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On the Elasticity of Demand for Income in Terms of Effort

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1. It is a generally accepted proposition of theoretical economics that the effects of a change in the terms on which incomes from work can be obtained depend upon the elasticity of demand for income in terms of effort.[1] If the elasticity of demand for income in terms of effort is greater than unity, then the effects of a tax or a fall in wage rates will be a diminution of work done and the effects of a bounty or a rise in wage rates will be an increase in work done. If it is less than unity, then the opposite movements are to be expected.

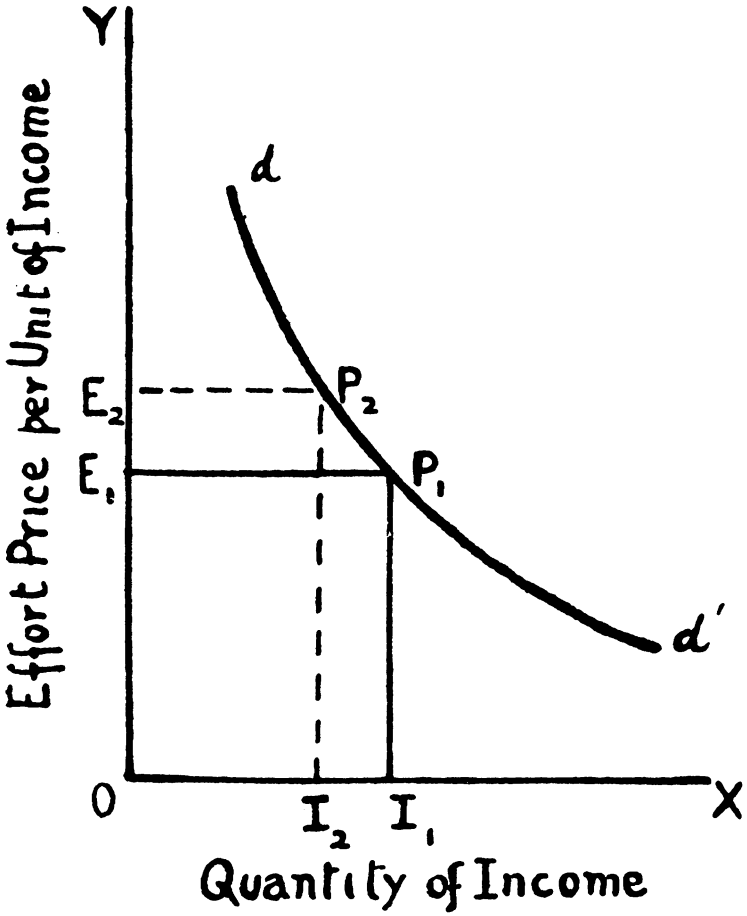
2. These propositions are capable of demonstration by the familiar geometrical constructions of either (*a*) unit or (*b*) integral demand curves. The only difference between the constructions relevant here and those of commodity price analysis is that the prices exhibited will be, not money, but effort prices.

(*a*) Thus, if we employ the unit demand apparatus, we measure quantity of income demanded along O X and the effort price of income along O Y. The curve d d^1 exhibits the conditions of demand, and the quantity of work done for any given income will be shown by a rectangle formed by erecting perpendiculars on O X and O Y to cut any point of equilibrium (P) in d d^1. If e.g. the effort price of income is O E_1 then the quantity of income which will be earned will be O I_1, and the amount of work done will be O E_1 P_1 I_1.

The effects of a change in the terms on which income can be obtained can be shown by shifting E. Let us suppose for instance the imposition of a uniform income-tax which shifts E from E_1 to E_2. Then the quantity of income earned will shift

[1] See Dalton, *Public Finance*, Second Edition, pp. 100-108, or Robertson, *Banking Policy and the Price Level*, Chapters I and II *passim*. It is possible, of course, to reformulate this proposition in terms of the elasticity of supply of effort, and for some purposes it is convenient to do so. But there is much to be said for exhibiting all psychological variables as phenomena of demand. See Wicksteed, *Commonsense of Political Economy*, Book II, Chapter IV, and "The Scope and Method of Political Economy," *Economic Journal*, 1913, pp. 1 *seq.*

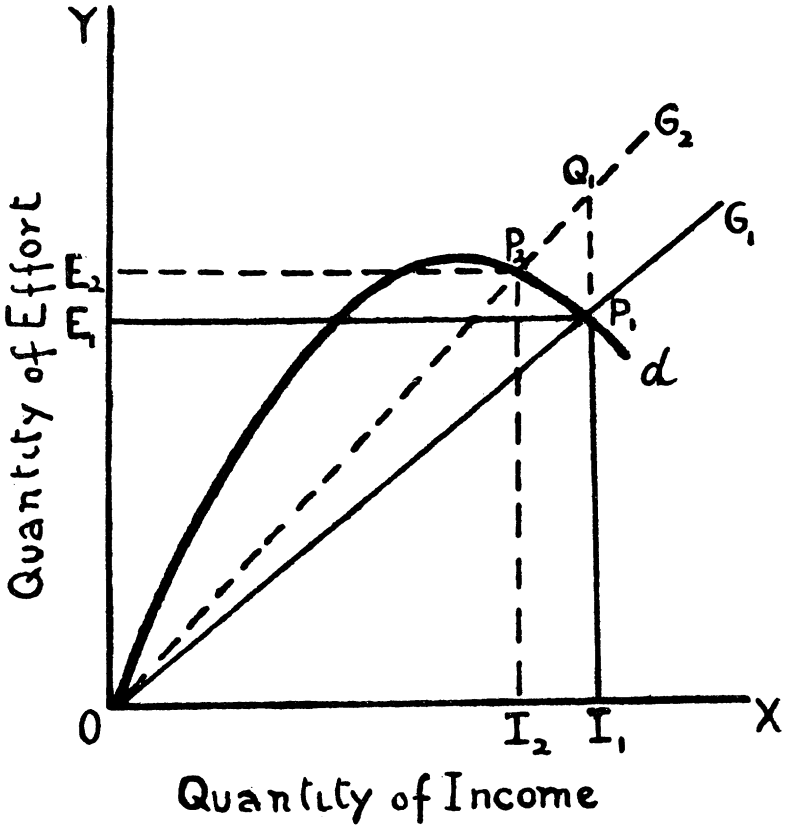
to I_2 and the change in the amount of work done will be shown by the difference between E_1 P_1 I_1 O and E_2 P_2 I_2 O. If in this region d d^1 shows an elasticity greater than one this difference will be negative (i.e. less work will be done). If it is less than one the difference will be positive (i.e. more work will be done).



(*b*) The integral apparatus shows the same result with even greater clarity. Along O X we continue to measure quantity of income. Along O Y, however, we measure the total amounts of effort which will be expended for different quantities of income. (That is to say, what was a rectangle on the unit apparatus has become a line on this apparatus.) O d is the total demand curve. The terms on which income can be obtained will

evidently be an angular magnitude, the tangent of the angle G_1 O X. Thus in the case depicted if the terms on which income can be obtained are represented by tan G_1 O X we get equilibrium at P_1 with O I_1, income earned for an expenditure of O E_1 effort.

Now suppose a tax imposed. We may represent this by



swinging O G to the left. (To get O I_1 before, it was necessary to expend I_1 P_1 effort. Now it is necessary to expend I_1 Q_1.) Equilibrium is re-established at P_2 with O I_2 income earned and O E_2 income expended. Since, in this region, O d shows an elasticity less than unity, O E_2 is greater than O E_1 (i.e. more work is done).

3. The propositions thus analysed are purely formal in character. They explain what will happen if the conditions of

demand are of a certain nature. To discover what the conditions of demand are in any particular case, it is generally supposed that we must rely upon observation. We cannot predict *a priori* what the effects of a change in taxation or of a change in wage rates will be; we must ascertain the probable elasticity of demand for income in terms of effort of the taxpayers or wage-earners concerned.

4. In recent years, however, propositions have been advanced which suggest that formal analysis enables us to predict that the elasticity of demand in the case of effort demand for income must always be less than unity—that is to say that the imposition of a tax will always have the effect of making a man work more, and a rise in his wage rates will always make him work less. If these propositions were true, they would obviously be of the highest *practical* importance—the effect on output of higher taxation need have no terrors for needy Chancellors of the Exchequer—and since they have been advanced by authorities no less eminent than Professor Pigou and Professor Knight, they clearly deserve the very closest attention.

5. The arguments of both the authorities mentioned involve in one form or another implicit appeal to the " law "or assumption of the declining marginal utility of units of income. Now *prima facie* it is difficult to see how this "law" or assumption justifies the inferences which appear to be based on it. *The assumption that, as income increases, the utility to an individual of additional units declines, justifies us indeed in inferring that the curve which exhibits the condition of demand for income in terms of effort will slope downwards, but it does not seem to justify the assumption that this curve must always cut a rectangular hyperbola negatively (i.e. that it must show an elasticity less than one at all reaches).* The assumption or " law " lays it down that the final degree of utility diminishes, but it does not *prima facie* say anything about the *rate* of diminution.

6. But let us examine more closely the actual arguments concerned. Professor Knight's is the more general and will be taken first.

Professor Knight's argument concerns the effect of a change in wage rates. " In so far as men act rationally," he argues,[2] ". . . they will at a higher rate divide their time between wage earning and non-industrial uses in such a way as to earn *more money* but to work *fewer hours.*" And he justifies this proposition by the following reasoning. " Suppose that at a

[2] *Risk, Uncertainty, and Profit*, pp. 117-18.

higher rate per hour or per piece, a man previously at the perfect equilibrium adjustment works as before and earns a proportionately higher income. When, now, he goes to spend the extra money, he will naturally want to increase his expenditure for many commodities consumed and to take on some new ones. To divide his resources in such a way as to preserve equal importance of equal expenditures in all fields he must evidently lay out part of his new funds for increased leisure; i.e. buy back some of his working time or spend some of his money by the process of not earning it."

At first sight the argument appears overwhelmingly convincing, sufficient even to overcome the reflection of commonsense that, if it were true, it would follow that it would always be futile to offer rational men permanently higher wages if it was desired to elicit a permanently increased supply of work. But closer inspection seems to reveal a flaw. Professor Knight's argument assumes that the prices of the commodities constituting real income are unaltered. This is presumably true so far as money prices are concerned. But the relevant conception in this connection is not *money price* but *effort price,* and a change in the rate at which money income can be earned, money prices remaining constant, *constitutes* a change in the effort price of commodities. *The money price is the same but the effort price is diminished.* And, that being the case, the question whether more or less effort is expended on commodities is obviously still an open one. It depends on the elasticity of demand for income in terms of effort.

This may sound abstract, but if it is thought of in concrete terms, it becomes very simple. If real income be conceived as consisting of a flow of one commodity, say, bananas, and the process of producing bananas as an exchange of effort for income, then it is surely clear that, if for some reason the effort price of bananas (real income) diminishes (a change equivalent to a rise in money wage rates), it is entirely a matter of elasticity of demand for bananas (real income) whether more or less effort is given for them, just as, if the money price of bananas changes, it is entirely a matter of elasticity whether more or less money is given for them.

The same objection can be put yet another way. In Professor Knight's example leisure is purchased by sacrificing income. We may therefore conceive—as he does—of a *real income price of leisure.* Now when the money rate of wages rises (commodity prices remaining the same) the real income price of leisure (the

cost of leisure in terms of real income sacrificed) rises. And when the price of leisure (or anything else) rises it is not at all clear that more will be bought even out of an increased real income. Again it is all a matter of elasticity.

7. Professor Pigou's proposition[3] relates to the effect of a tax on the willingness to work of an individual. " Since a part of his income is taken away, the last unit of income will be desired more urgently than the last unit of income that would have been left to him if there had been no taxation. But the last unit of energy that he devotes to work will not affect him differently from what it did, consequently there will be a tendency for him to work a little harder. . . ." Elsewhere this is put even more succinctly. " Since income is taken away from taxpayers the marginal utility of money to them is raised but the marginal disutility of work is unchanged. Hence, unless they are somehow impeded, they will increase the amount of work done."[4]

With very great deference it is submitted that this mode of argument proves much too much. This can be seen very readily if the argument be made completely general.

Suppose a man to be in receipt of a constant flow of exchangeable goods of any sort, say, corn. (The constant flow here is equivalent to the constant flow of disposable time which is assumed when variations in the supply of work are being considered.) Suppose that he is in the habit of exchanging some of this corn for a constant flow of some other kind of goods, say, coal (the constant flow of coal procured is equivalent to the constant flow of real income which can be earned by exchanging time for product). Suppose now that a tax is imposed which makes the price of coal higher (i.e. which makes the effort price of real income higher), would Professor Pigou acquiesce in an argument which ran as follows? The marginal utility of coal (real income) is now greater. But the marginal disutility of parting with corn (time) is unchanged. He will therefore, unless impeded, part with more corn (time). Such an argument seems plainly fallacious : it implies that the elasticity of demand for *any* commodity is less than unity. But in what way is it different from the argument on which Professor Pigou is relying?

Is it not clear that the relevant circumstance in the case of the imposition of a tax or the raising of a price is the change in the terms on which exchange is possible? The marginal

[3] *Economics of Welfare* (First Edition), p. 593.
[4] *Public Finance*, pp. 83-4.

utility of real income (or the single good) changes with changes in the quantity possessed as before. It is the terms on which income (or the single good) can be obtained which alter and it is the magnitude of this change together with the rate at which the utility of income diminishes which determines the nature of the new equilibrium. The flaw in Professor Pigou's argument seems to be due to an ambiguity in the expression "marginal utility" used in this connection. If a tax is imposed the utility of the marginal unit of income will rise, but the utility of the income derived from a unit of work at the original point of equilibrium may rise or fall, for it depends on the rate of exchange as well as on the utility of the marginal unit.[5] When the price of anything rises, we are entitled, if we assume diminishing marginal utility, to infer that, in all but exceptional cases, less of the commodity will be bought. But we are not entitled to infer that more money (or more effort) will necessarily be spent on it.

8. If these considerations are valid we are left with the conclusion, reached earlier, that any attempt to predict the effect of a change in the terms on which income is earned must proceed by inductive investigation of elasticities. The attempt to narrow the limit of possible elasticities by *a priori* reasoning must be held to have broken down.

[5] Mr. Hicks, to whom I am greatly indebted for assistance in framing the above criticism, has formulated the point symbolically thus:

If u = Utility of income earned,
v = Disutility of work done,
x = Amount of work done,
y = Amount of income received,

then in equilibrium $\frac{du}{dx} = \frac{dv}{dx}$ If the same amount of work is done, then $\frac{dv}{dx}$ remains unchanged, but $\frac{du}{dx}$ may vary in either direction; for $\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx}$. $\frac{du}{dy}$ must increase but $\frac{dy}{dx}$ must diminish, and the change in $\frac{du}{dx}$ will therefore depend on their relation, i.e. on the elasticity of demand for income.—Q. E. D.