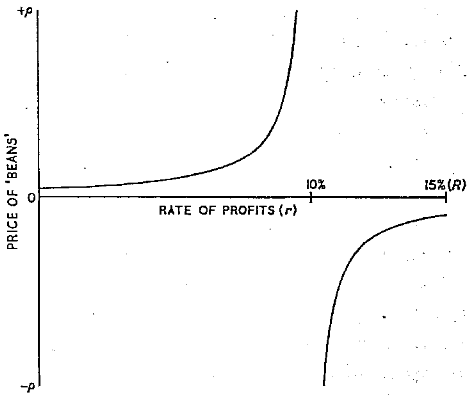

Fig. 9

The situation in which the price of beans p becomes infinite (at 10 % rate of profits) can also be described, if beans are taken as the standard of price, as one in which the price of every other commodity is zero: this gives a formal solution of the equations. But if we take as standard of price a basic commodity, it is impossible for every other price to become zero, since there must be at least one other commodity in the means of production of which that basic commodity enters. So that the corresponding situation, in which the price of a commodity becomes negative by passing through infinity, cannot occur in the case of a basic.

It is perhaps as well to be reminded here that we are all the time concerned merely with the implications of the assumption of a uniform price for all units of a commodity and a uniform rate of profits on all the means of production. In the case under consideration, if the rate of profits were at or above 10% it would be impossible for these conditions to be fulfilled. The 'beans' could however still be produced and marketed so as to show a normal profit if the producer sold them at a higher price than the one which, in his book-keeping, he attributes to them as means of production.

APPENDIX C

THE DEVICE OF A 'BASIC SYSTEM'[1]

This is a footnote to §§62–3, intended to explain briefly why, in constructing the Standard product for the multiple-product equations, it has been found advisable to transform these, as a preliminary, into Basic equations, rather than operate directly on the original system.

The object of the exercise is to identify the particular value of R which is appropriate from the economic standpoint. Once the non-basic commodities are eliminated (as is done through the Basic equations) this can be defined as the lowest of the possible values of R.

If, however, the elimination were not effected, additional values of R would arise owing to the presence of such non-basics as entered both the product and the means of production. Values of R of this type would have the peculiarity that the corresponding prices of all commodities would be zero (with the exception, for each value of R, of the prices of one non-basic or of a group of interconnected non-basics). Such values of R are meaningless from the standpoint of an economic system and must be rejected. One of them, however, might be the lowest of all (as in the example given in Appendix B in connection with the single-products system) and the mere possibility of this would invalidate the criterion by which the economically relevant value of R is identified. To get over this it would be necessary to distinguish the two groups of values of R on the grounds of the peculiarity described above; a procedure which seemed even more cumbrous than the one adopted in the text.

[1] Cf. §63n.

APPENDIX D

REFERENCES TO THE LITERATURE

1 The connection of this work with the theories of the old classical economists has been alluded to in the Preface. A few references to special points, the source of which may not be obvious, are added here.

It is of course in Quesnay's *Tableau Economique* that is found the original picture of the system of production and consumption as a circular process, and it stands in striking contrast to the view presented by modern theory, of a one-way avenue that leads from 'Factors of production' to 'Consumption goods'.

A method devised by Ricardo (if the interpretation given in our Introduction to his *Principles* is accepted)[1] is that of singling out corn as the one product which is required both for its own production and for the production of every other commodity. As a result, the rate of profits of the grower of corn is determined independently of value, merely by comparing the physical quantity on the side of the means of production to that on the side of the product, both of which consist of the same commodity; and on this rests Ricardo's conclusion that 'it is the profits of the farmer that regulate the profits of all the other trades'. Another way of saying this, in the terms adopted here, is that corn is the sole 'basic product' in the economy under consideration.

(It should perhaps be stated that it was only when the Standard system and the distinction between basics and non-basics had emerged in the course of the present investigation that the above interpretation of Ricardo's theory suggested itself as a natural consequence.)

Ricardo's view of the dominant role of the farmer's profits thus appears to have a point of contact with the Physiocratic doctrine of the '*produit net*' in so far as the latter is based, as Marx has pointed out,[2] on the 'physical' nature of the surplus in agriculture which takes the form of an excess of food produced over the food advanced for production; whereas in manufacturing, where food and raw materials must be bought from agriculture, a surplus can only appear as a result of the sale of the product.

[1] In Ricardo's *Works and Correspondence*, I, xxxi–xxxii.

[2] *Theorien über den Mehrwert*, I, 36 and III, 134, note.

2 The conception of a standard measure of value as a medium between two extremes (§ 17 ff.) also belongs to Ricardo[1] and it is surprising that the Standard commodity which has been evolved from it here should be found to be equivalent to something very close to the standard suggested by Adam Smith, namely 'labour commanded'[2] (§ 43), to which Ricardo himself was so decidedly opposed.

3 The notion of a Maximum rate of profits corresponding to a zero wage has been suggested by Marx, directly through an incidental allusion to the possibility of a fall in the rate of profits 'even if the workers could live on air';[3] but more generally owing to his emphatic rejection of the claim of Adam Smith and of others after him that the price of every commodity 'either immediately or ultimately' resolves itself entirely (that is to say, without leaving any commodity residue) into wage, profit and rent[4]—a claim which necessarily presupposed the existence of 'ultimate' commodities produced by pure labour without means of production except land, and which therefore was incompatible with a fixed limit to the rise in the rate of profits.

4 The plan of treating what is left of fixed capital at the end of the year as a kind of joint-product may seem artificial if viewed against the background of the continuous flow of industrial production, but it fits easily into the classical picture of an agricultural system where the annual product, in Adam Smith's words, naturally divides itself into two parts, one destined for replacing a capital, the other for constituting a revenue.[5] Adam Smith, however, excludes fixed capital from the annual product.[6] It was only after Ricardo had brought to light the complications which the use of fixed capital in various proportions brings to the determination of values that the plan in question was resorted to. It was first introduced by Torrens in the course of a criticism of Ricardo's doctrine. In explaining his own peculiar theory according to which 'the results obtained from the employment of equal capitals are of equal value', Torrens shows by means of examples that his theory is verified if only 'the results' are regarded as

[1] See *Works*, I, xliv.

[2] *Wealth of Nations*, bk. I, ch. v; Cannan's ed., I, 35.

[3] *Capital*, vol. III, ch. 15, sec. ii, Kerr's ed. p. 290.

[4] *Capital*, vol. III, ch. 49, pp. 979, 981 ff., referring to the *Wealth of Nations*, bk. I, ch. vi; Cannan's ed., I, 52.

[5] *Wealth of Nations*, bk. II, ch. iii; I, 315.

[6] Bk. II, ch. ii; I, 272.

including, besides the product in the ordinary sense of the word, e.g. 'the woollens', also 'the residue of the fixed capital employed in their manufacture'.[1]

Thereafter the method was generally adopted, even by the opponents of Torrens's theory: first by Ricardo in the next edition of his *Principles*,[2] then by Malthus in the *Measure of Value*[3] and later by Marx,[4] but afterwards it seems to have fallen into oblivion.

[1] 'Strictures on Mr Ricardo's Doctrine Respecting Exchangeable Value', in *Edinburgh Magazine*, Oct. 1818, p. 336; cf. *An Essay on the Production of Wealth*, by Robert Torrens, 1821, p. 28.

[2] In a passage in which the value of the 'corn' is compared with that of 'the machine and cloth of the clothier together', 3rd ed. (1821), (Ricardo's *Works*, I, 33).

[3] Published in 1823, p. 11; see also the posthumous 2nd ed. of Malthus's *Principles of Political Economy* (1836), p. 269.

[4] *Capital*, vol. I, ch. 9, sec. i, Moore and Aveling transl. p. 195, quoting Malthus; and cf. the quotation from Torrens in *Theorien über den Mehrwert*, III, 77.