THE THEORY OF CAPITAL

Proceedings of a Conference
held by the
International Economic Association

by

F. A. LUTZ

Chairman of Programme Committee

D. C. HAGUE

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Lutz and Hague

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INTRODUCTION

BY FRIEDRICH LUTZ

THE Round Table Conference on the Theory of Capital held on the Island of Corfu from September 4-11, 1958, was the ninth such conference organized by the International Economic Association. The economists listed on pp. vii-viii attended the meeting and eleven 3-hour sessions were held. In accordance with what is now established practice, the papers were not read during the meeting, as they had been circulated beforehand. After the author had briefly mentioned the most important points in his paper, a rapporteur summarized his impressions of the paper and added such critical comments as he wished to make. The meeting was then open for general discussion. A summary record of the discussions, which is the work of D. C. Hague, will be found on pp. 289-403.

In view of the renewed interest which the problems of capital have aroused during the last decade, with the efforts of economists to develop a theory of economic growth, The Theory of Capital was chosen as the topic for the 1958 meeting. There was, however, general agreement that empirical studies in the field should also be included, and the statistical papers submitted to the meeting greatly stimulated discussion.

In line with the preoccupations of perhaps most economists today, the macro-economic aspects of capital theory received rather more attention than micro-economic ones, although the latter were frequently touched upon. The programme can perhaps be best explained by setting out the main headings under which the papers can be grouped. Apart from the first paper, which was intended as a summary survey of the field, they are :

(1) the problem of measuring capital ;
(2) the capital-output ratio, its meaning and its actual behaviour ;
(3) the classification of technical innovations according to their labour-saving or capital-saving character ;
(4) the influence of the interest rate on investment ; and
(5) capital in dynamic growth models.

The problem of measuring capital and related aggregate concepts such as social income was discussed both on the theoretical level (Professor Hicks' and Professor Samuelson's papers) and from the statistical side (Dr. Barna's paper). The suggestions of the theorists as to how such aggregates should be measured understandably filled the statisticians with despair, even if the theorists were themselves aware that the statisticians could not live up to the exacting demands which their theories imposed. None the less the confrontation of the analytical conclusions with the concrete possibilities of measurement is a very useful one for both makers and users of statistics. It helps the former to see what they should try to approximate and guards the latter against drawing wrong conclusions from the available figures. And those theorists who have not devoted special attention to these problems of measurement will also greatly benefit from the perusal of the three papers just quoted. The analysis they contain — apart from being interesting and stimulating in itself — may help model-builders to avoid the pitfalls of excessively crude aggregation.

Several sessions were devoted to the capital-output ratio, which ever since the appearance of the Harrod-Domar model of economic growth has played an important rôle in dynamic theory. Two papers (Professor Domar's and Professor Hoffmann's) deal with this ratio on the empirical level, presenting not only figures showing the movement of overall capital-output ratios over time in the United States and Germany, but also figures for capital-output ratios in different industries. The concept of the capital-output ratio itself can be interpreted in many different ways; Dr. Goldsmith's statistical material distinguished no less than 19 different ratios. Which one of them should be used depends of course on the problem in hand. In view of the assumption of a stable overall capital-output ratio which has characterized some of the best-known models of dynamic growth, the question of the stability of this ratio was thoroughly discussed at the meeting. Here, strangely enough, the participants, arguing on the basis of the same empirical evidence, could not reach agreement. It was generally acknowledged, of course, that year-to-year changes in the ratio are strongly influenced by the position of the year in the trade cycle, years of depression showing a relatively high, and years of boom a relatively low capital-output ratio. For the purpose of determining whether the ratio tends to stability or not, averages over longer periods must be taken. But here the agreement ended. The ratio of the Net Capital Stock to the Net National Product calculated by Kuznets for the United States (see Professor Domar's paper) moved from 3·2 on the average over the twenty years 1869–1888 to 3·4 over the next twenty years and 3·6 over the following twenty. After that (in the period 1939–1955) it came down to 2·9. Is this movement (which is much smaller than that shown by some of the ratios calculated by Dr. Goldsmith) small enough to justify the notion that the capital-output ratio is, roughly speaking, stable ? Or does a change of 24 per cent from maximum to minimum invalidate the stability hypothesis ?

Some participants were inclined to minimize the significance of such 'small' changes over such long periods. Others argued — to use Professor Domar's words — that 'by its very nature the average capital coefficient is a sluggish animal', because even large changes in the incremental ratio will produce only small changes in the average ratio, so that changes of 24 per cent in the latter must be considered 'large'. I myself would side with the second group, with the additional reason that I have not so far been able to find a convincing theoretical explanation of why the capital-output ratio should remain stable. This episode in the discussions was not without its disturbing side. If we cannot agree on the interpretation of the empirical evidence, how can we know what we should try to explain : the stability or instability of the capital-output ratio ? Or, alternatively, how can we agree on what is a proper hypothesis on which to base our theoretical models ?

The capital-output ratio also entered the discussion in connection with the classification of inventions according to whether they are labour-saving, neutral or capital-saving. Professor Fellner in his paper takes the position that changes in the capital-output ratio in response to inventions cannot indicate to which of the groups a particular invention belongs. He holds that *every* invention — once we assume the supply of capital and labour to be given — must raise the output-labour ratio in exactly the same proportion as it raises the output-capital ratio. We are bound, therefore, to base our classification on the effect of the invention on the relative shares of labour and capital in the social net income — a conclusion which seemed to meet with the approval of most (or even all) participants in the discussion.

Although, as a matter of doctrinal history, capital theory developed in connection with, or even as a by-product of, interest theory, the programme committee did not solicit a paper on capital theory as an integral part of interest theory. Some reference to the connection is made in the introductory paper ; but since a thorough treatment of this aspect of capital theory would easily have provided the material for a Round-Table Conference of its own, the committee decided to select only one special problem out of the many that might have been discussed under the heading 'Interest Theory and the Theory of Capital', namely, the influence of the rate of interest on investment. Mr. Thalberg's paper treats this problem with special emphasis on the speed of reaction of investment to changes in the interest rate. Professor Barrère's paper, which deals with the factors determining the combination of productive inputs in the individual firm, although not especially focused on the influence of the interest rate on this combination, may also be regarded as a contribution on the same subject.

In this connection as in others — for instance, in the discussion of the first paper — the problem as to what the entrepreneur should maximize came up for discussion. The participants seemed to reach some kind of agreement on this question. It was generally admitted that in the complex world in which we live, different firms, or the same firm at different times may, for good reasons, follow different criteria of profit maximization. At the same time it was agreed that if we *had* to generalize, we should do best to base our theoretical analysis on Fisher's criterion of maximizing the present value of future profits (discounted at the market rate of interest) rather than on Wicksell's criterion of maximizing the internal rate of return, both criteria giving of course the same result in a stationary state.

Mr. Thalberg and Professor Barrère chose to stay in the sphere of 'pure' theory. The reader may therefore miss in this volume — a feeling that I share — a paper dealing with those more practical aspects of the problem of the influence of the interest rate on investment which are so important in connection with monetary policy, for example. Views on this question seem to change in cyclical fashion, between the opinion that the interest rate is effective and the opinion that it is not. In order to give a final answer to this problem, the theorist would need to take account of the actual practice followed by business men. He would need to know, for example, whether business men calculate with a rate of 'interest' that is customary in their industry and is — at least within certain limits — independent of the market rate. He would also have to introduce a larger number of variables into his theory than is customary. Attention should be given to the influence, on the effectiveness of the interest rate as a regulator of investment, not only of uncertainty, which Mr. Thalberg's paper treats, but also of the tax system, and of the quantitative availability of funds as contrasted with the level of the interest rate.

Three papers (those of Professor Champernowne, Mr. Kaldor and Professor Solow) are devoted to dynamic theory. This is not the place to go into the contents of these difficult papers, which evoke our admiration for the powerful theoretical analysis they present. Perhaps I may venture a few remarks on the problem of dynamic theory in general. Each of these and other models undoubtedly gives us some new insight into the process of dynamic growth. At the same time, it is clear that we can go on building theoretical models indefinitely. No matter whether we are enquiring into the conditions for smooth economic growth, or whether the question we are asking is how the economy will develop given certain initial conditions, we can produce a great variety of answers, depending on the assumptions we make. Exercises in dynamic theory are admittedly intellectually fascinating, but they show a tendency to follow the principle 'l'art pour l'art' which, in my view, is not a principle appropriate to economics. We shall probably never be able entirely to avoid an element of personal judgment as to which variables are important, what are the shapes of the functions (such as the investment function) we use, and how far we may legitimately go in the direction of aggregating economic quantities. But it seems to me that if the theory of dynamic growth is not to lose contact with reality, empirical research is now more than ever necessary in order to allow us to distinguish the more-fruitful hypotheses from the less-fruitful ones. The question mentioned above — whether the capital-output ratio is stable or not — is a case in point. (If, however, as in the discussion at Corfu, we cannot agree on the interpretation of the statistics when we have them, then not even empirical investigation can help us much!)

In the discussions, more than in the papers themselves, the marginal productivity theory was strongly attacked by what was, however, probably a minority of the participants. The theory is doubtless open to criticism, particularly as it applies to the determination of income from capital. Professor Marchal, in his paper, without rejecting the marginal productivity theory outright, seems to lean towards a theory which explains distributive shares as a result of the power of the groups participating in the economic process, although he does not fully develop this theory. Those who voiced their opposition to the marginal productivity theory did not — and could not of course — elaborate a distribution theory of their own during the meeting. So, before passing a final judgment, we must wait until we have the alternative theory, which is to replace the marginal productivity theory, before us in a fully-developed form. It is to be hoped that the discussion in Corfu will stimulate some of the participants to try their hand at such a formulation.

Chapter 1

THE ESSENTIALS OF CAPITAL THEORY

BY

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To write on the 'essentials' of capital theory, as it unfortunately fell to my lot, necessarily means making a value judgment. In selecting from among the almost infinite number of things that have been said about capital only what can be pressed into a short paper, I cannot hope to make the choice that others would make. It obviously is not possible for me to offer anything like a comprehensive survey of the main currents of thought on capital theory.

Most economists will probably agree that capital theory was thoroughly treated within the framework of static analysis for the first time by Wicksell — whose work incorporated part of Böhm-Bawerk's theory — and was later embroidered upon by Wicksell's followers, Akerman, Lindahl and Hayek. They would further agree that interest in capital theory then remained dormant for some time, until it was revived in connection with the theories of economic growth which are now so fashionable. Perhaps the most natural procedure therefore is to look first at the findings of the group of neo-classical writers just mentioned, and at the difficulties which they encountered, and then to ask what can be learned from them for the analysis of economic growth.

This programme means that I shall omit at least two important aspects of capital theory. The first, the determinants of the choice made by the income recipient between consumption and saving, had played a paramount rôle in the capital theory of such writers as Böhm-Bawerk, Fisher, Fetter, etc., but was not discussed in much detail by Wicksell and his followers. More recent authors have excluded it, by more or less general agreement, from the 'essentials' of capital theory, and I shall not deal with it here either. It is with the productivity aspect of capital, and not with the supply side, that I shall be mainly concerned.

The second aspect of capital theory that I shall disregard is the part played by capital in the theory of the trade cycle. Any attempt to treat this problem would obviously take me beyond the confines of a short paper, though of course the rôle that is assigned to capital in trade cycle theory is partly the result of the 'essentials' of general capital theory.

I. NEO-CLASSICAL CAPITAL THEORY

The problems which the Wicksellian school regarded as the 'essential' problems of capital theory were two: (1) the determination of the investment plans of individual entrepreneurs; and (2) with the help of the solution to this problem, the determination of the effect of the volume of capital on the interest rate and the wage rate, or more generally on the whole relative price structure of the economic system.

This attempt to work out a general equilibrium analysis for a stationary economy contrasted with the much more limited aim of another set of 'capital theorists', among them Irving Fisher. These writers contented themselves with explaining one price — the interest rate — as the result of the totality of the investment plans of individual entrepreneurs and the savings plans of income recipients, without entering, for instance, into the problem of the effect of capital accumulation on the wage rate.

Wicksell and his followers were able to give a clear-cut answer to the first problem. But their general equilibrium analysis, which made use of such aggregate concepts as the total volume of capital and the average period of production for the economy as a whole, was unsatisfactory in a number of respects. This fact induced one of the group, Hayek, to abandon the attempt to develop a capital theory for a stationary state, and it induced others outside the group to reject the whole Böhm-Bawerk-Wicksell theory.

In treating the first problem, the determination of the investment plans of the individual firm, the group considered it an important part of their task to explain not only the 'breadth' but also the 'depth' of investments. This fact, of course, distinguishes them from economists of both earlier and later generations who left the 'depth' of capital out of account. Two questions had to be answered simultaneously. What determines the periods of investment which the entrepreneur will choose for his inputs? And how many inputs will he invest? To answer these questions, the Wicksell school made use of a production function which contained time as one of the variables. The function was written, for instance, as $p = f(a, b, t_a, t_b)$ for the simple case where the product, p, is the result of two inputs, a and b, with two variable investment periods, t_a and t_b. Assuming that the prices of inputs (among which the services of long-lasting capital goods, Wicksell's *Rentengüter*, may also figure) and the prices of outputs were given to the individual entrepreneur, they proceeded to determine the investment periods and the quantity of each input used, under the condition that the entrepreneur aimed at maximizing the internal rate of return (profit rate) on his investment.

The production function here used presumes, of course, that the investment periods of inputs can always be determined. For the inputs which enter into durable goods this cannot be done. Akerman thought that he had discovered the rhythm with which inputs embodied in durable goods became outputs, and concluded that he had found the 'correct' depreciation formula. Wicksell observed that this was an illusion : 'for the annual uses successively following one another constitute a kind of joint supply and it is just as absurd to ask how much labour is invested in either one or the other annual use as to try to find out what part of the pasture goes into wool and what part into mutton'.[1] Depreciation methods are mere conventions adopted for the purpose of enabling us to make up balance-sheets : none of them has any foundation in the economic calculus. For the main purposes of capital theory, however, no such method is required. We do not need one in order to solve the problem of the optimum durability of a capital good ; all that we need to know is the cost of the good as a function of its durability, i.e. how much additional expenditure is necessary in order to lengthen its lifetime by an extra unit period.

So long as the assumption is retained that the entrepreneur maximizes the internal rate of return, both the 'breadth' and 'depth' of investments depend exclusively on the prices of the inputs and outputs. The important result follows that in response to a rise in the ratio of input prices (wages) to output prices the investment periods of inputs and the durability of capital goods will be lengthened. As between two investments of different, but in each case invariable, length, the longer one will as a rule be contracted less in 'breadth' than will the shorter one. This result reflects what Hayek later called the 'Ricardo effect' according to which a rise in the real wage rate causes a more capital-intensive process of production than before to be adopted over the system as a whole, and the other way round if the real wage rate falls.

It must be emphasized that the criterion of maximizing the internal rate of return on capital which these authors adopted is not

[1] Wicksell, *Lectures on Political Economy* (London, 1935), vol. i, p. 260.

the only one that can be chosen ; nor has it dominated the literature. It is one of the surprising things about capital theory that no agreement seems to have been reached as to what the entrepreneur should maximize. The adoption of the criterion chosen by Wicksell leaves the 'breadth' and 'depth' of the investment plans of entrepreneurs independent of the market rate of interest. Some authors, such as Irving Fisher and Keynes (the latter of whom was not interested in the 'depth' problem), determined the volume of investment by the rule that the marginal rate of return on it should equal the interest rate, a rule which is equivalent to maximizing the difference between the present value (at the going interest rate) of future returns from the investment and the present value of the costs of the investment. This criterion gives us quite different results for both the volume and the 'depth' of investment that the entrepreneur should choose. Here the interest rate plays a decisive rôle, whereas under the alternative criterion it has no influence at all. Contrariwise, a change in input prices under this second criterion does not affect the 'depth' of investment,[1] as was pointed out long ago by G. F. Shove. It affects only the 'breadth', and the entrepreneur has no incentive to act in such a way that the 'Ricardo effect' materializes.

Since the optimum degree of capital-intensity — in all the senses in which this is variable — is different according to which criterion is used by the entrepreneur, it seems that the question of which one should be adopted ought to be settled once and for all. To me Fisher's criterion (which is identical with the maximization of the profit rate on the cost of the capital invested by the existing owners of the business) seems the more realistic, as well as preferable on theoretical grounds. For if we do not suppose that new capital is absorbed by the system in response to a cheapening of the terms on which capital is available on the market, how can we explain how capital accumulation affects the real wage rate (or the ratio of input to output prices), and hence the profit rate (or the internal rate of return) ? And how therefore can the capital-intensity of the productive process be increased ? There seems to be a link missing

[1] In the simple point-input point-output case, e.g. trees, this can be easily demonstrated. Let $R=f(t)$ be the revenue from the tree, i the market rate of interest and C the given cost of planting a tree. Then the maximization of the difference between the present value of the revenue and the cost, i.e. of $V-C=f(t)e^{-it}-C$ requires that $i=\frac{f'(t)}{f(t)}$, an expression which is independent of the magnitude of C. On the other hand, the average internal rate of return is r in the equation $f(t)e^{-rt}=C$, giving $r=\frac{1}{t}\log\left(\frac{f(t)}{C}\right)$ where r is clearly dependent on C. (The maximizing condition is that the average rate should equal the marginal rate.)

from the argument. If with Böhm-Bawerk — and perhaps Wicksell, although he is less consistent in this respect — we make the assumption that the additional capital is already distributed among the entrepreneurs, the problem is of course solved in some way, however arbitrary. We can then imagine the entrepreneurs — with the additional capital already in hand — going into the market, bidding up the real price of labour and thus drawing down the profit rate ; so the rise in the wage rate would explain the increase in capital-intensity.

It seems in fact to be necessary here to suppose that all the new capital formation is financed from retained earnings, indeed an occasional tendency for the literature to adopt just this assumption has been visible. If, however, we want to allow that at least part of the new saving takes place outside the firms, it seems that we need to show through what mechanism it finds its way into the entrepreneurs' hands ; and the only possible medium would seem to be a fall in the interest rate. It then appears that it is with Fisher and his followers that we have to side in our choice of the criterion of profit maximization. It is not sufficient to point to the fact that in stationary equilibrium, where each entrepreneur earns the going rate of interest on his investment, the internal rate of return must equal the interest rate. What we need to demonstrate is how this equivalence between the two rates is reached — if it ever can be reached.

It was when Wicksell and his followers moved on from the analysis of individual investment plans to general equilibrium analysis that — by their own admission — they got into serious trouble. In this analysis they made use of the average period of investment of all the inputs that enter into the consumer goods coming forth during a given period. This concept is open to two main objections. First, as we observed above, the investment periods of inputs entering into durable goods cannot be determined. Second, even if we are prepared to adopt a certain convention as Lindahl does — in effect it is Akerman's depreciation formula — for describing the time sequence in which these inputs ripen out, there remains a still graver objection.

Wicksell was already aware that if the average period is calculated, as it should be, by weighting the inputs with their individual investment periods on the basis of the compound interest formula, the average period becomes dependent on the interest rate and thus can no longer serve as an independent variable helping to determine that rate. The theory can dispense with this average period only if the stock of capital, as a sum of exchange value, is assumed to be given from 'outside' as it were. But the value of the capital stock must itself be calculated either by capitalizing its future returns at the interest rate ruling in the market, or by summing the costs of production of the capital goods in the stock. In either case the interest rate enters into the calculation. Capital (in the sense of a sum of exchange value) can thus not be used to determine the interest rate ; it is itself determined by that rate. The theory is short of one equation, as Wicksell himself realized. He takes as a datum sometimes capital and sometimes the average period of investment;[1] and Lindahl, in his system of equations, decides to take the average period as a 'given factor'.[2]

This static theory ran into yet another difficulty. It had to assume that capital had from the beginning the exact time structure, or composition, which would be required for the stationary equilibrium that was to be determined with the help of the datum 'capital'. As Lindahl expresses it : 'In the theoretical treatment of the static problem the realisation of this time structure (i.e. the time structure required for equilibrium)[3] must be assumed, as well as the circumstance that the circulating capital has the form required for stationary conditions'.[4] In other words, the existing capital goods must be of the same nature as those which it will be profitable to reproduce. If one were to start out with an historically given, arbitrary collection of capital goods (which, as Hayek suggests, can only be described in terms of the alternative output streams which they may be made to render) there is, I think (along with Hayek), no way of proving that a stationary state will ever be reached. For we cannot assume that the form and composition of the capital goods will remain the same ; we cannot treat the capital supply in its concrete physical form as a datum as we are able to do, for example, in the case of the labour supply. We are necessarily pushed into some kind of analysis of a process of continuous change, and only within such a framework can we take care of the effect of net new capital accumulation which — even if we can give the latter no exact definition — we all regard as one of the main problems of capital theory.

But once we have accepted this position, the Wicksell theory still has its positive contribution to offer. If we are not too purist — as I am afraid we cannot be — about what we mean by the quantity of capital, surely we must admit that the broad results obtained by Wicksell and his followers for the system as a whole are true. For example the 'larger' is the 'amount' of capital, the longer will be the investment periods of inputs, the more durable will be durable goods, the lower will be the interest rate, and the higher will be the real wage rate.

[1] Wicksell, *Lectures*, vol. i, pp. 179 and 289.
[2] Erik Lindahl, *Studies in the Theory of Money and Capital* (London, 1939), p. 308. [3] The insertion in brackets is mine. [4] *Op. cit.*, p. 304.

II. MACRO-ECONOMIC CAPITAL THEORIES

In the literature there are two entirely different approaches to dynamic analysis, the micro-economic Swedish approach which grew out of the Wicksell tradition and the macro-economic approach which has one of its roots in Keynes' work and is followed in the many models of economic growth that have been developed during the past decade. The two approaches differ radically in their treatment of capital. In this section I shall take up the approach which proceeds in terms of aggregates, leaving the Swedish theory to the last section.

The macro-economic models vary widely in purpose and in assumptions and hence in the results to which they lead. Some authors have sought to discover what conditions must be fulfilled if an economy is to grow uninterruptedly, and in their models the problem of the sufficiency of aggregate demand plays a dominant rôle. Others, in whose opinion the problem of the sufficiency of aggregate demand is essentially a short-run problem, postulate full employment as a continuous condition. They proceed on the assumption 'that in a growing economy the general level of output at any one time is limited by available resources and not by effective demand'.[1] Some disregard the influence of factor prices on the choice of techniques while others take this influence into account. Some arrive at the conclusion that if the economy slides off ever so slightly from the theoretical path of uninterrupted growth it will experience either increasing unemployment on one side or prolonged inflation on the other. Others see the system as tending always towards the stable equilibrium rate of growth.

It is of course impossible adequately to review here the manner in which capital is treated in the large number of models which by now exist. All that I can do is to look briefly at two problems connected with capital in such growth models : (1) the measurement of capital ; and (2) the notion of the capital-output ratio.

The measurement problem, which had already plagued Wicksell, is important whenever capital is given an operational significance, that is, whenever it plays a rôle as an independent variable influencing other quantities (such as output or the interest rate) in the system. In some models nothing is said about how capital is measured ; it simply appears under the initial K. Solow avoids the problem by assuming that only one 'composite' commodity is produced, which can serve as an investment and a consumption good,

[1] N. Kaldor, 'A Model of Economic Growth', *Economic Journal*, 1957, p. 593.

so that the real problem of capital measurement, namely the fact that the 'composite' commodity is always changing its composition, disappears. Kaldor wants to measure capital by the amount of steel that is embodied in capital goods, and others have suggested horse-power as the unit. Among recent writers, Joan Robinson has paid more attention to this problem than most others, and I shall have occasion to comment on her view shortly. It should be added that in the majority of cases the measures adopted by the various authors do not seem to matter very much, since in the actual development of the model the term K usually takes over in the functional relationships or equations, and, more often than not, the reader, forgetting that the definition was a problematical one, henceforth thinks of K as an unequivocally determined magnitude. In some cases we seem bound to forget the author's own definition of capital because it appears to be against the logic of his argument. Thus, for example, Solow's assumption that the technical coefficients between capital and labour are variable seems to contradict the notion of a capital goods complex of fixed composition.

The capital measurement problem has perhaps received most emphasis in connection with the production function applied to the system as a whole. Wicksell never used the term K for this purpose, but always inserted the term T on the grounds that it is by allowing labour to use roundabout, time-consuming processes of production that capital raises the productivity of labour and thus is itself productive. What really mattered was the period of investment and this could not be represented by capital in the sense of 'a sum of exchange value'. For Wicksell was aware that an increase in capital, measured in terms of a constant price level, would be partly absorbed by higher wages and not only by an increase in the time-dimension of the productive process, so that K in this sense could not be directly substituted for T. A lengthening of the average investment period, T, required a more than proportionate increase in capital, K.

Now while the use of t in the production function relating to individual processes is unobjectionable, the use of T in the production function for the system as a whole is not. For in the latter case we have to take some average T which, as remarked earlier, becomes dependent on the interest rate. This means that we should have to draw a separate curve representing the productivity of labour as a function of T for each different level of the interest rate. The production function for the system as a whole, written as $f(T,L)$ or as $f(K,L)$ if we define K in such a way that it is interchangeable with T, therefore makes no sense unless the interest rate is already given, and it cannot be used to determine the factor prices.

Joan Robinson, who measures capital in terms of past labour-time compounded at a given interest rate and develops the concept of the 'real capital ratio' (i.e. 'the ratio of capital reckoned in labour-time to the amount of labour currently employed when it is working at normal capacity'),[1] comes to this same conclusion that a separate productivity curve for labour equipped with varying amounts of capital thus defined must be drawn for each level of the interest rate. But if we measure capital in terms of labour-time compounded at a given rate of interest, this can tell us nothing as to what will be the productivity of labour combined with that capital in the future, unless we assume that a stationary state prevails in which the average length of the current production process will be an exact replica of that of the past process. With an arbitrary collection of capital goods, the past labour-time embodied in them has no relevance for the future productivity of labour. We are no further advanced than was Wicksell and, it would seem, a good deal less far ahead than was Hayek with his description of the arbitrary collection of capital goods in terms of the output streams which they could be made to yield in the future.

Turning now to our second point, there is no doubt that the modern 'empirical' discovery of a constant capital-output ratio — and here we must ignore the problem of measurement — conflicts with what traditional capital theory would lead us to expect, except in certain rare circumstances. The Harrod-Domar model is based from the start on a constant capital-output ratio. Its constancy follows from these authors' assumption that only 'induced' investments take place which are a given multiple (the accelerator) of their output, and that technical innovations, if they occur, are 'neutral' — leave the capital-output ratio undisturbed. Kaldor, on the other hand, thinks that he can find forces which drive the economic system along a growth path which keeps the capital-output ratio constant.

The constancy of the capital-output ratio, combined with the assumption that savings are a given percentage of income, means that output must grow at an exponential rate if capacity is always to be fully utilized. Since this rate of growth may be different from the rate of growth of the labour force, it appears that there is a dichotomy between the two rates, which can create difficulties. However, these difficulties arise only if we assume either that the proportions in which capital and labour can be used in the system are fixed or, if they are not, that the wage rate and interest rate are fixed. The flexibility of all these elements was once regarded as being among the 'essentials' of capital theory, and I think it should

[1] Joan Robinson, *The Accumulation of Capital* (London, 1956), pp. 122-3.

still be, particularly when we are dealing with problems of long-term economic growth. But if we do assume flexibility of factor prices and proportions — and we must remember here the point stressed by Hicks [1] that an important dimension of the variability of factor proportions consists in changes in the 'industry mix' — and if we suppose that capital increases faster than labour, we must conclude that capital grows not only in 'breadth' but also in 'depth'. Then the capital-output ratio must necessarily rise, unless technical inventions are of such a nature that they exactly offset the effect of the 'deepening' and so keep the average ratio constant, a result which would be nothing short of a miracle.

Kaldor reaches his result, where the economy tends to grow in such a way that the capital-output ratio always remains constant, by a combination of assumptions which are certainly different from those postulated by the 'classical' theory. One of them concerns his investment function which is based on 'the desire to maintain the capital stock in a given relationship to turnover, modified by any change in the rate of profit on capital',[2] which therefore — in the first of its two components — casts the shadow of the result he wants to obtain. Another basic assumption on which that result depends concerns the shape of his 'technical progress function'. In constructing the corresponding curve, Kaldor rejects the old distinction between the shift to 'new' technical methods of production within the given framework of technical knowledge and the adoption of new methods opened up by technological progress. He establishes a single relationship between the growth of capital and the growth of productivity which incorporates the influence of both types of change in technique. The curve showing the annual percentage growth in output per man as a function of the annual percentage growth of capital per man rises at a declining rate. Shifts of this curve are brought about by 'variations in the flow of new ideas and in the readiness with which they are adopted'.[3] I find it difficult to follow Kaldor in his belief that the shape of the curve he draws is plausible. But this shape is essential to the result he finally obtains, namely a tendency to a long-period equilibrium rate of growth with a constant capital-output ratio. Nor does it seem possible to draw a sharp distinction between those events which should be included in the original curve (and which give it its shape) and those which should be allowed for by shifting the curve. The 'technical progress function' seems to be a weak basis for the far-reaching conclusions which he draws. Is there anything to be gained, I would here ask, by giving

[1] J. R. Hicks, *Theory of Wages*, p. 187 ff. [2] Kaldor, *op. cit.* p. 604.
[3] Kaldor, *ibid.* p. 596.

up the distinction — which is at least a logically clear one and was previously considered an 'essential' of capital theory — between alternative methods with a given state of technical knowledge and alternative methods provided by new inventions ?

There is, as Solow [1] has shown, one way of demonstrating that under certain assumptions concerning the shape of the production function the system tends to a constant capital-output ratio, provided we exclude technical innovations altogether. The gist of the argument, translated into non-mathematical terms, is as follows. Suppose that capital and the labour force both grow, but the first at a faster rate than the second, so that the capital-output (and capital-labour) ratio rises. Suppose further that throughout the development process savings are a constant proportion of income. Then the rate of increase of output per worker will decline as the capital-labour ratio rises, and the rate of growth of savings in the community will also fall, until finally it becomes equivalent to the rate of growth in the labour force. From then on the capital-output and the capital-labour ratios will both remain constant.

This argument, which abstracts from innovations, cannot of course be used as an explanation of the alleged *empirical* constancy of the ratio. If we allow for innovations, the ratio can remain constant only on the basis of the purely fortuitous circumstance that innovations are of just the right type to keep it so. The 'essentials' of capital theory certainly give us little help in explaining how the constancy of the capital-output ratio — if, despite all the measurement difficulties, such constancy can be held to exist — comes about.

III. THE SWEDISH CONTRIBUTION TO DYNAMIC CAPITAL THEORY

Turning to the Swedish attempt to construct a micro-economic dynamic theory, I shall take Lindahl's [2] work as sufficiently representative. His purpose is not to find the conditions for uninterrupted growth of the economy. It is rather to show how prices and quantities are determined when the factors affecting them are changing over time.

The future is, in this analysis, sub-divided into small unit-periods, and it is assumed that changes in the factors determining prices take place at the transition from one period to another. Lindahl then

[1] R. Solow, 'A Contribution to the Theory of Economic Growth', *Quarterly Journal of Economics*, February 1956, pp. 68 ff.

[2] Lindahl, *op. cit.* pp. 318 ff.

develops, under the assumption of perfect foresight, a system of equations which determines for the whole series of periods up to the horizon (at which for simplicity's sake a stationary state is assumed to be reached) the following magnitudes: (1) The quantities of consumers' goods produced in each period; (2) their prices; (3) the supply of 'original' services in each period; (4) their prices; (5) the interest rate in each period; and (6) the total quantity of each 'original' service invested in each period in the production of each of the consumers' goods maturing in the current period or in succeeding periods.

This analysis, unlike Lindahl's and Wicksell's analyses of the conditions for stationary equilibrium, can and does start out, at the base date, with an arbitrary collection of capital goods whose services are considered 'original' services when used in the first period and 'saved-up' services when used in later periods. We find no explicit mention of such concepts as aggregate income, total capital, or aggregate savings and investment, which are essential quantities in the macro-economic growth models. Capital goods as such do not even appear in Lindahl's equations. The only goods for the prices or quantities of which equations are set up are consumers' goods. Capital goods are, in Wicksellian fashion, conceived of as services of original inputs, saved-up and on their way to becoming consumers' goods. Consumption and investment plans are made at the base date for the whole series of future periods, with full knowledge of when the inputs invested can be made to mature; and these plans are co-ordinated from the beginning for each period by the fore-known 'equilibrium' prices and interest rates which the perfectly flexible price mechanism yields. Investments in each period are identified with those inputs which are directed towards the production of consumption goods that will ripen out at various times in the future, the length of these investment periods being subject to the choice of the investors. This is clearly a different world from that of some of the aggregate models, where relative prices play no rôle and where investments are considered as an undifferentiated mass of capital goods added to the existing stock, a mass which will bring forth more or less output according to the magnitude of the capital-output ratio.

This does not mean that aggregate concepts, such as income, capital, savings and investment, are entirely absent from Lindahl's system. They can be calculated by a summing process from the quantities and prices which are the solutions to the system of equations. Lindahl himself carries out this exercise, which yields the result, for instance, that under perfect foresight the amount of total net savings in each period will be equal to the increase in the value of capital. He does not, however, attempt to frame his description of the dynamic process in terms of the aggregates and functional relationships which are found in the more recent models of economic growth to which we referred in the previous section.

When imperfect foresight is assumed, the same scheme can be used. At the base date, plans are made for the whole series of future periods and the co-ordination of these plans can be described by the same type of equation as under perfect foresight. If unforeseen events occur (at the transition from one period to the next) expectations are revised, a new system of equations can be set up for the remaining periods, and so on for all subsequent revisions. In this case capital values do not change exclusively in response to positive or negative savings ; and the amount of net *ex ante* savings need no longer be equal to the *ex post* increase in the value of capital. Inequality is a sign that expectations have not been fulfilled. The equality of *ex ante* savings and investment as a condition for uninterrupted growth has been explicitly introduced into some of the macro-economic models ; and even in those models where it is not specifically mentioned it must be an implicit condition for uninterrupted growth that entrepreneurs' expectations should not be disappointed.

It is not our purpose here to criticize Lindahl's conception of perfect foresight, or the details of his system of equations. What it is important to notice is that his approach shows that, in principle, it is possible to treat within a dynamic framework the 'essentials' of capital theory as they have been handed down to us by the neo-classical school and which many of us at least see no reason to discard. Following in Lindahl's steps, we must begin with a given collection of capital goods capable of rendering various alternative output streams. We must define, on the assumption of given expectations, the investment plans of entrepreneurs and the savings plans of both households and entrepreneurs. We must analyse (using Hicks' method as set out in *Value and Capital*) their effects on the system as a whole in the current period. Then, on the basis of the capital stock that emerges at the end of that period, we must proceed to the analysis of what happens in the next period.

This type of analysis is, of course, complicated and tedious ; for practical purposes we are bound to look for aggregates. But our task must be to find the right aggregates and functional relationships, and to include enough of them in our model, to ensure that we do not lose sight of — or mistake the character of — any of the important effects of capital accumulation. It seems to me highly doubtful whether the selection of just two aggregates, capital and income, related by the capital-output and the savings-income ratios, can satisfy this purpose.

IV. MEASUREMENT OF CAPITAL

Among the 'essential' capital problems we have touched upon in the foregoing discussion, that of measuring capital (or investment) does not admit of a precise solution. Like other quantities which we continuously use, such as real income, capital can be accurately measured only in a stationary economy, and only in such an economy can we know precisely what is meant by 'keeping capital intact'. So long as the economy is undergoing any sort of change, and capital goods are altering their form, we cannot know exactly to what extent capital is increasing or decreasing. And if the changes are small, as in the case discussed by Hayek where we start off with an arbitrary collection of capital goods and let the system gradually approach a stationary state, without, however, being sure that it will ever be reached, we cannot even say whether the shifting composition of the capital stock during the process involves a change in the quantity of capital or not. Notwithstanding any refinements that may still be possible, we must, I am afraid, be content with only a vague notion of what is meant by 'more' or 'less' capital. We must be glad if our measure allows us to distinguish to our common-sense satisfaction between two different quantities when the difference is large enough. In our theoretical models, we are obliged none the less to proceed *as if* capital were precisely measurable. The sin we commit by so doing is of the same kind (though it may perhaps be greater in degree) as that of which we are guilty when we speak of an 'increase' in real income or a 'rise' in the real wage rate.

Perhaps, then, we shall have to give up the search for the 'true' measure of capital, just as we had to abandon the search for the 'ideal' index number, and the 'correct' depreciation formula. The hopelessness of this search is a penalty of the very existence of economic change. This primary essential of capital theory, as it was considered in the past, may thus have to be regarded as not so essential after all.

Other essentials that belonged to the same school of thought have, however, I think, been too wantonly discarded by many of the exponents of dynamic model building. We cannot, I suggest, hope to construct models which give us a reliable pointer to the effects of capital accumulation — however vague this term is — unless we include variations in factor and commodity prices and in the degree of capital-intensity in the system. What we require are growth models based on something like the old production function which took care of these flexibilities. It is a sign that this feeling is shared by others that some recent model builders seem to be moving back in this direction.

Chapter 2

THE MEASUREMENT OF CAPITAL IN RELATION TO THE MEASUREMENT OF OTHER ECONOMIC AGGREGATES

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I. INTRODUCTION

I PROPOSE in this paper to discuss the measurement of capital from what seems to have become a rather unfashionable angle. The time-references of a capital stock are themselves a source of so much difficulty that it is tempting, when considering them, to avert one's eyes from other difficulties and, by some violent simplification, to assume that differences in 'life' or in prospective date of completion are the only causes of heterogeneity that fall to be considered. By such means, to use Joan Robinson's phrase, the 'index-number birds' may be scared off. I have myself come to the problem of capital measurement as part of the general problem of measuring economic aggregates *in real terms*; so I fear that I may be taken, on this occasion, to be one of the birds who were to be scared. But it is by no means my intention to be destructive. I belong to the party which is still looking to find, at the end of its journey, a rehabilitation of the so-called 'Production Function' $P=f(L, C)$ in some form or other; what I am looking for is a concept of capital which will ultimately allow us to think, more or less, in those terms. But I think it is clear, once the question is posed in that way, that it will be impossible for any concept of capital to be used in that manner unless it is provided with a concept of Product which is correlative with it. The measurement of capital and the measurement of product are at bottom two aspects of the same problem; what has been learned about the one matter must be relevant to what has to be learned about the other.

It is, however, convenient to begin from a rather obvious difference between the problem of capital measurement and that of measuring those income flows where the principles of measurement in real terms are more fully understood. It is a characteristic of the flow of real income that most of the items out of which it is composed are goods and services that are currently valued upon the market. Index-number comparisons, which are based on these market values, do therefore seem (even on a quite unsophisticated interpretation) to make very good sense. It is indeed true that there are some things which we may want to include in income, though they are not market-valued, or not clearly market-valued : a difficulty which some would evade by restricting their concept of product — to the production of consumption goods for the private consumer, or to the gross product of private industry. But even if one insists on a broader definition, it remains true that the difficulty which is raised, on the income side, by these awkward cases is not so very serious. We can still rely for the most part upon market values ; there are relatively few items values for which we have to *impute*.

The position is entirely different when we turn to capital measurement. Though there will be some capital goods which have changed hands, or are about to change hands, at dates that are fairly near to the moment of time at which the capital measurement is taken, this linkage with market values is now not the rule but the exception. (Even when there is a linkage with market values at approximately their right date, it is usually complicated by an economically significant transformation of the good between the moment of measurement and the moment of purchase, or sale.) Thus it seems true to say that while the valuation of income goods is characteristically a market valuation, the values of the goods which enter into the capital stock are characteristically imputed values. We cannot take over a market valuation for them ; we have to set values upon them ourselves.

The distinction which I have just been making marks the great practical distinction between the problem of measuring capital and the problem of measuring income ; but I have taken care, in the way I have stated it, to make it clear that it is not a difference in the nature of the two problems — it is a distinction in degree, not in kind. Though the question of imputation is central in the case of capital measurement, it is not absent in the case of product measurement, unless indeed we take deliberate steps to restrict our concept of Product (or Income) so as to exclude it. Tempting as it is to take that line, it now appears that it will carry the penalty of cutting off our concepts of income and capital from contact with one another. The alternative procedure, of using the hard cases on the income side as a bridge between the two problems, begins to look as if it would be more promising.

That, in any case, is the line which I propose to take in this paper. I have myself given a good deal of thought, from time to time, to the question of imputing values, as that question arises on the side of income measurement.[1] I shall begin here by setting out some broad principles which have seemed to me to compel acceptance in relation to that other problem ; I shall then go on to enquire how far these same principles can be of help to us in the capital field also.

II. IMPUTATION IN THE MEASUREMENT OF INCOME

In a perfect market, prices are proportional to marginal utilities, and to marginal costs. Prices may therefore be regarded as reflecting *either* marginal utilities *or* marginal costs ; if all the goods which entered into an economic aggregate were sold upon perfect markets, the prices at which they were sold could be taken to stand for marginal utilities, or for marginal costs, indifferently. If markets are not perfect, utilities and costs may diverge ; price may diverge from either, or from both. If there is no market there is no price, but a valuation in terms of utility, or in terms of cost, may still be possible, though there is now no reason why those valuations should coincide. When we seek to set values on non-marketed commodities, it is the utility value, or the cost value, that we must be setting, but these will not (in general) be the same. In general (if we look away, as we must now do, from the case of the perfect market, where the two valuations are brought into equality) there are two principles of valuation, a utility principle and a cost principle, answering at bottom quite different questions.

Let us consider, for a moment, the easiest application — to the consumption goods sector of the Social Product. We may then compare one output of consumption goods with another, either from the standpoint of its capacity to satisfy wants (utility), or from the standpoint of its producibility from given resources (cost). Of these, it is the utility measure which has received by far the greater amount of attention in the literature, where the point that the want-satisfaction criterion, which has a fairly clear meaning when the Product is being equally divided among a homogeneous population, loses its precision when there is unequal distribution, has been elaborated at least as much as it deserves (and perhaps more). I fully admit the validity of this objection, though I have reasons, which I have stated at length in a recent article,[2] for holding that its importance has been

[1] Since the days of my 'Valuation of the Social Income', *Economica*, 1940, p. 105.

[2] 'The Measurement of Real Income', *Oxford Economic Papers*, 1958, p. 125.

much exaggerated. But I shall not enlarge upon that point here. For though, as we shall see before we have done, there are some things to be said about the measurement of capital from the utility side, it is the cost measure which appears in this case to have the greater relevance to practice. And it will, I think, turn out that we shall get further if we begin from cost than we should do if we started from a utility measure right away.

How exactly do we define this cost measure, or cost criterion? Since the definition is to extend beyond the price-system, it must (I think) make no direct reference to prices; it must be such as to have a meaning in any economy, whether it is a market economy or not. It is, in the first place, to give us a test whereby we can say that the set of goods which composes the Product of an economy at time (or place) A is larger than the set which composes the Product at B; it is to do so with reference to resources applied. The test must therefore run as follows. We are to say that 'A' is larger than 'B' if the B-set is such that it is within the possibilities of production with A-techniques, from the resources applied at A; while the A-set is *not* within the possibilities of production, with B-techniques, from the resources applied at B. (That is to say, if the matter is symbolized on the conventional two-commodity diagram, 'B' is inside the production-possibility curve of A, while 'A' is outside the production-possibility curve of B.)

Thus the test has two parts (as is not surprising by analogy with the ordinary index-number comparisons); and it must be admitted that it is quite a question whether the two parts will fit together. If the resources applied are at all specific, it seems only too likely that 'A' will lie outside the B-curve, while 'B' lies outside the A-curve, so that neither will show itself on this test to be larger than the other. It can of course happen with any index-number comparison that the Laspeyre and Paasche indices give different verdicts; we are used to the existence of a band of indeterminateness which separates the significant changes (in an upward or downward direction). The trouble, in the present case, is that this band may be extremely wide. But this is hardly a matter which it is useful to discuss in such general terms. I shall go ahead, for the present, assuming that there is a sufficient amount of substitutability in the system for the double test to display a reasonable amount of 'coherence'. How far that can possibly be so in particular applications is better studied when we have the particular applications before us.

The next thing which I have to point out is that the cost comparison of outputs, which has just been described, is in principle only one of a whole family of related comparisons. The production-possibility curves (or hyper-surfaces) which we have so far been considering are nothing more than 'sections' of that more general relation — or 'Production Function' in another sense of that expression [1] — which connects all inputs with all outputs under given technique. This 'Production Function' can be used in a similar way for many other purposes.

For instance, instead of fixing inputs and varying outputs, we can fix outputs and vary inputs. Then, granted (as always) that there is enough substitutability, we can ask whether the A-inputs lie within the 'curve' of factor-endowments that are sufficient to produce B-outputs, and whether the B-inputs are similarly sufficient to produce A-outputs ; by this means we can use the same technique to give us a measure of input as a whole.[2] And we need not stop there.

For there is no formal distinction between the ways in which inputs and outputs enter into the 'Production Function'. We do not have to hold all inputs constant, or all outputs constant ; we can vary some inputs, holding others constant, or some outputs, holding others constant, just as we like. Thus, on the same principles, we can measure a section of output, say, Food Output, at cost, by asking whether the A-outputs of Food could have been produced, by B-techniques, from B-inputs, while B-outputs of Non-Food were being produced simultaneously. Or we could measure a section of input, say Labour Input, by asking whether A-labour inputs, combined with B-non-labour inputs, would be sufficient (with B-technique) to produce B-outputs as a whole. We can draw the line between the things we vary, and the things we hold constant, as we like.

This is the basic idea, derived (so far) from quite static analysis, which I am going to maintain can be used to clear up some issues of principle about the measurement of capital.

[1] Though the mathematical economists (for instance, Tjalling Koopmans in *The State of Economic Science,* first essay) have lately been teaching us superior ways of putting it, it will suffice for present purposes if we think of this general Production Function in the old-fashioned form

$$F(x_1, x_2, \dots x_m, y_1, y_2, \dots y_n) = 0,$$

where the x's are outputs and the y's inputs.

[2] This measure of input as a whole is not the same as the measure of output as a whole, as might perhaps be supposed at first sight. In the one case we should be asking whether A-outputs could be produced from B-inputs with B-techniques ; in the other whether B-inputs would be sufficient to produce A-outputs with A-techniques ; and vice versa for the other limb of the comparison. If all went well, the relation between the measure of output and the measure of input ought to give us a measure of the improvement in technique — or, as it might be better to say, a measure of the *efficiency* with which resources are combined on the one occasion compared with the other.

III. INCOME AND CAPITAL MEASUREMENT IN TIME

The point has now come when we must endeavour to translate our static schema into an explicitly dynamic (or time-conscious) form. When we think of the production process *in time*, what is it that corresponds to the 'Production Function', or general relation between inputs and outputs, which we have just been using? The correct translation, I am going to maintain, is that which naturally arises from the Social Accounting framework. We must look at the production process during a *period* of time, with a beginning and an end. It starts, at the commencement of the Period, with an Initial Capital Stock; to this there is applied a Flow Input of labour, and from it there emerges a Flow Output called Consumption; then, there is a Closing Stock of Capital left over at the end. If Inputs are the things that are put in, and Outputs are the things that are got out, and the production of the Period is considered in isolation, then the Initial Capital Stock is an Input, a Stock Input to match the Flow Input of labour; and further (what is less well recognized in the tradition, but is equally clear when we are strict with our translation), the Closing Capital Stock is an Output, a Stock Output to match the Flow Output of consumption goods. Both input and output have stock and flow components; capital appears both as input and as output.

Between these four items (and between all components of the four items), there exists (with given technique) a production relation with much the same properties as the general 'production function', of static theory. If, with given (stock and flow) inputs, there is an increase in consumption (or in some component of consumption, other components being unchanged), then there must be a reduction in some component of the closing stock of capital — and so on. There is accordingly no reason why we should not use this relation as a basis for a system of measurement, in the same way as we used the static 'function' in the static application.

It is immensely tempting, because the capital stock appears as input and as output in the same 'production function', to suppose that there is some way by which we can re-shape that function so as to embody a direct comparison between the opening and closing stocks that it contains. But that, on the principles we are here elaborating, is a thing which cannot be done. We can measure capital either way, as input or as output; but between one capital stock regarded as input and another regarded as output there can be no comparison. The capital which appears as end-stock, and the capital which appears as beginning-stock, of the same productive process, cannot be compared by a reference to any property of that productive process itself.

What we can do is to take the end-capital which appears as output of a particular process, and make an *output* comparison between it and the beginning-stock of the same process; but then the beginning-stock must itself be treated as an output, naturally (of course) as the output of another process. Or we can take the beginning-stock of our process, and make an *input* comparison between it and its end-stock; but the end-stock must then be treated as the beginning-stock of another process. It is always necessary that there should be a process for each stock; and each stock must always have the same relation to its own process, if comparability is to be maintained.

We are now in a position to make a direct attack on the problem of comparing the capital stocks of two economies, or of the same economy at different dates, on the principles we have been laying down. We must decide whether we want to make an output or an input comparison. Suppose it is an output comparison; we then proceed as follows.

We must *prefix* to our capital stocks (A and B) processes of production (of, I think, the same calendar length) which can terminate in the actual stocks at A and at B. We then ask whether the B-stock lies within, or outside, the possibilities of production from the A process, if the A-flow of consumption is also to be derived from the A-inputs; and whether A is similarly within or outside the possibilities of production from B. That is the principle of the output comparison.

If we are to make an input comparison, we proceed in the same way in the other direction. We must *adjoin* to A and to B processes of production (of equal length) which are such that they can start from the actual capital stocks at A and at B. We then ask whether the application of A-labour to the B-stock would make possible the production of the A-flow of consumption goods, with the A end-capital left over at the end of the period; and, again, vice versa.

This is the principle of the comparison; but it is evident that there are many further questions to be answered before we can proceed from so general a principle to anything which gets within sight of being applicable to a particular case.

I begin with some questions about the processes which we are to 'prefix' or 'adjoin'. Are they actual or hypothetical? It would appear to be possible to take the actual experience of the A-economy, in the year (say) which preceded A as a matter of history, and use that as the process which we prefix to A in the output comparison; and similarly for B. This looks all right for the 'direct' A-process, that which we prefix to A, and treat as belonging to A; but what of the 'cross-process', which we need for the cross-comparison? This process, which is (say) to be based upon A-inputs, to produce A consumption outputs, but to leave over the B end-stock (as nearly as possible) as its end-capital, is without doubt a purely hypothetical construction. But on exactly what hypothesis is it constructed? If the comparison is to be fair, the cross-process must not only use A-inputs, it must also use A-techniques; but what are A-techniques? During the actual 'year' preceding A, techniques may have changed; the production possibilities of the earlier techniques will then be different from those of the later; which are we to use? There is no possibility of defining a cross-process which uses the *same* technique as that employed in the 'A-year'. Besides, we must always remember that what we want to compare with the B-stock is the capital stock as it actually is at A; technical changes which occurred somewhere further back, in the history which actually led up to A, are for this purpose irrelevant. I accordingly conclude that it is, in strictness, not the actual history of the process which led up to A which we want to prefix to A; it is a process, leading up to A, which maintains unchanged, over its whole course, the technology of the time A itself.[1]

It would accordingly appear, from this consideration alone, that the process to be prefixed to A is not the actual historical process which preceded A; it is a hypothetical process, more closely associated with the state of affairs at the precise moment A, at which the measure is taken. But if that is granted, it would seem that the process still requires more definition. Is there any reason why we should not prefix to A any sort of process of production, with unchanging technology, under the sole condition that it can terminate with the actual A-capital as its end-stock? Such a process could be constructed, which would terminate with A, in many ways. Thus it might have started from a much smaller capital stock, and have worked up to the A-stock by large input of labour and small

[1] We can speak of the *technology of time A*, in spite of the fact that (in a technically advancing economy) some of the capital goods existing at time A will be obsolete; they would not have been made in that form if A-knowledge had existed at the time of their original construction. The actual physical make-up of the A-capital stock is always a datum. When we are measuring it as output, it is the technology of *making* those capital goods which is relevant; their obsolescence does not enter into question. When we are measuring it as input, it is the use which can now be made of these actual goods which matters; it does not matter that better ways of making them, or superior substitutes for them, have been discovered. Obsolete equipment looks like being a formidable difficulty, only because it is so easy to get input capital and output capital mixed up.

consumption output (high saving); or it might have got there by still larger input of labour and less restricted consumption; or it might have started from a capital stock very similar to that at A, so that neither labour input need be notably large nor consumption output notably small. (I use quantitative expressions, for purposes of illustration; but the point is that the other inputs and outputs which enter into the 'production function' may be of many different amounts — and perhaps kinds.)

Of these various alternatives, there is one which stands out as having a special relation with the capital stock at A; and perhaps therefore as having a special claim to be regarded as *the* process which we ought to prefix. This is the stationary process, which begins with a capital stock that is identical with the A-stock, and which replaces, in its course, exactly the same stock as that from which it started.[1] It is possible to work out a theory on that basis, and I have myself spent some time doing so; but I have concluded, in the end, that it is not a procedure which offers any substantial advantage. It is by no means clear (when one allows, for instance, for such things as 'indivisibilities') that the capital stock of an economy need be such that it would be *possible* to prefix a stationary process to it. One must, I now think, take another line of escape.

If there is sufficient substitutability within the system, it should not matter very much (for the purposes of our comparison) how the prefixed process is varied, within limits, in the ways we have just been varying it. A comparison on the basis of a rather more labour-using process, and one on the basis of a rather less labour-using process (for instance), should, if the comparison is made consistently, give very similar results. If there is not enough substitutability for this to be true, then (as we saw previously) it is only too likely that the whole comparison will break down. So it seems that we must just have faith (or hope) that enough substitutability for the purpose does in practice exist. I believe that this is what economists do, without fully realizing it, when they attempt to measure capital. All I am doing here is to make more explicit some assumptions that were always there somewhere underneath.

IV. THE TIME-PATTERN OF THE PRODUCTIVE PROCESS

Nevertheless, having got to that point, there is a further question. Are there any steps which *we* can yet take to assure ourselves of the

[1] It should be noticed that we are not committed, by the mere assumption of stationariness, to any particular size of labour force to be used with the stationary stock. It is still possible that the capital may be used more or less intensively.

substitutability which, on more than one count, we now so sorely need? Are there any arbitrarinesses still remaining in our construction which we can yet fill in, and by so doing make the chance of 'coherence' greater, and the comparison a little easier? One matter which has still been left quite arbitrary is the time-length of the productive process which we are to prefix (or adjoin, as the case may be). There are several points which can be brought under this heading which remain to be made.

Some of the difficulties about technical change which have just been worrying us would have their sting taken out of them if we were prepared to admit that the period could be a very short period. The idea of measuring capital at time t by reference to a process of infinitesimal length ($t - dt$ to t, or t to $t + dt$) is mathematically most attractive; it would enable us to exclude the possibility of changes in technique within the period. But, for our general purpose, it is quite clear that it will not do. For though the difficulties of the *direct* process are (perhaps) reduced to a minimum by such shortening, those of the cross-process are aggravated to such an extent as to be intolerable. If we are to be able to make the cross-comparison, we must give time for the A-stock to be transferred, by some process of saving (or dissaving) and replacement, into the B-stock; or (what may be much easier) for an initial stock, which is not identical with either the A-stock or the B-stock, to be transformed, on the one hand into A and on the other into B. If the A- and the B-stocks differ at all seriously, it is surely impossible even to conceive of such a transformation unless much time is allowed. We cannot manage with too short a process.

But if we elongate the process, we run into another difficulty. I have so far allowed myself to talk as if the *flows* (of labour input and consumption output) were simply composed of sets of quantities (of various sorts of labour and of products) which themselves had no particular time-reference, except to the process as a whole. If the period is short enough, this may be adequate; the only variations in the consumption flow (say) which we may need to consider are variations in the total amounts of each sort of consumption good made available over the whole period. But when we lengthen the period, this must soon cease to be the case. Postponement of a particular output from early to late (or, more generally, the transference of a marginal unit from a date at which its availability is more, to one where it is less *convenient*) makes a difference which, if we pay no attention to the time-shape of the income flow, we are neglecting. This neglect, when the argument is translated into price-terms, amounts of course to a neglect of interest.

In the face of this complication, there are three courses which could conceivably be adopted.

The first is boldly to maintain that a cost measure of capital cannot pay attention to the time-shape of the consumption flow ; a flow with one time-shape, and one with another shape, have to be treated as the same for this particular purpose. In spite of the somewhat Marxist flavour of this alternative, I do not think we should reject it out of hand. It is a simple way out, and if the period is not too long, it may not do much harm. But if the period is stretched out (as we have found to be rather necessary when the structure of the capital stocks that are to be compared is decidedly different), the neglect of time-shape may lead to serious trouble. For if we take a *direct* flow that has some definite time-shape (either the actual historical time-shape, or perhaps some approximation to constancy over time — and it is hard to see how we can prefix a flow that does not have some definite time-shape) ; and if we then set against it a cross-process that is not obliged to keep to any definite time-shape in the distribution of its outputs, we are giving an unfair advantage to the cross-process. We are then in danger of getting the opposite sort of incoherence to that which had troubled us previously ; we are liable to find that each capital stock is larger than the other from its *own* point of view. And that is worse than the other sort of incoherence ; it is bad to get no answer, but to get an answer which makes nonsense is worse still.[1]

I am, however, inclined to look upon this first alternative with less disfavour than might perhaps have been expected, because the most obvious alternative to it is itself incapable of providing us with a practical way out. We might insist that the cross-process is to have the same time-shape as the direct process : that not only must the same consumption goods be produced as in the direct process, but that they must be produced at the same times. However, this is a requirement which is surely, by its nature, quite incapable of fulfilment. The goods which can be produced, in the short run, from a given capital stock, are surely related, very closely related, to the composition of that stock ; we cannot start from a given stock and set it producing, *at once*, a bill of goods for which it is quite unsuited. This second alternative has to be considered, but the objection to it is such that it must be completely ruled out.

[1] The same point may be put in another manner. If we prefix a short period, we are (effectively) treating the 'capital cost' of the end-stock as being attributable to the beginning-stock ; if we are including that, it does not much matter if we attribute all the remaining cost to labour, and neglect any further 'waiting'. But if the process has to be elongated, to permit transformation, the neglect of waiting will lead us much more seriously astray.

Is there any third alternative? I cannot see that there is, so long as we keep strictly to 'cost' principles, and avoid any reference to utility. Only if a reference to utility is permitted, can we cut the knot. For we can then stop saying that the cross-process is to produce exactly the same goods *at the same times*, as the direct process. All that is necessary is that the one flow should be *equivalent*, in utility terms, to the other.

That, as is only too well known, raises all the conundrums about inter-personal comparisons, which have led so many economists into a quagmire. Here, where the question is not only one of a choice between goods, but also (indeed particularly) of a choice between dates, so that we have to raise the issue in the particularly awkward form of the comparability of time-preferences, these horrors will be especially acute. But this is no occasion for a sally into that arena. I will merely observe that if we can bring ourselves to accept some formula for utility comparison, we can not only [1] get a coherent theory of income (or consumption) measurement, we can also overcome the worst obstacle in the way of a theory of the measurement of capital. But though, in the end, I am driven to adopt this utility way out, the efforts which we have been making to construct a theory of capital measurement which avoids utility will not have been wasted.

For even if the theory, which we can now envisage, does not in the end avoid all reference to utility, it need only refer to utility for a subsidiary purpose. It remains true that capital may either be measured in a forward-looking, or in a backward-looking manner. The backward-looking test still asks whether B-capital could have been produced from an A-process: starting from an original equipment that could terminate with the A-stock, with the same (A) labour applied to it, and after turning out a flow of consumption goods 'at least as good' as that which was turned out in the A-process. In other (and more familiar) words the question is whether the B-capital could have been produced out of the labour and saving that were needed for the A-capital. Alternatively, on the forward-looking test, we ask whether the B-capital could produce, with the co-operation of A-labour, a stream of consumption goods which is 'at least as good' as the stream of consumption goods similarly producible from the A-capital. (It is, in this case, hardly necessary to do more than to look at the flow of consumption goods, for we can elongate the period, so as to push the end-capital so far into the future that it becomes of little present importance.) These are the two ultimately very simple questions which seem at the end to stand out.

[1] As I tried to show in the *Oxford Economic Papers* article above cited.

V. CONCLUSION: THE PRODUCTION FUNCTION

I return, in conclusion, to the Production Function, in the other sense of $P=f(L, C)$. It would be helpful to have a name which would distinguish this from the other ; I suggest that this function (of which Cobb-Douglas is of course a special case) might be called the 'Reduced Production Function'. How far do the concepts of capital which I have been elaborating enable us to make sense of this Reduced Function ? There are, I think, two useful things on this matter which one can now say.

It is clear, in the first place, that if we take a forward-looking concept of capital, and introduce it into a Reduced Production Function, we are making that Function a mere tautology.[1] For the forward-looking concept measures capital in terms of the Product which it can produce with the aid of given labour ; a larger capital so applied *must* then produce a larger product. It is its capacity to produce a larger product which makes it larger. Such an interpretation gets us no further on. Or so it would seem : but there is one way in which it gets us further on, for it reminds us that the concept of Product, which is needed to make sense of the Reduced Function, is itself correlative with the forward-looking concept of capital. It is not the quantity of consumption goods, produced in any given period, which has the functional relation with capital. It is the whole stream of consumption goods, extended into the indefinite future, reduced to homogeneity by reference to a scale of preference, including time-preference. Income, in the Lindahlian sense of the term, will stand for that concept of Product very well.

The concept of Capital which does enter, non-tautologously, into the Reduced Production Function, is the backward-looking concept of capital. When capital is taken in that sense, and Product in the sense just described, the Marginal Productivity of Capital has a clear meaning.

For let us take our stand at time t ; and let us both prefix to t a process leading up to t, and also adjoin to t a process leading on from t (the technique appropriate to time t being preserved, in the way it should be, throughout each of these processes). Then C is a reflection of the backward-looking process, and P is a reflection of

[1] It would be more correct, as was pointed out to me privately by M. Malinvaud after I had read this paper, to say that the Production Function is tautologous on the capital side. Even if capital is measured in a forward-looking sense, the marginal productivity of labour retains a meaning ; it is the marginal productivity of capital which becomes meaningless, once we have defined capital in such a way that there is always more capital if there is more product.

the forward-looking process. If C had been larger, that would mean that more labour or saving had been applied in the former process; if P is larger, that will mean that the flow of consumption goods is larger (as a whole) in the latter process. So that if capital has a positive marginal productivity, an increase in input, or a reduction in output, in the former process makes possible an increase in output (without reduction in input) in the latter process. The Marginal Productivity of Capital is the Marginal Productivity of Roundaboutness, after all.

Chapter 3

THE EVALUATION OF 'SOCIAL INCOME': CAPITAL FORMATION AND WEALTH

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I. INTRODUCTION

AN earlier paper [1] dealt with problems of interpreting aggregate data on production and consumption. The present paper explores theoretical problems raised for national income calculation by investment, technological change, and uncertainty windfalls. It purposely works with simple and strong models so that the intrinsic difficulties will not be obscured by observational difficulties of detail. Government expenditure and taxes are throughout ignored.

The simplest neo-classical capital model is that of Ramsey. In my version, it assumes a fixed amount L of 'primary' factor that I shall call labour, but that could also be interpreted as unaugmentable land or a composite of natural resources and labour. A homogeneous flow of gross product called $\dot{Y}$ per unit time is produced by labour and by a physically homogeneous stock of capital K according to the production function

$$\dot{Y} = F(K, L), \tag{1}$$

which obeys the classical laws of constant-returns-to-scale and diminishing returns to variations in capital unaccompanied by any change in labour.

The current flow of product is assumed to be allocable [2] between (i) current consumption flow of product $\dot{C}$ and (ii) gross capital formation per unit time of capital, which I call $\dot{K}$: or

$$\dot{Y} = \dot{C} + \dot{K} = F(K, L). \tag{2}$$

To arrive at (iii) net capital formation, dK/dt, we must subtract from gross births of capital the current rate of capital deaths, called $\dot{D}$

[1] P. A. Samuelson, 'Evaluation of Real National Income', *Oxford Economic Papers* (New Series), 1950, pp. 1-29.

[2] Equation (2) has assumed for simplicity (what is dropped in Fig. 3) that the consumer and capital goods industries always use labour and capital in the same proportions.

to refer to current rate of depreciation or capital consumption. If capital of all ages is to be really homogeneous, we must assume that depreciation involves the physical destruction of capital at a rate independent of age and proportional to the existing total stock of capital. This involves a life table for any given stock of capital showing a radioactive rate of decay in an exponential fashion.

We can summarize our capital consumption postulate in the relations

$$\dot{D} = mK$$
$$\frac{dK}{dt} = \dot{K} - \dot{D} = \dot{K} - mK, \tag{3}$$

where m is a positive constant representing the percentage rate per unit time of capital depreciation. The average length of life of capital will then be $1/m$.

We may now summarize our net national product flow $\dot{y}$ and its allocation between consumption and net capital formation as follows :

$$\dot{y} = \dot{Y} - \dot{D} = F(K, L) - mK$$
$$= \dot{C} + \frac{dK}{dt} = f(K, L) \tag{4}$$

where the new net product f function is defined by (4).

II. NET VERSUS GROSS CAPITAL FORMATION

The above relations between net and gross national product and gross and net capital formation are the familiar ones met with in the literature of national income. Within the framework of a purely theoretical model such as this one, I believe that we should certainly prefer net national product, NNP, to gross national product, GNP, if we were forced to choose between them. This is somewhat the reverse of the position taken by many official statisticians, and so let me dispose of three arguments used to favour the gross concept.

First, there is the argument that estimates of depreciation are conceptually and statistically inaccurate so that $\dot{Y}$ is more accurate than $\dot{y} = \dot{Y} - \dot{D}$. Within our simple model, we know precisely what depreciation is and so for our present purpose this argument can be provisionally ruled out of order.

Second, there is the argument that GNP gives a better measure than does NNP of the maximum consumption sprint that an economy could make by consuming its capital in time of future war or emergency. If we are speaking *ex post* of an emergency economy, the recorded level of consumption $\dot{C}$ does tell us how much it has been consuming for peace and war and no further measure is needed. If consumption exceeds NNP, the positive difference does accurately measure the rate of recorded capital disinvestment. If the level of gross capital formation was recorded to be zero, then the emergency consumption did indeed equal the whole of the recorded GNP.

None the less, it is misleading to apply a *present* non-emergency level of GNP in forming an estimate of *future* emergency consumption potential. Once we recognize that depreciation depends on intensity of use as well as on mere passage of time, we realize that future emergency consumption is not limited by *present* GNP but rather by the maximum future GNP that the system can squeeze out through more intensive using-up of its capital.

This means that the size of GNP producible at any time with given technology, capital and labour is *not* a solid figure ; it depends on our volition and is what we make it ; it is in this respect unlike NNP whose maximum value we cannot better without somehow getting more inputs or changing production functions. A striking example of my contention is the strong case where all capital is like a storage battery or an inventory of chocolates. In any very short period, our rate of chocolate or electricity consumption can be as great as we wish provided only we run down our capital fast enough. This means that we can in any emergency period think of our decisions as increasing m, the force of mortality of capital, to any desired level. As a result, the level of emergency consumption — and emergency GNP — can be much greater than the usual peace-time GNP.[1]

Our summarizing equations (4), in writing $\dot{y}=f(K, L)$, seem to assume implicitly that m is a given technical constant which is not one of the economic unknowns of our problem. This seems to imply that we recognize depreciation due to the passage of time but deny the existence of depreciation due to service or to 'user cost'. Actually, I am willing to entertain the assumption that current gross production can be increased by deferring maintenance and increasing intensity of capital use as measured by m. Thus, we can write $\dot{Y}=F(K, L, m)$; but we do suppose that for each given L and K,

[1] Our model has not distinguished between (i) inventory or circulating capital and (ii) fixed capital or capitalized items. The accountant arbitrarily uses one year or some other criterion to determine which way any item is treated. The result of a change in such an arbitrary decision is an arbitrary change in the level of GNP as we double count more or less in the total. However, and this is one of the most important arguments for NNP, the net magnitude remains invariant under changes in accounting conventions. Incidentally, with inventories *not* capitalized, we definitely can have $\dot{C}>GNP$ because gross investment $\dot{K}$, inclusive of negative inventory change, can be negative. Thus, emergency GNP computed along official lines does not correctly measure maximum emergency consumption. Nor does it give much insight into the range of feasible time profiles of emergency consumption.

there has already been selected an optimal intensity m so as to maximize net product, $\dot{y} = F(K, L, m) - mK$, as determined by the condition

$$\frac{\partial \dot{y}}{\partial m} = 0 = \frac{\partial F(K, L, m)}{\partial m} - K. \tag{5}$$

We assume that this optimum value of m has been determined and substituted into $F(K, L, m) - mK$, which can therefore be written as our function of K and L alone, $f(K, L)$.[1]

A third argument favouring a gross rather than net product figure proceeds as follows: new capital is progressively of better quality than old, so that net product calculated by the subtraction of all depreciation and obsolescence does not yield an ideal measure 'based on the principle of keeping intact the physical productivity of the capital goods in some kind of welfare sense'.[2] The next section's separate argument will deal with problems of technological change and uncertainty leading to quality changes. Within our simple theoretical model, depreciation is correctly calculable with no obsolescence problems arising. So we have no reason to think that our gross product $\dot{Y}$ is a better approximation to true net product than is our exact net product $\dot{y}$ itself.

III. QUALITY CHANGES AND CAPITAL GAINS

The 1935 debate between Pigou and Hayek as to the meaning of maintaining capital intact can be pointed-up by an extension of our simple model. Suppose in addition to K a new capital good K_2 is

[1] If war makes us want $\dot{C} > \dot{y}$, we at first find it advantageous to stay with our optimal peace-time m. The excess $\dot{C} - \dot{y}$ is at first matched by positive $D - \dot{K}$, and the old optimal m gives us the minimum of disinvestment of capital compatible with our emergency consumption needs. However, after consumption goals have been so increased as to make $\dot{K} = 0$, we cannot on most interpretations imagine making *gross* investment actually negative. But from now on we let $\dot{C} = F(K, L, m)$ with $dK/dt = -mK$; and by increasing the intensity with which capital is used still further, we can further increase current consumption. But with m varying from its optimal peace-time value, we meet the following paradox: to enjoy high sprints of consumption, we greatly increase GNP, deliberately letting NNP as measured by $f = F - mK$ (slightly) decrease. The paradox is resolved when we reflect that $\dot{C}$ can only be added algebraically on a one-to-one basis to dK/dt in a meaningful valuation provided society really can achieve 1 extra C for each extra unit of disinvestment. When $\partial f/\partial m = (\partial F/\partial m) - K \neq 0$, because we have exceeded the optimal peace-time m, each new unit of $\dot{C}$ really costs (and is worth) more than each sacrificed unit of capital formation: so a correct emergency NNP would be computed by giving greater than unity weight to $\dot{C}$ relative to dK/dt; and this new correct measure would not be sub-maximal in the emergency sprint. (See Fig. 2's curvature beyond d for an illustration of all this.)

[2] Quoted from Richard Ruggles, 'Concepts, Sources, and Methods of United States National Income Accounts', *Econometrica*, 1952, pp. 469-70, by E. F. Denison, 'Quality Change, Capital Consumption, and Net Capital Formation', *National Bureau Conference on Research in Income and Wealth*, October 9-11, 1953.

discovered which has twice K's productivity in *every* use but which is produced by exactly the same production function as K (same costs) and with the same life expectancy as K. Then we get

$$\dot{C}+\dot{K}+\dot{K}_2=F(K+2K_2,\ L)$$

$$\dot{K}=\frac{dK}{dt}+mK\geq 0,\ \dot{K}_2=\frac{dK_2}{dt}+mK_2\geq 0.$$

Competition will immediately insure that $\dot{K}=0$ as the old capital good becomes 'obsolete' in the sense that its current value at once drops to half its cost of reproduction: its algebraic net investment will be $dK/dt=-mK$ for ever afterwards.

How shall we write NNP (i) ever afterwards; (ii) before the new K_2 is at all known; and (iii) the 'instant' or period it becomes known? We have already given the answer to (ii) in (4) — but if the new K_2 were 'suspected' in advance but not known as to exact date of its invention and form, what would the answer be? Question (iii) involves the issue of what financial revaluation you wish to put in NNP or GNP. The important question (i) would seem best answered by going from the indisputable relation

$$\dot{C}+1\frac{dK_2}{dt}+\tfrac{1}{2}\frac{dK}{dt}=F(K+2K_2,\ L)-mK_2-m\tfrac{1}{2}K$$

$$=g(K+2K_2,\ L),\ \frac{dK}{dt}=-mK,$$

to the definition

$$NNP=\dot{y}=\dot{Y}-1\dot{D}_2-\tfrac{1}{2}\dot{D}. \tag{6}$$

Operational difficulties of statistical measurement aside, my solution may be more nearly akin to Hayek's than Pigou's (or Denison's): the depreciation I subtract from GNP to get NNP evaluates the physical capital being used up not at its historic (bygone) production cost, nor at its current reproduction cost if it is in fact not being produced, but at its current competitively-bid used market price.[1] Note that the post-invention NNP has risen, whatever it may be supposed to have done in the transition when learning the 'bad news' of the new invention was causing anguish and revaluations by owners of old K. The immediate post-invention GNP has not changed at all, even though society's 'prospects' are now

[1] Suppose in a decade better K_2 is replacing K but with $dK_2/dt=-dK/dt$ so that the national income statistician is registering zero capital formation. Then using my measure of capital formation $\frac{1}{2}dK/dt+1dK_2/dt$, positive capital formation is really going on. Add the value integral of this over the decade to initial $\frac{1}{2}K+1K_2$ and call the result V. Then the new product will be given by $F(2V,\ L)$, reflecting a higher GNP, and where the factor 2 must be written in the old production function to portray correctly the new production possibilities.

definitely improved. While the physical capital-output ratio K/f will at first remain constant, the value capital-output ratio pK/f will immediately halve; the incremental capital-output ratio, in the relevant form $[\partial(dK_2/dt)/\partial K_2]^{-1}$ will be half that of the old $[\partial(dK/dt)/\partial K]^{-1}$ as the interest rate doubles.

IV. FACTOR PAYMENT IMPUTATION OF PRODUCT

I have assumed conditions most favourable to viable competition — namely constant returns to scale. Therefore, competitive pricing of the services of labour and capital can in the absence of uncertainty be relied on to create a state of zero (excess profits) and a price of consumption and investment flow exactly equal to unit cost of production as measured by the sum of wages and capital rents per unit of product. Thus

$$p\dot{y} = p\left(\dot{C} + \frac{dK}{dt}\right) = wL + rK, \tag{7}$$

where (p, w, r) are respectively (i) the price (in money or any other *numeraire*) of current flow of output, whether consumption output or equivalent investment output; (ii) the wage of labour per unit time, which is to say the rent per unit time for the services of human population; and (iii) the rent per unit time for the use of capital K, it being understood that the capital is returned intact to its owner — or what is the same thing that the gross rental rate R has subtracted from it an allowance for depreciation before the capital owner can reckon his net rental r. This means we can write down a factor payment breakdown of gross national product as follows:

$$\begin{aligned} p\dot{Y} &= p(\dot{C} + \dot{K}) = p(\dot{y} + \dot{D}) = p\left(\dot{C} + \frac{dK}{dt} + \dot{D}\right) \\ &= wL + RK. \end{aligned} \tag{7)'$$

In our simple theoretical model, the rents of all factors can be immediately recognized as equivalent to the value of their marginal products. Thus, on a net and gross basis, we have

$$\begin{aligned} r &= p\frac{\partial f(K, L)}{\partial K} \\ R &= p\frac{\partial F(K, L)}{\partial K} = p\frac{\partial f(K, L)}{\partial K} + pm = r + mp \\ w &= p\frac{\partial f(K, L)}{\partial L} = p\frac{\partial F(K, L)}{\partial L}. \end{aligned} \tag{8}$$

These relations show us that the subtraction from gross rent R to get net rent r equals the percentage depreciation of capital per unit time m multiplied by the price of capital p: i.e. $R - mp = r$ in our model.

Euler's theorem on homogeneous functions is of course applicable under our neo-classical assumptions of constant returns to scale; this assures us of the compatibility of the exhaustion of product assumptions of (7) and (7)′ with the marginal productivity equivalences of (8).

Table 1 gives a symbolic presentation of the usual two views of national income or product: on the left-hand side we have the cost

TABLE 1

Factor Payments or Cost		Flow of Product	
'Profits'	0		
+ Wage Rents	$wL = p\frac{\partial f}{\partial L}L$	Consumption Product	pC
+ Capital Net Rents	$rK = p\frac{\partial f}{\partial K}K$	+ Net Capital Formation	$p\frac{dK}{dt}$
Total of Net National Income or Product	$wL + rK = pf$	Total of Net National Product	$p\left(C + \frac{dK}{dt}\right) = pf$
+ Depreciation or Capital Consumption	$pmK = pD$	+ Difference between Gross and Net Capital Formation	$p\left(\dot{K} - \frac{dK}{dt}\right) = pmK$
Total Gross National Product	$wL + RK = pF$	Total Gross National Product	$p(\dot{K} + C) = pF$

or factor payment view; on the right-hand side, the flow of consumption and investment product.

The above view regards value of product as being composed on the cost side completely of current factor rents: rents for use of the stock of capital; rents for the use of stocks of primary factors, in our case human population. Rents for the use of physical capital goods are commonly met in modern economies, so this corresponds to usual usage. More bizarre is the appellation 'rents' for what is usually called wages. But reflection shows that under most systems of jurisprudence, human population is the one factor that *cannot* be bought outright: unless slavery is permitted, labour services must be rented.[1]

[1] Because we assume a fixed total of L, no problems of depreciation of human capital here arise. But even in a steady population, if we let individuals die and be

However, if the factor labour could be capitalized and titles to labour transferred, we could calculate the total value of all stocks of factors, including such primary factors. The total value of net product could now be regarded not as total factor *rents* but rather as total *interest* on the capitalized factor wealth (account being taken of 'capital gains' magnitudes).

Because labour is usually not capitalized, we can on the left side of Table 1 leave in wage rents. At the same time we may replace capital rents by interest on the capitalized value of physical capital assets.[1]

But where do we get (*a*) the instantaneous interest rate per unit time i, and (*b*) the capitalized value of the physical capital K?

One advantage of our simple capital model is the ease with which it can answer these questions. As long as there is positive gross capital formation going on, so that $\dot{K}>0$ and both $\dot{C}$ and $\dot{K}$ are being simultaneously produced, the current market price p quoted for consumption and investment flows provides us also with the current value of the stock of capital K. We are in the following fortunate position: *A current reproduction cost of the capital that is being produced always gives us an unambiguous market value for all capital.*

We can now work backwards: what interest rate i will multiply into the value of capital pK to get us the same non-wage net rental income as does rK? Clearly, we must have

$$i=\frac{rK}{pK}=\frac{r}{p}=\frac{\partial f(K, L)}{\partial K}, \tag{9}$$

where the last marginal productivity relation comes from our earlier derived equations (8). Among the factor payments on the left-hand side of Table 1, we can now replace rents of capital goods rK by the equivalent interest return on value of capital $i(pK)$.

born and if we do not define our units as clans that maintain the same age distribution, there does arise for the individual problems of valuation of lifetime earnings and of depreciation. A so-called 'personal income tax' that treats the perpetual earnings of securities the same as the earnings of a doctor or actor in the prime of life chooses to ignore a substantive difference.

[1] Adding some rents to some interest may seem illogical. If so, current statistical practice is still more illogical. Even the items that can all be capitalized are treated either as rents or as interest depending upon the accidental institutional form in which families happen to choose to hold titles to the productive assets used by business. An even greater heterogeneity is introduced by lumping together into corporate or unincorporated enterprise 'profit' a *mélange* of implicit wages, rent, or interest earned on owner-supplied factors and various returns that result from lack of 'perfection' in all economic markets. Of course, the statisticians are not to be criticized because the world happens to depart from simplified conditions. (See my p. 48 footnote for the demonstration that factor rents differ from interest by a fundamental 'capital gains' term.)

V. OWN-RATES OF INTEREST AND PRICE CHANGES

A rate of interest is a pure number per unit of time. How then can (9) make i equal a physical marginal productivity? If we rewrite $\partial f/\partial K$ in the form $\partial(dK/dt)/\partial K$, we recognize that the dimensionality K cancels out, leaving us with a pure number per unit of time. We can interpret $\partial f/\partial K$ as an 'own-rate-of-interest' *in natura*: it gives the 'net productivity of capital', the rate at which extra capital stock can produce extra rate of growth of *itself*.[1]

So long as the price of capital goods is constant over time, the own-rate-of-interest i is the same thing as the prevailing rate of interest on money loans i_m. Thornton, Marshall, Wicksell, Fisher, Keynes, and others have known that the own-rate and the money rate of interest must diverge by a term equal to the percentage price change of the good in terms of which the own-rate is measured. Thus,[2] equilibrium requires that

$$i_m = i + \frac{dp}{pdt}. \tag{10}$$

This says: When the price of K is rising in a foreseen way, the money rate of interest will exceed the own-rate of interest by the foreseen percentage rate of price inflation.

Statisticians are alert to the possibility that general price changes may introduce revaluations into their measured profits. Under conditions of certainty, price changes will necessarily introduce themselves into contractual interest as well. Under these conditions, a typical firm that owns physical capital financed completely by borrowed money can 'afford' to pay the prevailing high market rate of interest only by reckoning as an addition to its current sale receipts a capital revaluation term equal to $K(dp/dt)$. The resulting augmented total of calculated 'receipts' will just equal its wage-rent payments plus money-interest payments, leaving it contented with zero residual 'profits'.

If the statistician accepts such income statements and consolidates them for all businesses, he will arrive at a total greater than the NNP of Table 1 whenever prices are rising in a foreseen fashion. This total will then be $p\dot{y} + K(dp/dt)$. If he tries to eliminate this

[1] If all the fruits of capital were ploughed back into investment, $i = \partial f/\partial K$ would measure the slope of product growth plotted against capital: i.e. $i = (df/dt)/(dK/dt) = (d\dot{y}/dK)$ if $\dot{C} = 0$; it is then K''/K' the slope (or percentage rate of growth) of net investment plotted against time on semi-log paper. Whether or not $\dot{C} = 0$, i is the reciprocal of the marginal capital-output ratio $\partial K/\partial f(K, L)$.

[2] See P. A. Samuelson, 'Some Aspects of the Pure Theory of Capital', *Quarterly Journal of Economics*, May 1937, pp. 469-96, particularly equation (10) of Math. Appendix I.

element of capital revaluation, he can do so only by refusing to count in the $K(dp/dt)$ receipt that the firm has been relying on in determining its prudent borrowing. Only if the statistician adopts the rule that firms are making 'negative residual profits' equal to their current cash deficit $p\dot{y} - wL - i_m pK$ will he be able to get the left-hand factor payment side of Table 1 to add up to the same total as the right-hand side. It will no doubt come as a surprise to the firms to learn that when they are just breaking even according to their prudent calculations, they are really enjoying negative adjusted profits. Yet unless a firm is selling off some of its physical capital to families, it cannot 'realize' the capital revaluation term and will be having to do new borrowing to finance its cash deficit. If the rentier families try to spend all their money interest on consumption, they will find it necessary to reduce their ownership of physical capital.

Three facts require emphasis. Although we speak here of capital gains, there is nothing of the 'windfall' character in them; they are completely foreseen and counted on in validating all decisions. Secondly, these capital gains are not necessarily temporary or 'non-recurring', or unmaintainable. On the contrary, if the balanced rate of inflation were known to last for a century, the capital gains would occur in every year.

Thirdly, different kinds of assets and contracts must all create the same instantaneous yield per unit of time, equal in money to i_m times capitalized market value V; but each different contract will have this common yield made up in different proportions of (*a*) current coupon receipt or dividend and (*b*) capital appreciation. In a perfect capital market, the fundamental identity between current net receipts N, value V, and interest i_m is

$$i_m = \frac{N}{V} + \frac{dV}{Vdt} \tag{11}$$

holding for all assets.[1] This means that a $2\frac{1}{2}$ per cent coupon bond maturing in 1970 will sell at a discount compared with a 5 per cent coupon maturing in 1970. The changing amount of the discount is such that their yields are equal in every period. The U.S. and U.K. governments treat much of the yield of the old discount bond as so-called capital gain and tax it more lightly or not at all: the national income statistician must not — at least under conditions of certainty — make this same mistake.

The price changes discussed in the last few paragraphs have all been of a generally inflationary type, with consumption goods changing in price by the same amount as physical capital goods.

[1] *Ibid.* p. 471, equations (7).

However, if we widen our model so that there are many kinds of capital goods, it is easy to produce cases where the mere accumulation of capital depresses interest rates and raises capital prices relatively to consumption good prices. Investors can then truly feel that their capital gains increase their command over consumption goods.

VI. EQUALITY OF CAPITALIZED VALUE AND REPRODUCTION COST

We have deduced a market price for all physical capital from reproduction cost. Is this necessarily consistent with its value as calculated in Irving Fisher's way by summing ***present discounted values*** of all future receipts? The answer can be shown to be, Yes.

Employing money units for all valuations, we must use the compound interest formulas appropriate to instantaneous interest rates that *change* according to the foreseen function $i_m(t)$. Then[1] value of K at $t = t^0$ is

$$
\begin{aligned}
V &= \int_{t^0}^{\infty} r(t) \exp\left[-\int_{t^0}^{t} i_m(T)dT\right]dt \\
&= \int_{t^0}^{\infty} \frac{\partial f[K(t),L]}{\partial K} p(t) \exp\left[-\int_{t^0}^{t} \frac{\partial f[K(T),L]}{\partial K} dT - \int_{t^0}^{t} \frac{d \log p(T)}{dT} dT\right]dt \\
&= p(t^0) \int_{t^0}^{\infty} \frac{\partial f[K(t),L]}{\partial K} \exp\left[-\int_{t^0}^{t} \frac{\partial f[K(T),L]}{\partial K} dT\right]dt \qquad (12) \\
&= p(t^0) \int_{0}^{\infty} e^{-x}\, dx \\
&= p(t^0).
\end{aligned}
$$

The proof in (12) of the equivalence between capital's reproduction cost and present discounted value has made use of our earlier (8) relating rent and marginal value productivity and of (10) relating money interest to the own-rate $\partial f/\partial K$.

VII. SUMMARY OF SIMPLEST CAPITAL MODEL

Figs. 1 and 2 summarize the salient features of the simplest capital model. With primary factors L fixed, Fig. 1 shows the level of Net National Product (y) producible by each level of capital (K).

[1] See P. A. Samuelson, 'Some Aspects of the Pure Theory of Capital', *Quarterly Journal of Economics*, May 1937, p. 485, equations (31).

On the vertical axis, the brackets indicate the factor payment breakdown between (i) the imputed rental share of capital, $K(\partial f/\partial K) = Kr$, which has been shown to be equal to the interest return on capitalized value of capital or reproduction cost, and (ii) the imputed wage or rent return to labour and other primary factors.

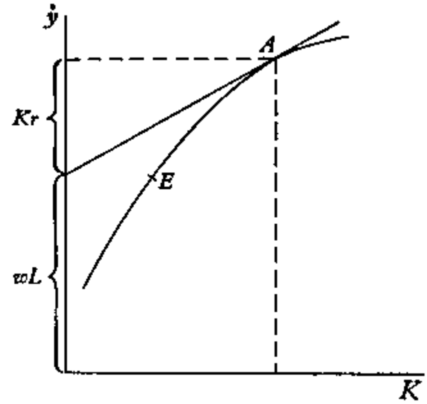

FIG. 1

Fig. 2 shows the production possibility curve, depicting society's alternative choices between current consumption ($\dot{C}$) and net capital formation (dK/dt). The production possibility schedule is a straight

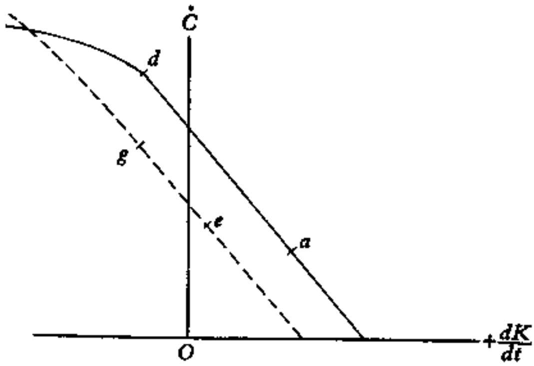

FIG. 2

line with $-45°$ slope because of the special assumption that consumption flows and capital formation are infinitely substitutable from the production viewpoint. This is true even when net capital formation becomes negative, as capital depreciation exceeds gross capital formation. However, beyond the point d, where gross capital formation is zero, further expansion of consumption at the expense of investment is possible only by heavily milking capital; therefore, the curve becomes convex from above, as shown.[1]

VIII. IS INCOME CONSUMPTION?

We may now dispose of a terminological question of purely historical interest. Irving Fisher, perhaps the shrewdest single writer in the field of capital theory, always insisted on defining income so that it is identical with 'benefits', i.e. with 'consumption', or with psychic utility of consumption.

Thus, compare in Figs. 1 and 2 the situation prevailing at A and a with the situation that prevailed some time earlier at E and e. In the elapsed time, net saving and investment has increased the stock of capital from the point E to the point A. The flow of producible output, which we have called $\dot{y}$ or Net National Product, is also seen in Fig. 1 to have been increased. In Fig. 2, the $-45°$ line ad is north-east of the parallel line ge: at the later date more consumption *and* more investment are *possible* than at the early date.

The usual definition of income implicitly assumed by us until now, and associated with the names of Haig and Marshall, would say that income has increased over time. Irving Fisher would object, saying, 'Earnings have indeed increased; but since income ought to be defined so as to be synonymous with consumption, we must determine where society is on the two schedules before passing judgment. If in the early period, society was at the point marked e, and later at a enjoying less consumption than at e, then we must conclude that income (i.e. consumption) has fallen.'

Few have followed Fisher in his terminology.[2] More popular is

[1] Our earlier footnote to equation (5) covers this case. Mathematically, maximum $\dot{C}$ does equal $f(K, L) - dK/dt$ so long as $dK/dt \geq -mK$. But if we want still more $\dot{C}$, we must maximize with respect to m, $F(K, L, m) - mK - dK/dt$ subject to $dK/dt = -mK$. And this gives $\dot{C} = F(K, L, m)$, $dK/dt = -mK$ as the parametric equations of the curve north-west of d, defined for m greater than the value satisfying (5). (From now on I treat p as unity.)

[2] Arguments as to what income really is remind one of the ancient pseudo-question: 'How do we know that Uranus is really Uranus?' The problem becomes important only when Fisher insists that an income tax should tax what he chooses to call 'income' rather than what he chooses to call 'earnings'. The substantive merits of a consumption spending tax as against an earnings tax should not be appraised in such purely semantic terms, nor even in terms of the semantic question of whether the latter involves 'double taxation' of current earnings as well as the later fruits of those earnings. Single taxation has no unambiguous meaning, and double taxation may be no more objectionable than heavier 'single' taxation. To be sure, A. C. Pigou (*Public Finance*, Part II, ch. x) and others have correctly insisted that a consumption tax uniform throughout time *may* involve less dead-weight distortion of saving-investment decisions than an equivalent earnings tax, which is a substantive rather than a philological point.

the definition of Haig, which defines income as 'net accretion to . . . economic power' (including whatever part of the accretion is used for consumption).

The Haig-Marshall definition of income can be defended by one who admits that consumption is the ultimate end of economic activity. In our simple model, the Haig-Marshall definition measures the economy's *current power to consume* if it wishes to do so. The amount that it can consume in this period, without impairing its capital and destroying its power to consume in the future,[1] is indicated in Fig. 2 by the vertical intercept of the $-45°$ line. In the later period, society could, if it wanted to, consume more than it could in the earlier period; regardless therefore of how much it actually chooses to consume in either period, the Haig-Marshall definition says that income is higher in the later period.

Note that in our simple model, one $-45°$ line is definitely out-and-beyond the other. Hence, we could alter the Haig-Marshall definition in many ways and still come out with the same answer. Thus, if we defined income as a maximum net capital formation possible if consumption were zero, we would be using the horizontal intercept as our measure and would come out with exactly the same qualitative and quantitative comparisons. The same would be true if we generalized the definition and defined income as an arbitrarily weighted sum of producible consumption and investment.

Fig. 3 for the first time depicts an alternative capital model, where capital formation and consumption flow are not infinitely substitutable. Instead, as we desire more current consumption, we must sacrifice even more investment at the margin. How would (i) Fisher, (ii) Haig-Marshall, (iii) Hicks, or (iv) an income statistician, compare and measure income at *a* and *b* in Fig. 3?

Fisher would find consumption higher at *b* than *a* and would therefore say income was higher at *b*.

Some statisticians would, I think, tend to measure incomes by the vertical intercepts of the tangent lines through *a* and *b*. On their definition, *b* would involve more income than *a*. The statistician might defend his measure as being most nearly in accord with the Haig-Marshall definition: Add in the 'value' of consumption goods and net capital formation.

(iii) Neither Haig nor Marshall have told us exactly how they would evaluate and compare *a* and *b* in Fig. 3. Certainly some economic statisticians would interpret them as follows: Money

[1] J. R. Hicks, *Value and Capital* (1939 and 1946), in effect defines income as the maximum level of permanently maintainable consumption. Capital is required to be maintained intact only so that consumption can be permanently maintained.

national income is meaningless; you must deflate the money figures and reduce things to constant dollars. To deflate, apply the price ratios of *b* to the *a* situation and compare with *b*; alternatively, apply the price ratios of *a* to the *b* situation and compare with *a*. If both tests give the same answer — and in Fig. 3 they will, because *a* lies outside of *b* on straight lines parallel to the tangent at either *a* or at *b* — then you can be sure that one situation has 'more income' than the other. If these Laspeyre and Paasche tests disagree, reserve judgment or split the difference depending upon your temperament.

(iv) Others (e.g. Hicks of the earlier footnote) want to measure income by comparing the vertical intercepts of the curved production possibility schedules passing respectively through *a* and *b*. This is

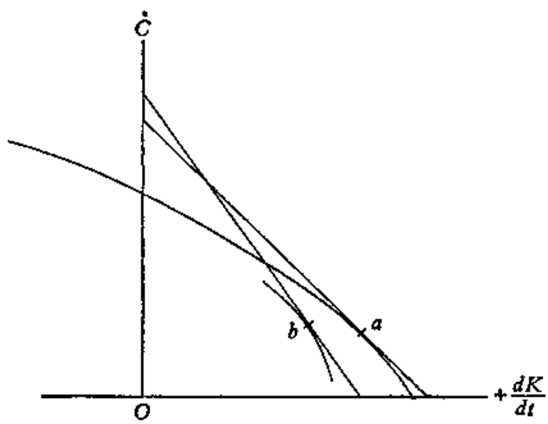

FIG. 3

certainly one attractive interpretation of the spirit behind Haig and Marshall. The practical statistician might despair of so defining income: using market prices and quantities, he could conceivably apply any of the other definitions; but this one would be non-observable to him. An economy that has historically been doing positive investment will not, in the absence of gigantic controlled social experiments, reveal what its full consumption potentialities really are.[1]

Fig. 3 has been drawn in such a way that the intercept of the curve through *b* might conceivably fall outside or inside the intercept of the curve through *a*. A statistician, who cannot command society

[1] Occasionally, by lucky accident, we may infer something about the relative positions of the intercepts: thus, if *a* were north of the vertical intercept of *b*'s *tangent* line, we could infer, from convexity of the curves, that the *a*'s intercept was north of *b*'s.

to perform controlled experiments, could observe the points *a* and *b* and their slopes, but could not observe the shapes of the curves away from these points.

Why did not the problem of making investment goods dK/dt and consumption goods $\dot{C}$ commensurable arise earlier, in Fig. 2 and in our first capital model ? It did not because of our arbitrary simplification which made dK/dt and $\dot{C}$ infinitely substitutable along a $-45°$ line. We can always tell unambiguously how much nearer to or farther from the origin one parallel line is than another.

Actually the problem did arise in our model in a concealed way. In Fig. 2 the production *loci* cease to be $-45°$ straight lines when consumption becomes so great that we rapidly milk capital. To the left of the points *d* and *g*, the *loci* in Fig. 2 are curved. I have drawn them in such a way that the *loci* cross, showing the *ge locus* to be capable of a greater emergency consumption sprint than is the *da locus*, even though the latter *locus* generally lies outside the former. Every statistician, Haig-Marshall, Hicks, or anybody else, would have been inclined to judge income to be higher in the latter situation than in the former. However, if we defined income as 'capacity to produce emergency consumption' — and why shouldn't we ? — income along the broken curve will be the higher.

Our dilemma is now well depicted. The simplest economic model involves two current variables, consumption and investment. A measure of national income is one variable. How can we fully summarize a doublet of numbers by a single number ?

You might answer :

'Even in a Crusoe static one-period world, an economy involves more than one variable : e.g. bread and wine. We boil these down into a single measure of income by (i) taking certain linear sums of their values, using as coefficients in the summation one or another situation's relative prices. This often gives a good approximation to (ii) the indifference-curve ordinal welfare evaluation of bread and wine to Crusoe, or to (iii) the production potentialities of the economies being compared, as measured by the 'distance' outward from the origin that their respective production possibility schedules lie. See Samuelson's lengthy and complex discussion in the *Oxford Economic Papers* of the index-number problems and dilemmas involved and references there to the work of Pigou, Hicks, Kuznets, and others.

'Why not apply the same index-number reasoning to the simple dynamic model, combining investment and consumption in the same way as you combine bread and wine in the static model ?'

I am forced to answer that this suggestion will not do. We do not ordinarily think of capital formation, dK/dt, as being desired for its own sake. I am anxious that the U.S. should have plenty of pig-iron and machinery, not because I care about them, but only inasmuch as they later permit me to have more bread, wine or defensive guns.

To see that our earlier methods fail, consider the strong case shown in Fig. 4*b*, where the situation containing *a* is 'clearly better' than the situation containing *b* in that more of both consumption *and* net investment is possible in the first situation. (Fig. 4*b* consists solely of $-45°$ lines, but if the *loci* were curved, only trifling modifications in my argument would be called for.)

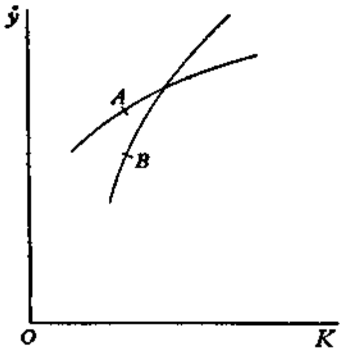

FIG. 4*a*

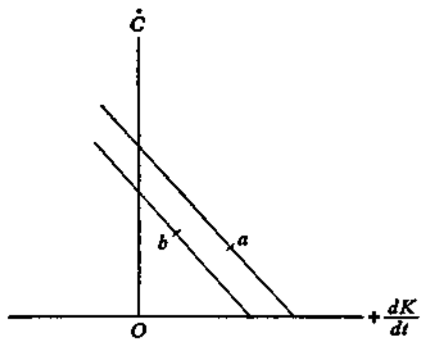

FIG. 4*b*

Fig. 4*a* shows the two different production functions of two different societies whose national products we are interested in comparing. That Society *B* produces less *currently* than Society *A* is shown by both diagrams.

If $\dot{C}$ and dK/dt were ultimate goods that could be treated like bread and wine, no one could stop us from conceptually drawing in indifference curves on Fig. 4*b* with points of tangency at *a* and *b*. Supposing the social indifference curves to be regularly convex, who could deny (i) that the point *a* is 'better' than *b* according to either society's valuation or (ii) that the production potentialities of *A* are uniformly 'better' than those of *B*? If $\dot{C}$ and dK/dt were replaced by bread and wine, we could be sure that citizens in *B* would gladly trade places with citizens in *A*.

Can we make the same inference in this dynamic context? Following the Hicks [1] interpretation, we should certainly have to

[1] The earlier reference to Hicks should not be construed as a criticism of him. His *Value and Capital* discussion is directed *against* the income concept, and his formulation of income as 'level of consumption flow permanently attainable'

argue that income is higher in A — as measured by its high vertical intercept depicting level of permanently maintained consumption.

But note that neither in Society A nor Society B is the representative citizen choosing to maintain capital intact: in both cases he is doing net saving and increasing the society's capacity to consume. One might be tempted to give preference-weighting to dK/dt as well as $\dot{C}$ and conclude that both citizens would feel that point a is better than point b.

However, Fig. 4*a* shows that no such inference is justified. It may be legitimate for us to regard the $\dot{C}$ of one society as consisting of the same consumption good(s) as for the other, or at least as being capable of being made commensurable through the use of each

reflects the typical definition of his predecessors. See, e.g., F. Hayek, 'Maintenance of Capital', *Economica*, August 1935, and a long chain of earlier writers.

Incidentally, if positive saving will always go on and if the capital marginal productivity curve will always be inelastic, then the capitalistic class as a whole can count on no above-zero level of permanent consumption. So some might argue that their net income is throughout literally zero. Or supposing that there existed one, essential, irreplaceable, exhaustible resource, then society as a whole might be regarded as having zero net income.

If labour (or better still land) in $f(K, L)$ were perpetual and capitalizable, and if it were known that positive dK/dt were going to take place at some definite rate, then the interest rate would fall. This would not affect the value of each unit of K since its net rental would fall in the same measure; but it is definitely not true that the unit value of all assets could not rise. Actually the capitalized value of L, the price of the asset labour or $P_L L$, would go up for two reasons: more K will raise the rent of L; and lower interest means less discounting of future rentals. So the total asset value of society (measured in $\dot{C}$ and K units) will be rising through time. At any moment what is the total interest yield on these assets? It is $[K\partial f/\partial K+0]+[L\partial f/\partial L+d \log P_L/dt]$ where the last term comes from (11)'s statement about capital revaluations. This total exceeds f by the last revaluation term. Now actually, after we have subtracted from this expression the dK/dt that the K owners have decided to save, the remainder will *not* be truly available for consumption — since the whole expression does exceed $f=\dot{C}+dK/dt$ by the 'spurious' revaluation term.

A simple example may show that NNP regarded as $f(K, L)$ *cannot* be regarded as the interest yield on any meaningful capital value magnitude, but must instead be corrected by a capital gains term. Suppose f is such that primary L is redundant until K reaches some larger magnitude than today's K. Then all of current f or $\dot{y}$ goes as rent on scarce K. But suppose dK/dt is going on and we are known to be reaching the time when capital's net productivity falls to a new plateau at half its previous level. After that time, primary L will command a positive rental, which will be right now capitalized in the markets for L. Since the days of those revenues are approaching, P_L will *now* be rising in a known way. Equation (11) shows that this foreseen capital gain must be taxed if we are to treat investors in land the same way we treat investors in K; but if we do this, our base for the income tax is $(rK+0)+(0+Ld \log P_L/dt)=\dot{y}+$ an irremovable capital gains term.

[In a trivial sense $\dot{C}(t)+dK(t)/dt=\lambda(t)\ K(t)$, where λ is the *average* productivity of capital $f(K, L)/K$. So K can be trivially regarded as the present discounted value of all future consumption, the rate of discount being not the interest rate $i(t)$ — the *marginal* productivity of capital goods — but $\lambda(t)$ — the *average* productivity of capital goods. It is only in this trivial sense that income $\dot{y}$ can be regarded as the interest fruit λ on the reproducible goods of society K. Such a formulation imputes, in a way that is meaningless from the market or social viewpoint, all production to K alone. Such a non-labour, non-land theory of value has little relevance.]

society's indifference contours. But what is the commensurability of dK/dt in the two situations? Unit by unit they both have current unit opportunity costs in terms of the (commensurable) $\dot{C}$ of the two societies. True, but what of that? Society A's capital is subject to much faster diminishing returns *in the future*. Very soon, at the rate both are saving, will Society B forge ahead of Society A.

If you asked Citizen B which he would rather live in, he might reply: 'My society is currently poorer, but I prefer it to A.'

If you asked Citizen A which he would rather live in, he might reply: 'B's technology is currently poorer than mine, as measured by the half-dozen earlier proposals of Haig-Marshall, Fisher and others, but I too would prefer it to my own society.'

For note that Fisher's current consumption, as measured in Fig. 4*b* by the latitudes of *b* and *a*, is just as bad as a measure of welfare as is any Haig-Marshall or statistical definition. Current consumption in Society A exceeds that in B, yet both citizens agree on the welfare primacy of B!

How do we know that both citizens prefer B? To answer 'Both have told us so' is superficial, and causes us to rephrase the question: 'How does each know that he prefers the set-up of B to that of A?'

His answer would be: 'I consider my ordinal preferences between present-day consumption and consumption *of all future dates*. I perceive that B, with its more slowly declining productivity curve for capital, permits me to enjoy a time-profile of consumption that lies definitely higher on my indifference contours than does any time-profile of consumption feasible in A.'

The question is resolved in terms of the indifference contours plotted in what space? The $\dot{C}$ and dK/dt two-dimensional space? Definitely not. Rather, *in the space of all present and future consumption*! Reverting to discrete time periods with future consumptions given by $[C_0, C_1, C_2, \ldots, C_t, \ldots]$ we envisage the citizen as choosing, from the feasible menus of consumption implicit in Fig. 4*a*'s two technologies, the time-profile of $[C_0, C_1, \ldots]$ he likes best.[1]

One might feel that this formulation makes our task all the more impossible. Given three time-periods, we seek to evaluate and summarize in a single variable — index of welfare — a triplet of numbers. Given n future periods, we seek to reduce to a single number an n-tuple of variables.

[1] If we stay with the continuous model, we must set up an ordinal index that is a 'functional' of the whole time-profile of $\dot{C}(t)$ from now to as far in the future as is relevant. The value *sums* of our later discussion would then be written as *integrals*. If we truncate the time-horizon, we must subjectively evaluate $(C_0, C_1, \ldots, C_t, S_t)$ where S_t is the terminal vector of society's capital.

However, things are not really so bad. Even in the one-period statical case, we really have thousands of different goods. Having 3 or n times as many consumption goods does not particularly add to our theoretical complexity. To the extent that we can set up meaningful statical index-number comparisons — and the reader of my earlier paper will remember how delicate and complex this procedure is — we can also set up meaningful comparisons of time-profiles of consumption. That is, we can do so on our heroic assumptions of foresight of the future and transitive ordinal tastes.

What we need for our index-number comparisons are market-price ratios reflecting intertemporal substitution choices: i.e. we need interest rates and future money or relative prices. With competitive market prices and quantities given, we can hope to set up index-number comparisons that will sometimes tell us (i) whether the consumption profile of A is better or worse than that of B in the estimation of a consistent set of ordinal preferences, or (ii) whether the consumption profile of one of the situations is capable of being bettered *in respect of every good and every time-period* by the other situation.

Irving Fisher, if this were all explained to him, would break in with the remarks, 'You have cast scorn on my semantic suggestions that current consumption be called current income. Instead you have embraced as a definition of income what I call current earnings and what Haig, Marshall, and most legislators (wrongly) call income. You have then gone on to show that no interpretation of such an income concept can be given a meaningful welfare connotation. (And in all honesty I must add that you have also shown that my own income concept of current consumption cannot be given the desired welfare interpretation.)

'None the less, you are now veering toward another of my important concepts. What you really seem to be proposing as your welfare measure is something close to what I (and others) call *wealth*. By wealth I mean the present discounted value of all future *consumption* (and not, mind you, the present discounted value of future earnings or Haig-Marshall income).'

Fisher would be right. Our rigorous search for a meaningful welfare concept has led to a rejection of all *current* income concepts and has ended up with something close to wealth.

Specifically, in complete analogy with the statical one-period case, we shall want to set up the rather complex index-number comparisons of the following types.

For simplicity, we work with discrete time-periods $(0, 1, \ldots, t, \ldots)$. Let the consumption of each time-period be made up of the total expenditure on m different goods of each period, evaluated at market prices. Thus

$$C(t) = p_1(t)q_1(t) + p_2(t)q_2(t) + \ldots + p_m(t)q_m(t) = \Sigma p(t)q(t),$$

for $t = 0, 1, \ldots,$ etc. Again for simplicity, let $p_1(t)$ always be unity, the first consumption good being our *numeraire*. Let i_t be the interest rate of the t^{th} period expressed in terms of the *numeraire* good.

Then wealth in any situation is defined as

$$W = \frac{C(0)}{1} + \frac{C(1)}{1+i_1} + \frac{C(2)}{(1+i_1)(1+i_2)} + \frac{C(3)}{(1+i_1)(1+i_2)(1+i_3)} + \ldots \tag{13}$$
$$= \sum_{t=0}^{\infty} \{\beta(t)\Sigma p(t)q(t)\},$$

where

$$\beta(0) = 1,\ \beta(1) = (1+i_1)^{-1}, \ldots, \beta(t) = \{(1+i_1)(1+i_2)\ldots(1+i_t)\}^{-1}.$$

More compactly, we can call each good at each different time a different good, letting capital Q's represent such goods. The relevant prices of these goods are their discounted prices: thus, if Q_{999} corresponds to $q_{13}(17)$, its relevant price is $\beta(17)p_{13}(17)$, which we may write as P_{999}. Then we may rewrite wealth compactly as,

$$W = P_1Q_1 + P_2Q_2 + \ldots = \Sigma PQ.$$

The wealth in Situation A is of course ΣP^aQ^a. The wealth in Situation B is ΣP^bQ^b. Clearly, there is no meaning in comparing the *money* wealth of one situation (i.e. time and place) with that of another situation. Even if our money or *numeraire* consists of the same initial period's consumption good, a comparison of $\Sigma P^bQ^b \gtreqless \Sigma P^aQ^a$ is meaningless.

It is not meaningless to compare the Q's of one situation with those of another provided we use the *same prices* and *interest rates* in the comparison. Thus

$$\Sigma P^bQ^b > \Sigma P^bQ^a \text{ or } \Sigma P^b(Q^b - Q^a) > 0 \tag{14}$$

does tell us that the real wealth in B is preferable (according to the tastes and time-preferences prevailing in B) to the time-profile of goods available in A. The reader can provide the similar, alternative interpretation in the case that $\Sigma P^aQ^a > \Sigma P^aQ^b$.[1]

[1] If we had observed $\Sigma P^bQ^b < \Sigma P^bQ^a$, we could have concluded something about B's capacity to produce consumption: namely, B was not capable of producing what A was actually enjoying. This and other statements made here can be verified to follow from the analysis of my cited 1950 paper.

IX. THE TRUE NATURE OF THE COMPARISONS

Our comparisons are not of wealth directly, but of wealth-like magnitudes. I must stress and restress that although they offer no difficulties in theoretical principle as compared with the statical case, the national income statistician is very far from having even an approximation to the data needed for these comparisons. A vital difficulty is the hard and unchangeable fact of uncertainty. Futures markets might enable us to salvage something even in the presence of uncertainty ; but futures markets are themselves of little quantitative importance in present-day economies.

This may sound pessimistic. After all we do have estimates of national wealth. Could not a magnitude like $\Sigma P^a Q^a$ be approximated by summing the capitalized value of productive factors ? [1] In theory, yes. We could similarly approximate from market valuations the actual wealth in B, $\Sigma P^b Q^b$.

But none of this is any help. We have agreed that the only meaningful comparisons are not those of the type between $\Sigma P^b Q^b$ and $\Sigma P^a Q^a$, but rather those of mixed type involving $\Sigma P^a Q^b$ and $\Sigma P^b Q^a$ in comparison with the actual wealths. I know of no way of even approximating from market valuations of factors what the values of consumption quantities Q^a at P^b prices would be.

We are left with the pessimistic conclusion that there is so much 'futurity' in any welfare evaluation of any dynamic situation as to make it exceedingly difficult for the statistician to approximate to the proper wealth comparisons. Reflection shows that this is inherent in the nature of things. An appraisal of an economy's situation does involve implicitly or explicitly an appraisal of its future prospects. The current consumption or earnings of the present instant are as nothing against the prospect of the near and far future.

In real life, all decisions are decisions about wealth. Closely examined, no decisions appear to be in terms of current instantaneous magnitudes. If we use as our income period the present minute or day, this truth becomes more obvious. It is only the calendar year, which some accountants and primitive aborigines sometimes regard as fundamental, that blinds us to this fact.

Adam Smith is often these days criticized for writing about the

[1] Note that labour or other primary factors must be included in the capitalized total : otherwise we may miss a full three-quarters of it. Note too that ordinal disutility of labour or other factors is not taken account of in the total as stated : but in principle, we can introduce inputs as negative outputs ; this could reduce the new total $\Sigma P^a Q^a$ to zero or less, but the comparison $\Sigma P^a Q^a \gtreqless \Sigma P^a Q^b$ remains valid.

wealth of nations and not about their incomes. But the present discussion reveals that he was (inadvertently ?) right. Indeed it is revealing, from the present viewpoint to go back and read Professor Pigou's discussion of this point in *Wealth and Welfare* (1912) and, since 1920, in *The Economics of Welfare.* I cannot do better than quote verbatim his argument concerning proper definition of national dividend or income.

> 'The major part of this volume, however, is concerned . . . with causation. The general form of our questions will be: "What effect on economic welfare as a whole is produced by such and such a cause operating on the economic circumstances of 1920 ?" Now it is agreed that the cause operates through the dividend, and that direct statements of its effects must refer to the dividend. Let us consider, therefore, the results that follow from the adoption of those two conceptions respectively. On Fisher's follower's plan, we have to set down the difference made by the cause to the dividend, not merely of 1920, but of every year following 1920 ; for, if the cause induces new savings, it is only through a statement covering all subsequent years that its effect on the dividend, as conceived by Fisher's follower, can be properly estimated. Thus, on his showing, if a large new factory is built in 1920, not the capital establishment of that factory, but only the flow of services rendered by it in 1920, should be reckoned in the dividend of 1920 ; and the aggregate effects of the creation of the factory cannot be measured without reference to the national dividend of a long series of years. On Marshall's plan this inconvenient elaboration is dispensed with. When we have stated the effect produced on the dividend, in his sense, for the year 1920, we have implicitly included the effects, so far as they can be anticipated, on the consumption both of 1920 and of all subsequent years ; for these effects are reflected back in the capital establishment provided for the factory. The *immediate* effect on consumption is measured by the alteration in the 1920 dividend as conceived by Fisher's follower. But it is through total consumption, and not through immediate consumption, that economic welfare and economic causes are linked together. Consequently, Marshall's definition of the *national dividend* is likely, on the whole, to prove more useful than the other, and I propose in what follows to adopt it.' [1]

[1] A. C. Pigou, *The Economics of Welfare* (Fourth edition, 1932, London), pp. 36-7. I have omitted earlier passages in which Professor Pigou admits that Fisher's (current) consumption concept of income (i) gives a better objective index of correlation with 'economic welfare which a community obtains over a long

Careful reading of Professor Pigou's argument suggests to me that it does establish the following point. (i) Current consumption does *not* fully reflect the welfare effects of policies now being initiated. (ii) It is necessary, even though difficult, to consider effects on future consumption (suitably discounted ?) ; i.e. welfare changes are to be measured by wealth changes.

So far, this is all in accord with the results of the present investigation, which was led to wealth rather than income as the *desideratum* for the economic theorist. But Professor Pigou goes on to conclude : (iii) adding the rate of net capital formation to the rate of consumption does adequately measure the sought-for wealth. This does not, in my tentative judgment, follow ; and from the standpoint of the many important subjects discussed in *The Economics of Welfare*, it seems to me it would have been better to eschew both Fisher and Haig-Marshall income and to ask how any given policy change will increase the *wealth* of the nation.

I think this becomes clearer if one concentrates on the wording : 'What effect on economic welfare . . . is produced by . . . a cause *operating on the economic circumstances of 1920* ?' Why confine the question to the time period that I have italicized ? To make the point clearer, suppose we transform our time-dimensions and concentrate on 'The economic circumstances of the minute of noon, July 4th, 1920', or the 'circumstances of the Twentieth Century'. Are we to let dimensional change alter our substantive decision ? Are we to regard the calendar year as a privileged set of units ?

Page through the rest of Pigou's great book. Ask the important questions he asks, such as : Should factories be permitted to burn noxious chemicals in crowded cities ? Would it pay the community to introduce a given device for reducing smoke nuisance ? Should decreasing-cost industries be subsidized ?

To a cardinal hedonist like Professor Pigou or an ordinalist like myself, these questions are truly answerable only in terms of effects upon objective or subjective wealth. If the consumption prospect *over all relevant time* that every person can envisage will be deemed better after a given policy change than before, then it is a good one. Only by remote chance can such questions be answered by considering

series of years' and (ii) is more relevant to a country's temporary war-time potential. My earlier critical remarks concerning our ability to infer the latter potential from normal *ex post* income data will be seen to apply even more to Fisher's current-consumption concept than to the Haig-Marshall concept. As to the community's obtaining welfare over a long period of years, this seems to me to apply some 'interpretation' of utility over a period of time (on an *ex ante* or *ex post* basis ?) and to very clearly relate to what Fisher would have to consider dimensionally and conceptually a wealth or stock item rather than what he defines as his income flow per unit time.

Haig-Marshall definitions of current income; and in the singular cases where this is possible, we will know it to be so only by making the correct wealth calculation.

Note that my reformulation in terms of wealth rids us of a fundamental difficulty. My proposal requires us, in making policy judgments, to answer if we can that Situation *A* is better, worse or indifferent to Situation *B*. The current income concepts try in some sense to measure finite changes, or rates of change, of a wealth-like [1] magnitude between, say, *A*, the January 1, 1920, date and *B*, the January 1, 1921, date. Similarly, 1922 income measures some kind of change between *C*, the January 1, 1922, date and *D*, the January 1, 1923, date.

If the theorist gives his *imprimatur* to an income concept, who can blame the statistician for comparing 1922 real income with 1920 real income? But this is not methodologically like comparing Paris and London — or the Twentieth and the Thirteenth Centuries. Such questions usually turn out to mean: 'Given the choice, would you prefer to live in Paris or London?' 'Or live now or then?'

Comparing 1922 and 1920 income is more like asking the quite different and usually uninteresting question: 'Do you like Paris as much better than London as you like Salt Lake City better than Fresno?' [2] On the other hand, comparing the consumption prospects over all time subsequent to 1920 with those subsequent to 1922 does involve a wealth index-number problem and is comparable to a simple comparison of London and Paris. It is like Coué's statement: 'Every day in every way I'm getting better and better'. It does not involve the conceptually less interesting statement: 'Every day I'm getting better at an increasing rate'.[3]

[1] The shocking looseness of the Haig definition of income as the 'accretion to wealth' between two periods is of course to be modified — as I did earlier — to take into account consumption of the period in question. Therefore, I term the magnitude of which income can be regarded as the difference not wealth but a wealth-like magnitude. Dimensionally these are the same, and in the limiting case of zero consumption they would be numerically identical.

[2] The occasion for answering such a question scarcely ever arises. If forced to answer such a question, there are an unlimited number of assumptions and considerations by which one could give an answer rather than be burned at the stake. I should warn the reader that the ordinal fact that I am indifferent between (i) living in Salt Lake City, (ii) living in London, and (iii) a lottery ticket that with probability one-half determines whether I live in Paris or Fresno, *is an ordinal fact* about my reactions to stochastic situations. He misunderstands the Ramsey–Finetti–Savage–Neumann theories of subjective probability and utility who thinks that this approach verifies or refutes the views of an introspective arithmetical hedonist, even though it may be convenient to say that London and Salt Lake City have equal utilities that are half-way between the utilities of Paris and Fresno.

[3] Many of us do look at production statistics to determine whether a nation is accelerating in its physical indexes. We even redouble our efforts when the rate of growth falls below past average values. But this reflects an empirical inference about engineering and other potentialities, not any opinion about psychic geiger-counters.

X. SUMMARY

The present investigation can be briefly summarized :

1. When we work with simple and exact models, in which no extraneous statistical difficulties of measurement could arise, we find that the only valid approximation to a measure of welfare comes from computing *wealth-like* magnitudes not income magnitudes (of the Haig, Fisher or any other type).

2. In the absence of perfect certainty, the futures prices needed for making the requisite wealth-like comparisons are simply unavailable. So it would be difficult to make operational the theorist's desired measures. But operational practicality aside, if the theorist specifies in detail the dynamic technology of his model, he will meet none of the pitfalls that come from an attempt to summarize his model by various crude aggregations. The contradictions that result from over-crude aggregation should never be confused with the technical relations that hold at the firm and family level or with the market capitalizations which hold in competitive security and asset markets.

Chapter 4

APPRAISAL OF THE LABOUR-SAVING AND CAPITAL-SAVING CHARACTER OF INNOVATIONS

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I. IMPLICATIONS FOR MEASUREMENT

SPECIFIC observations on the difficulties of measuring the stock of capital will be made later in this paper. At the outset I will say merely this: the analysis assumes that the quantity of capital is measured by some method which enables us to distinguish between changes in this quantity and changes in the productivity of capital. The analysis assumes also that the nature of the technological processes exerts an influence on distributive shares, in other words that these shares are not uniquely determined by administrative decisions. Both these assumptions are implicit in the marginal productivity theory, but they are implicit also in other ways of looking at capital and at the determination of distributive shares.

Those specific sections of the paper which discuss statistically observable trends in capital-output ratios (i.e. in reciprocals of the average productivity of capital) are based on the measurement of the capital stock by its cost of acquisition to the investor, with correction for price changes and with allowances for depreciation and obsolescence. The capital stock is defined exclusive of stocks in the possession of households, but inclusive of residential buildings. This method contradicts neither of the two assumptions described in the first paragraph of this section. However, such measurements can be linked to a meaningful concept of capital — to a concept based on an articulate theory — only on sweeping simplifying assumptions. This is another way of saying that such measurements give trends in a time-series for the capital stock which broadly coincide with trends in a 'meaningful' quantity of capital only if the degree of deviation from certain simplifying assumptions does not change very much with the passage of time. I think there is reason to believe that the results obtained by such methods of measurement are not useless. To this I shall return later.

The analytical core of my paper does not depend on this particular way of measuring capital. It depends merely on measurement satisfying the two conditions described in the first paragraph of this section.[1]

II. TRENDS IN THE AVERAGE CAPITAL-OUTPUT RATIO CONNECTED WITH THE QUANTITY, NOT WITH THE CHARACTER, OF INNOVATIONS

Contrary to statements frequently encountered in current discussion, changes in capital-output ratios do not provide dependable indications of the capital-saving or labour-saving character of innovations. It is tempting, but quite wrong, to argue that when, say, 'automation' gains ground at the expense of other types of innovation, then the labour-saving (or capital-using) character of this change will express itself in higher capital-output ratios.

Unless there is a change in the resource inputs in the economy as a whole, it must be true that the upward shift of the production functions will get the economy as a whole into a position where, for the given resource inputs, aggregate output is greater than would have been the case with no innovation. This must be true regardless of the character of the innovations. Given the resource inputs in the economy, automation will raise Y/K for the economy as a whole in the same proportion as Y/L, where Y is output, K capital-input, and L labour-input ; and the 'wireless' type innovation,[2] too, will raise Y/K in the same proportion as Y/L. Which of the two innovations raises Y/K and Y/L more depends on which innovation is stronger in an overall sense (alternatively expressed, it depends on which type of innovation comes in 'greater quantity', is 'more plentiful'). But the answer does not depend on which innovation is more labour-saving or capital-saving. The Y/K ratio is of course the reciprocal of the capital-output ratio.

These statements assume that the resource inputs (thus also K/L) are not affected by the character of the innovations. It is reasonable to make this assumption when we frame our concepts, because while

[1] Much of the analysis of this paper is based on the present writer's *Trends and Cycles in Economic Activity* (Henry Holt, New York, 1956). See also John H. Power, 'An Economic Framework for the Theory of Growth', *Economic Journal*, March 1958. Statistical sources are estimates by Kuznets, Goldsmith and Budd which are discussed in detail in the present writer's *op. cit.*, particularly in the Appendix to Part III.

[2] I am using the wireless as an illustration for a capital-saving innovation.

one of our analytical results will be that the character of the innovations *may* change the path of inputs in the economy as a whole or in individual industries, this potential result must not be defined into the concepts from which we derive it. In other words, the character of the innovations is disclosed by concepts which show the changes brought about by new technology for given resource inputs, even though in certain cases the innovations will alter the input quantities. There exists no way of defining a functional relationship of reasonably general validity which would indicate just when and how a change in the distribution of the labour- and capital-saving effects will alter the path of resource inputs in the economy as a whole.

Innovations would always have to be said to 'alter the path of resource inputs' if the 'effects of innovations' were interpreted broadly enough to include in these effects the subsequent increase in capital inputs via an increase in output and via a positive marginal propensity to save. It seems obvious from the outset that this subsequent change in resource inputs (due to greater supply) should be left out of account in defining the quantity and the character of the preceding innovating activity. As for the innovating activity of the subsequent periods, this must be defined as making use of the resource inputs of those periods, without regard to the question whether part of the capital input can or cannot be linked to preceding innovations, indirectly through higher outputs and higher savings. However, innovations may, in certain circumstances, alter (distort) the path of resource inputs quite directly, by limiting the capacity of the economy to absorb given labour or capital under the new technology. My point here is that even this potential indirect effect must not be defined into the concept of one or the other type of innovation, but it too must result, as a potential consequence, from analysis employing previously defined concepts. We shall see in the next section why it is advisable to proceed in this fashion.

We conclude that we must look for a definition of the 'character' of innovations on the assumption that the resource inputs (thus also K/L) are given for the economy as a whole. The effect of innovations on Y/K (or on Y/L) does not show their capital-saving or labour-saving character, but shows their overall strength or quantity.

III. MISLEADING APPEARANCES

The misleading *prima facie* impression that, for example, labour-saving innovations raise K/L, and perhaps also K/Y, derives, I think, from one's inclination to select very extreme and unrealistic illustrations, relating to special cases in which the time-path of resource inputs would be changed promptly and in a predictable way. For example, if every industry adopted exceedingly strong automation devices, and if this were the only innovating activity in the economy, then moving along the new production functions to a point where the labour input stays unaffected would presumably involve a significant lowering of real wages. In the event of a floor to real wage rates — institutional or even merely biological — unemployment would develop. This is the kind of 'non-Keynesian' unemployment attributable to the rigidity of real wages (or to the low level of the equilibrium wage rate due to the insufficiency of the capital stock). Such unemployment would alter the path of resource inputs directly and promptly. The ratio of capital input to labour input would surely rise. The capital-output ratio, too, might rise.

On the other hand, if innovating activity consisted exclusively of the adoption by all industries of devices like the wireless — simple equipment making use of a free resource with great effectiveness — then moving along the new production function to a point where the capital input stays unchanged would presumably involve a very significant lowering of rates of return on investment. If the risk premiums required prevented such a lowering, then, with rigid money wage rates, the Keynesian type of unemployment would develop ; both the capital and the labour input would be reduced, and there exists no general rule that would tell us what would happen to the capital-labour ratio and the capital-output ratio.

If labour-saving or capital-saving innovations reduce the capacity of the economy to absorb given supplies of labour or of capital, then the labour-saving or capital-saving character of innovating activity *does* express itself in different effects on the capital-labour ratio and perhaps also in different effects on the capital-output ratio. But to define the 'character' of innovations on the assumption of distorted paths of resource inputs would result in a confusing analytical structure, because any statement that can be made about the probability and the extent of these distortions would have to be very imprecise. When innovating activity moves in the real world from the 'wireless' towards 'automation' or vice versa, then this invariably involves *gradual* changes in the character of the 'innovation-mix'. The predictable effect of such gradual changes in the character of the 'innovation-mix' is a steepening or a flattening of the secular uptrend in real wage rates, and a less or a more favourable trend in rates of return to investors. At the end of such a steepening or flattening of trends we could encounter the Keynesian difficulty of insufficient rates of return ; or we could encounter the contrary difficulty of a down-trend in real wage rates such as bumps into a wage floor. This is why in the long run both workers and the owners of capital may have an interest in avoiding a consistent slanting of the character of innovating activity towards one extreme — even towards the extreme that initially favours their interests. But it would be awkward to define the possibility of these delayed disturbances into our working concepts, instead of discussing the possibility of such disturbances with the aid of concepts that are kept clean of them.

So far our conclusions are the following: when framing our concepts, we should regard the aggregative K/L ratio as unaffected by the innovations; the effect of the innovations on Y/K should be regarded as a measure of their overall strength or quantity; and the measure of their character should be sought elsewhere, along lines first suggested by Hicks in his *Theory of Wages*.

IV. CAPITAL-SAVING RATHER THAN LABOUR-SAVING INNOVATIONS OR VICE VERSA

The 'wireless' type of innovation differs from the automation type in an essential respect, even if they are equally strong. Given the time-path of resource inputs, the wireless exerts a favourable influence on the relative share of labour in the national income, while automation has the contrary effect. Let P stand for profit income (aggregate income from the ownership of capital); and let wL stand for the real wage rate times labour input, that is, for labour income. Then we may conclude that the wireless type of innovation raises wL/Y and lowers P/Y, while the automation type exerts the contrary influence. We may then define an innovation as relatively capital-saving, rather than labour-saving, if it lowers P/Y and raises wL/Y; and as relatively labour-saving, rather than capital-saving, if it lowers wL/Y and raises P/Y. The definitions could be extended to a three-factor system with natural resources as the third factor.

The relative shares which serve as a basis for this definition apply to the economy as a whole. The effect of innovations on distributive shares in individual industries does not serve as a particularly convenient criterion of classification, because the innovation may very well express itself in the creation of a new 'industry'. But frequently it is not unreasonable or unusual to say that an innovation has merely changed the methods of production or the physical character of the product in specific 'industries' in the broad sense; and various types of innovation will then shift distributive shares, for given inputs, in the innovating industries in the same direction as in the economy as a whole. Furthermore, we may add that *for a single firm*, with all market prices given, an innovation *would* change K/L in one direction or the other, according as the innovation is relatively labour-saving or capital-saving. But this is of course not true of innovations which are macro-economically adopted for given macro-economic resource inputs.

V. THE CHARACTER OF INNOVATIONS AND TRENDS IN FACTOR RETURNS

A relatively capital-saving rather than labour-saving innovation lowers P/Y and it raises wL/Y; whether it actually lowers the profit rate (P/K), while raising the real wage rate (w), depends on how strongly the innovation raises Y/K and Y/L. If these average productivity terms are raised in a sufficient degree, a relatively capital-saving rather than labour-saving innovation may increase both P/K and w (not only w). This is because

$$\frac{P}{K} \equiv \frac{P}{Y} \cdot \frac{Y}{K}$$

$$w \equiv \frac{wL}{Y} \cdot \frac{Y}{L}.$$

Similarly, a relatively labour-saving rather than capital-saving innovation lowers wL/Y and raises P/Y; whether it lowers the real wage rate (w) while raising the profit rate (P/K) depends again on how strongly the innovation raises the average productivity terms in the foregoing equations.

In our terminology this is expressed by saying that both factor returns will be increased, not only that which the factor-saving effect of the innovation favours in relative terms, provided the *strength* (or 'quantity' or 'product-raising effect') of the innovation is big enough as compared to its relative factor-saving effect.

A good many automation devices, taken in isolation, and the wireless, taken in isolation, probably illustrate cases where *not both* factor returns are raised. Innovations consisting entirely of the wireless type would, for example, lower not only the relative share of capital (P/Y), but also the profit rate (P/K). Furthermore, getting more of these same innovations into previously unaffected industries would not improve the trend in the profit rate, because with the greater quantity, or greater product-raising effect, of these innovations we would also get more of the 'relative capital-saving effect' into the economy. But this illustration, based as it is on a single type of innovation, is not very revealing. Assume, instead, that the innovations of a period consist of the automation type and of the wireless type in some specific proportion and that we observe the trend in the profit rate and in the real wage rate resulting from this innovating activity which takes place all over the economy; [1] then we may conclude that, other things being equal, more innovations of both types in an unchanging proportion would render the trend in both the real wage rate and the profit rate more favourable. The quantity (or overall strength) of the innovating activity would increase, while the relative factor-saving effect would stay unchanged. The unchanging trends in P/Y and in wL/Y would be associated with a more favourable trend in Y/K and in Y/L.

VI. A STATEMENT USING THE MARGINAL PRODUCTIVITY THEORY

If we use the marginal productivity theory, with the production function [2] written as $Y = F(L, K,)$, then the terms used in the foregoing discussion acquire more specific meaning. The identities on page 63 change to the following equations for the profit rate (P/K) and the real wage rate (w):

$$\frac{P}{K} = \frac{K}{F}\frac{\partial F}{\partial K} \cdot \frac{F}{K}$$

$$w = \frac{L}{F}\frac{\partial F}{\partial L} \cdot \frac{F}{L}.$$

Since the changes which innovations bring about in the first term on the right-hand side of each equation express the relative factor-saving effects of the innovations, we arrive at the conclusion that *these effects are measured by the change in the proportionate gap between the average and the marginal productivity of K and of L,*

[1] In a two-factor model, at least one of these two trends must be more favourable than would be the case without the innovations. If the 'strength' (product-raising effect) of the innovations is great enough, then both trends will be more favourable. This does not apply to a model with more than two factors (although it does always remain true that the trend in at least *one* of the factor returns must be more favourable with than without innovations).

[2] By the 'production function', I mean one that is not corrected for innovations, in other words does not express the time-path of a dynamic economy. A function containing such a correction would have to be written as $Y = F(L, K, t)$, where t stands for time. See, however, the footnote on p. 69.

respectively (i.e. in the ratio of these two magnitudes). For example, if the gap between average and marginal product is increased for K, then the innovation is relatively capital-saving rather than labour-saving. The effect of the innovations on the second term of the right-hand side of each equation measures the strength (or quantity or product-raising consequences) of the innovating activity. These conclusions, too, could be extended to a three-factor model.

For some purposes it is enough to know that one innovation is 'either less relatively capital-saving *or* more relatively labour-saving' than another without distinguishing between these two cases.

VII. SIMPLIFYING ASSUMPTIONS OF THE HARROD-DOMAR MODEL

Even though the simplified Harrod-Domar assumptions will not have to be used here, I shall express their meaning in the foregoing conceptual framework.

If for some rate of increase of the capital stock, innovations are just sufficiently plentiful to prevent the capital-output ratio from rising, and if their relative factor-saving effects are distributed in such a way as to keep the relative shares in income constant, then rates of return to investors, too, will stay unchanged along that path. Hence, if the foregoing conditions are met for a sequence of net investments which matches the sequence of *ex ante* savings along a growth path, then the Harrod-Domar results can be put into effect by holding the rate of interest constant.

This is the meaning of the Harrod-Domar equations, in their initial simple form, for a two-factor economy in which the capital stock is rising at a greater proportionate rate than the labour supply. In general, the assumption concerning the character of innovations is *not* that they are 'neutral' in the sense of 'neither labour-saving *rather than* capital-saving nor vice versa'. The assumption is *not* this ; the assumption is that, given the elasticity of substitution of the factors, the character of the innovations is such as to keep distributive shares constant. This assumption reduces to 'neutrality' in the foregoing sense only if the elasticity of substitution is unitary.

If the supply of the two factors were rising in identical proportion, then the Harrod-Domar model would imply the absence of innovating activity (so long as the capital-output ratio is treated as a constant).

VIII. THE RECORD

The evidence obtainable for the United States points to the following.

(*a*) For the economy as a whole, the capital-output ratio seems to have risen from the 1870s to the early part of the present century, presumably through the second decade of the century. Subsequently, the capital-output ratio has shown a falling trend, and by about 1950 its value was no larger than in the 'seventies (perhaps somewhat smaller).

(*b*) It is certain that the rise of the capital-output ratio during the late part of the nineteenth century was caused, at least to some considerable extent, by shifts in the composition of output toward 'capital-intensive sectors'; during part of the more recent period of fall, the trend may have been somewhat promoted by shifts in the composition of output towards 'labour-intensive sectors', but the indications here are less definite. At any rate, for the whole eighty-year period the failure of the capital-output ratio to rise with a rapidly rising capital-labour ratio can certainly not be explained by changes in the composition of output. The conclusion here is that innovations have been sufficiently 'strong' (sufficiently plentiful) to maintain, and on balance perhaps slightly to lower, the capital-output ratio. Taking the eighty-year period as a whole, the consequences of 'diminishing returns' *and* of the compositional changes toward capital-intensive industries were fully offset for the average productivity of capital, even though they were not fully offset in the first half of the period when the compositional changes were especially significant.

(*c*) During the period in which the capital-output ratio was rising (from the 'seventies to about 1920), there is likely to have been a decline also in rates of return, because, aside from cyclical and other fluctuations, the relative shares in income showed no tendency to change. This conclusion concerning relative shares is based on Edward C. Budd's work, which applies a correction for the changing weight of the self-employed. The fact that during the forty-year period in question the changes in the composition of output favoured, on balance, the capital-intensive sectors, suggests that the compositional changes may have had a labour-share-reducing effect which was offset by a somewhat rising tendency of labour's share within the individual sectors of the economy.

(*d*) From 1920 to 1950, when there occurred a fall in the capital-output ratio, the relative share of capital declined, and the rate of return (P/K) probably did not change much. On the whole, there may even have been an insignificant rise in the P/K ratio from the beginning to the end of this period, but a smaller rise than would have been required to offset the preceding decline.

Intersectoral shifts favoured the share of labour during the period 1920–1950, taken as a whole, and the rise in the share of labour can probably be fully explained by *these* shifts (not by intrasector changes).

IX. EVALUATION OF THE RECORD

We may therefore conclude that while innovations have been sufficiently plentiful to prevent a fall in the Y/K ratio for the period from the 'seventies to the present, they probably were not sufficiently 'relatively labour-saving' *or* too 'relatively capital-saving' to prevent some degree of lowering of rates of return (before taxes). However, this lowering does not show consistently over all sub-periods, and in particular does not show for a comparison of 1950 with the 1920s. In general, therefore, the distribution of the factor-saving effects of the innovations has been such as to prevent any appreciable or consistent lowering of the P/K ratio, while wage rates have, of course, shown an appreciable uptrend.

The last statement takes as given the changes in the capital-labour ratio, the *quantity* of innovating activity (as measured by its effect on the Y/K ratio), and the intersectoral shifts during the period. The distribution of the factor-saving effects has so far come out 'just about right' in the foregoing sense, *when these other changes are taken as given*. The term 'just about right' is placed in quotes because it arbitrarily implies the norm of a horizontal trend in rates of return combined with an uptrend in real wage rates. Also the term 'just about right' should be qualified by the statement that by this criterion innovations tended to be somewhat too 'relatively capital-saving' (*or* not quite sufficiently 'relatively labour-saving') to prevent a fall in P/K over a period of eighty years.

It is tempting to attribute these results to the workings of an adjustment mechanism which may direct the character of inventing and innovating activity. To firms possessing monopsony power in factor markets relative resource scarcities are directly transmitted; in the footnote on p. 68 we argue that sufficiently strong overshooting in the character of innovations may well result in a kind of 'monopsonistic distortion' which tends to put an end to the overshooting and thus also to the monopsonistic distortion.[1] The introduction of new technology has come to be based on increasingly conscious and systematic efforts. The hypothesis of a somewhat imperfectly functioning adjustment mechanism therefore possesses a good deal of plausibility.

X. HYPOTHESES CONCERNING NEUTRALITY OF INNOVATIONS AND LESS THAN UNITARY ELASTICITY OF SUBSTITUTION

What has been said so far does not confirm or contradict the possibility that innovations may have been 'neutral'. They ***may have been*** 'neutral' in the sense of having been neither 'relatively labour-saving, rather than relatively capital-saving' nor 'relatively capital-saving, rather than relatively labour-saving' (see Section V and Section VII). This is what is meant by neutrality.

Assume, for example, that the elasticity of substitution along given production functions is unitary and that there are no changes in the product-mix. Assume also that the quantity of innovating activity is such that Y/K stays unchanged in spite of a continuous rise in the capital-labour ratio. In this case the real wage rate may be rising very gradually and the profit rate will show a horizontal trend provided that innovations are 'neutral' in the sense just explained. Consequently, in this case the system 'needs neutral innovations', and the neutrality of innovations is compatible with the statement that the relative labour-saving and relative capital-saving effects were 'just about right'. On these assumptions, which are implicit in the Harrod-Domar equations, 'neutrality' does not necessarily point to randomness, but it *may* result from an adjustment mechanism. Still, if neutrality can be established, randomness

[1] If many firms adopt innovations the characteristics of which overshoot in one direction, then the resulting trends in factor prices provide incentives to seek inventions with the contrary characteristics. The reason may be expressed in the following way. Significant overshooting in the capital-saving direction would create a degree of labour-scarcity which expresses itself in 'monopsonistic' awareness of the unavailability of labour even in a world of otherwise purely competitive firms. This is because every firm is aware of the fact that it cannot compete away employed labour of given varieties from other firms without offering higher wages. On the other hand, significant overshooting in the labour-saving direction creates 'monopsonistic' awareness of a capital-scarcity. Either kind of 'monopsonistic awareness' transmits a scarcity effectively to the individual firm which then seeks to direct its innovating activity towards alleviating the scarcity. Such a mechanism, however, cannot be expected to be very precise because between 'significant overshooting' in the one direction and such overshooting in the other direction there may exist a rather wide range of distributions of the factor-saving effects with no quasi-monopsonistic consequences.

emerges as an alternative interpretation, a possibility that cannot be brushed aside.

As was explained in Sections V and VII, we are on less hazardous ground when concluding that the distribution of the relative factor-saving effects of the innovations has resulted in certain observable trends than when we attempt to decide whether innovations were or were not neutral. However, very recently Robert M. Solow of the Massachusetts Institute of Technology has invented a method by which he could appraise the character of the American innovating activity for 1909–1949, and he concluded that the innovations of the period were neutral. The footnote below contains a sketch of the idea underlying the method which Solow used in his article in the *Review of Economics and Statistics* (August 1957). Obviously, any analysis of this sort is based on far-reaching simplifying assumptions.[1]

If we accept Solow's results, then the conclusions of Section IX still stand, but the following conclusions may be added:

(*a*) At least in recent decades, and possibly also earlier, the innovations seem to have been 'neutral' in the sense of having been neither 'relatively labour-saving rather than relatively capital-saving' nor the contrary.

(*b*) The elasticity of substitution along given production functions is probably smaller than one. This is a proposition which I should like to add as a corollary in the event that we accept Solow's

[1] On the assumptions of the marginal productivity theory we may distinguish that part of the aggregate output-increment per period which would be created by the additional factor inputs *along given production functions* from that part which is created by innovations. This is because the observable distributive shares reflect the partial elasticities of the given production functions with respect to the factor inputs. The output-increment *not* attributable to additional factor inputs (via this elasticity estimate) may then be interpreted as having resulted from innovations. This latter part of the output-increment of each period, when expressed as a percentage of the output itself, will not depend on the quantity of factor inputs at the time of the innovating activity if, and only if, the innovations are neutral. The reason for this is that neutral innovations, which by definition leave relative shares unchanged, do not alter the elasticities of the production function (note that these elasticities are ratios of marginal to average products). Consequently, neutral innovations shift up the production function by the identical percentage for alternative input combinations. The proposition which Solow established empirically for the United States is that for the period 1909–1949 the percentage upward shift due to innovations showed a time-shape which was independent of the time-shape of the factor inputs.

Mathematically, this implies that the production function, corrected for innovations, can be written in the specific form $Y=A(t) \, . \, f(L, K)$, where $A(t)$ expresses the innovation effect. The general form of such a 'production function corrected for innovations' would be $Y=F(L, K, t)$, as was seen in footnote 2 on page 64. The specific form $Y=A(t) \, . \, f(L, K)$ is valid only in the case of neutral innovations.

One reason why this very interesting and original piece of analysis is not truly *conclusive* is that *in the event of non-neutrality* other time-trends (for example, a trend in the quantity of innovations) may eliminate the change in the proportionate rate of increase in output which 'should' show with the passage of time when the capital-labour ratio changes.

neutrality suggestion (especially if we believe that neutrality dates further back than 1909).

The following is my reason for believing that if we accept proposition (*a*), then (*b*) is probably also valid. From the 1870s to the 1920s the intersector shifts, on balance, probably favoured the share of capital. Yet distributive shares showed an approximately horizontal trend. If this is so, and if innovations were neutral, the movements along the production functions presumably increased the relative share of labour. Consequently, along the functions, relative shares would have changed in favour of the factor that was becoming relatively scarce. This means less than unitary elasticity of substitution, since the capital-labour ratio was rising appreciably.

From 1929 to 1950 there was very little change in the capital-labour ratio, and the relative share of labour seems to have risen just about to the extent determined by the intersector shifts which in this period favoured labour's share. This is consistent with the suggestions expressed in the present section, and perhaps it lends Solow's results some additional plausibility (because the changes in distributive shares can be explained without bringing in the innovations of the period). However, the results of my own analysis are contained in Section IX which does not imply neutrality.

XI. CAN WE PLACE RELIANCE ON THE NUMERICAL RESULTS?

In Section I, I commented briefly on the specific method of measuring capital which is implied in the numerical appraisals of this paper. The analysis relating to the concepts themselves does not imply this specific method.

On grounds of 'pure logic' no convincing argument can be made for the method under consideration. The following appears to be the nearest it is possible to get to linking the method to purely logical reasoning.

If expectations always came out right, and if the value of the capital stock included no capitalized rents, then the reproduction value of the stock would be equal to its discounted net earnings. A time series of reproduction values would give us the same trend as a time series of discounted net earnings even if, with the passage of time, expectations merely stayed equally near to realized results, and if the relative weight of capitalized rents stayed unchanged. On these simplifying assumptions, reproduction values can be linked to the theoretically relevant concept of discounted net earnings.

However, to define a production function with 'capital in general' as one of the inputs, we should have to use some method of price correction in any event. This is because we are concerned with changing capital inputs in *real terms*. The physical character of the capital goods changes from period to period, and price correction therefore creates difficulties which cannot be solved really satisfactorily.

The difficulties are of the same general character as those which we encounter when deflating the value of a flow of consumer goods for price changes. The quality of consumer goods, too, changes all the time. In some sense the difficulties are even greater for capital goods than for consumer goods, because it is part of the inherent logic of the process that the physical character of the capital stock *must* change. It must change regardless of whether we move along given production functions or take account of innovations. On the other hand, it is perhaps more 'necessarily true' that changes in the physical character of the existing stock are very gradual than that the changes in the quality of the consumption flow are; and at any rate it does not matter much whether the water in which the logical purist drowns is ten feet or twenty feet deep.

When the physical character of goods changes (say, improves) and we nevertheless speak of 'the same quantity' of goods provided that the price-corrected value is the same, we draw a distinction which at least for capital goods we intend to make but which in practice we cannot make without resorting to arbitrary procedures. This is the distinction between a greater quantity of goods, and goods with greater productivity. The distinction is essential for the concept of the production function, unless each type of capital good is viewed as a separate factor. The procedure which yields this distinction as a by-product is, of course, arbitrary in that only crude common-sense criteria can be used for deciding which specific goods of one period are 'the same kind of goods' as specific goods of a preceding period (in the sense that if their price-corrected value has stayed unchanged, then their quantity has stayed unchanged, although their productivity usually has changed).

To summarize, on certain simplifying assumptions we can link a meaningful concept of capital to current reproduction values; but only with considerable arbitrariness can we arrive at a time-series of price-corrected reproduction values. What we really have is usually not a price-corrected series of reproduction values but a price-corrected series of original costs of acquisition with allowances for depreciation. The relationship between the trends in these two types of series, respectively, will again depend on the quality of foresight. We have come a long way from a meaningful concept of 'capital'.

However, the proper line of defence should be developed differently, and I feel that this can be done rather convincingly. A time-series of this sort is not useless if in its main outline it rather consistently shows a development which is confirmed by common observation, that is, if the main outline of the development is in accordance with the intuitive content of the concept which is being formalized. The price-corrected output series satisfy this condition rather well : in their main outline they behave 'as expected' in major depressions, in minor recessions, and over longer intervals. The capital series also behave rather well in this regard, and they 'make sense' also when considered in conjunction with labour-input series. If the crude and easily observable characteristics of a series satisfy this test rather consistently, one is perhaps justified in paying some attention also to those additional properties of the series which inherently are incapable of being confirmed or refuted by 'common observation'. To be sure, such an extension of confidence to the additional and finer properties of the data is not riskless. But this risk is inevitable in empirical investigations, regardless of whether they do or do not involve the concept of capital.

Chapter 5

ON MEASURING CAPITAL [1]

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I. INTRODUCTION

THERE are no statistics of capital corresponding to those of output, input and employment — and this for very good reasons. While it is possible to ask sensible and economically meaningful questions about output, input and employment, no such questions about capital are, or ever will be, included in censuses of manufactures. First, instructions for the answering of questions about capital cannot be sufficiently precise. Second, in the great majority of business firms, records of assets are not in a form in which they would yield data on capital as defined by economists.

There is, of course, a certain amount of information collected in censuses of manufactures which has been used for lack of anything better. First, there are statistics on motive power installed or in use. These give some measure of the degree of mechanization achieved in different countries in comparable trades but are less satisfactory for the comparison of different trades since motive power is only one specific form of capital. Second, in some countries censuses of manufactures have collected data on 'capital'. Although the precise meaning of the questions asked is not always clear, these data broadly correspond to the book value of assets. In the United States the authors of censuses warned against the use of these figures and they were eventually discontinued in 1922. In Canada, also, the data obtained were thought to be misleading rather than useful and were discontinued in 1956. In the United Kingdom, the post-war pilot census of production for 1946 included questions on the book value of assets (for income tax purposes) but the results were suppressed.

Admittedly statistics of output, input and employment are not

[1] This paper is based on research into the measurement of capital at the N.I.E.S.R., London. Grants in aid from Conditional Aid Funds and from the Rockefeller Foundation are gratefully acknowledged.

accurate. But on the whole the concepts used by statisticians are fairly close to the concepts of economic theory, and the questions asked should be answerable by an efficiently-run business firm. The inaccuracy of the data obtained is statistical and relatively unimportant. But as regards data on capital, conceptual divergences play a more important rôle than statistical errors. Data on the book value of assets are especially deceptive as they appear accurate and, sometimes, comprehensive ; but they may give a very misleading measure of capital as an economic variable.

This inherent lopsidedness in the body of statistics inevitably led to gaps in empirical economic research on productive relationships. Thus the statistics available induced an emphasis on input-output relationships and on output-per-man data rather than on the triangular relationship between capital, labour and output. To fill the gap deliberate efforts had to be made to construct data on the missing variable, capital.

The efforts to measure capital as an economic variable follow either the micro- or the macro-approach. With the micro-approach information is obtained from original sources specifically for the purpose of measuring capital. With the macro-approach, data already available are manipulated to give the required results. With the micro-approach, data can be obtained only through sampling but, on the other hand, with the macro-approach one is usually not quite sure of the conceptual basis. Hence, as regards accuracy, the sampling errors of one approach are to be weighed against the conceptual errors of the other.

I shall distinguish four methods, two following the micro- and two the macro-approach.

1. *Sample surveys of statistical data.* This is the method I have been following in an enquiry at the N.I.E.S.R., London. The essence of the method is to discover questions which are economically meaningful and which can be answered from the internal records of reasonably efficient business firms.

2. *Sample surveys of engineering data.* This is the method followed for most industries, though supplemented by other data, by the Harvard Research Project and by the RAND Corporation. The main sources of data are plans for new projects, such as those given in applications for government financial assistance.

3. *Estimates derived from book values.* The most prominent of recent estimates are those of the N.B.E.R., New York, including those for manufacturing by Creamer and Bernstein. In these estimates an attempt is made to correct book values for the changing

level of prices and, if possible, for revaluations. But no attempt is made to correct unrealistic depreciation policies.

4. *Estimates derived from series on capital expenditure.* This is the 'perpetual inventory' method pioneered by Goldsmith in the U.S.A. and followed more recently by Redfern in the U.K. and Krengel in W. Germany. Capital is estimated as the accumulated capital expenditure of a number of years expressed in constant prices, which may or may not be written down for depreciation.

In the following I give a brief account of my own research (1 above) [1] but I also include some comments on the perpetual inventory method (4 above). The latter method appears attractive and the one which could be most easily followed in a number of countries. However, the lengths of life of assets which are used in published estimates are based on convention and not on empirical observation, and this may be a source of important error in the results. I feel that a reconciliation of the results of the two methods should prove fruitful.

I do not propose to give a full account of the other two methods and the following comments are not offered as a balanced criticism.

As regards sample surveys of engineering data (2 above), there are three kinds of difficulties. First, almost all new projects are unbalanced in the sense that they allow for further expansion with relatively small amounts of additional outlay. Second, capital-output-labour relationships are continually changing; it is difficult to foresee the relationships that will emerge after experience has been gained in running the plant. Third, very often social factors (e.g. shift working) are important in determining the degree of utilization of plant, which can actually be only a fraction of the theoretical maximum. Hence engineering data need careful checking against statistically-observed relationships. There is, however, one great advantage in the use of engineering data: they refer to a single technique (the latest), while statistically observed relationships usually average-out old and new techniques. A good case can be made for deriving incremental capital coefficients for the latest techniques only.

As regards estimates based on adjusted book values (3 above), it is difficult to see how book values can be adjusted without much fuller information than is generally available. Book values are, as a rule, at original cost and it is impossible to bring them to a given level of prices without knowing the composition of the total by years of vintage. Moreover, assets are generally written off too fast, and

[1] The other methods are followed in the papers by E. Domar and W. Hoffmann.

assets completely written off, but in use, escape inclusion in the estimates whether a correction is made for changes in the price level or not. The fact that some business firms revalue assets, but the majority do not, must make the statistician's life more and not less difficult. All in all, I feel that estimates derived from book values have inherent defects. For instance, some cyclical variation in capital coefficients can be easily explained by the cyclical nature of the error in such estimates.

II. THE STATISTICAL DESCRIPTION OF CAPITAL

Capital being a multi-dimensional concept, it is simplest to proceed by describing several aspects of it. I suggest that the chief characteristics of capital pertaining to a particular industry are: (1) the gross (i.e. undepreciated) capital-labour or capital-output coefficient; (2) the mortality experience of different component parts of the stock of assets; and (3) the depreciation of the value of individual assets with age.

If complete information were available, such as is postulated in economic theory, measures of these three characteristics could be derived from it; and it could be shown that the measures are not independent of each other. In theory, the value of capital is defined as the discounted future income stream to be derived from it. With complete information on the future income stream and with a given rate of discount, it is easy to determine the three measures defined above and it can be seen that variations in the rate of discount would influence each.

In recent years, formulae have been devised by Champernowne and Kahn [1] and, under somewhat less restrictive assumptions, by Blyth,[2] to examine the effects of the rate of discount on the depreciation of assets with age; though these authors assumed that the length of life of assets is fixed, irrespective of the rate of discount. I feel that the empirical approach must be much cruder and that the three characteristics defined above could be measured as if they were invariant to changes in the rate of discount. If justification for this assumption is necessary, I suggest that the rate of discount implicit in empirical valuations is considerably higher than the market rate of interest, and that variations in it, either through time or between different countries, are small compared with variations in other

[1] D. G. Champernowne and R. F. Kahn, 'The Value of invested capital', *Review of Economic Studies*, 1953–54.

[2] C. A. Blyth, 'The theory of capital, and its time-measures', *Econometrica*, 1956.

factors which enter into the calculation, notably variations in entrepreneurial expectations.

I succeeded in obtaining sample data for British manufacturing industry under (1) and (2) above and gained some impressions under (3).

IIIa. REPLACEMENT COST[1]

The success in obtaining estimates of gross (undepreciated) capital in British manufacturing is due to the discovery that a sufficient number of business firms possess, and are willing to disclose, realistic data on the value of their assets. This valuation is different from what is given in balance-sheets or derived from stock exchange quotations, although the different valuations interact on each other, at least through psychological factors.

The 'replacement cost' valuation of assets is realistic in the sense that it is a suitable measure for the purpose of evaluating the contribution which capital makes to production. It refers to capital employed rather than to capital owned by the firm, and it can be separated by activity and by type of asset. Replacement cost is frequently computed for purposes of fire insurance and therefore it is fairly widely available. In fact, I asked firms whether the fire insurance valuation of fixed assets could be taken as a satisfactory measure of replacement cost, or if some adjustments were necessary, or if they had an even better estimate for the purpose.

The obvious answer to the search for 'realistic' valuation is market price. The difficulty is that market price is not always available or, when available, is not necessarily accepted as typical. Only a fraction of fixed assets are new, and as regards the second-hand market, there are seldom any regular quotations. There is always a suspicion that only in abnormal cases (e.g. forced sale) do assets pass through the market. But even though observed market prices are not always acceptable, in principle the equivalent of 'normal' price can be imputed. Such imputations are in fact made for purposes of fire insurance, adjustments to the balance-sheet, offers of purchase, or public price fixing.

Unlike book values, which are spuriously accurate, the expert valuation is frankly an estimate but is economically more significant. The expert tries to estimate the market price that would obtain under certain hypothetical conditions. He assumes, for instance, that the transaction takes place as between willing seller and willing buyer.

[1] For a fuller description of the results of this sub-section, cf. *Journal of the Royal Statistical Society*, 1957, Series A, Part I.

The general method followed in estimating replacement cost is, first, to estimate the cost of replacement with a brand-new asset and, second, to make an allowance for age, wear-and-tear, and obsolescence. The difficulty is that replacement, if it were to take place, would not be by identical units. As a result of inventions, capital undergoes an almost continuous improvement and hardly any two assets produced at different times are identical. The expert has to find a modern substitute for the existing asset and has to take account of differences in the profitability of the two types of asset. Lastly, when he takes age into account, he has to look forward and estimate the likely future life of the asset, rather than look into the past to determine expired life. Naturally different experts may arrive at different results, but this is no more than to say that the estimates are subject to error.

There is an important difference between the value of assets as installed in certain relation to each other and the break-up value when each asset is sold separately. For our purpose, assets should be valued at their worth to the existing management, on the assumption that the firm is to continue in business.

As stated above, the expert estimate is made in two steps and it follows that there are two concepts of replacement cost : replacement cost new, and written-down replacement cost. The second concept corresponds to the value of capital in economic theory but the first may be equally important in a study of productive relationships. The value of assets declines with age partly because of falling efficiency (in an economic or in a technical sense). It logically follows that value declines faster than efficiency, and indeed for important classes of assets efficiency does not decline at all. For this reason the relationship between replacement cost new and output may be more stable than between written-down replacement cost and output, and the first concept is more relevant in forecasting incremental requirements of capital.

It so happened that the firms included in my sample amply supplied me with estimates of replacement cost new but not with written-down replacement cost ; it should not be assumed that this will be the case in other samples also. Data were obtained from ninety firms, some of whom operated in more than one industry ; these firms accounted for 13 per cent of manufacturing employment and 16 per cent of fixed assets. The results are summarized in Table 2 and Fig. 5.

Note.—Fixed assets include buildings and machinery but exclude land and vehicles. Data other than for fixed assets are taken from the census of production. Since the presentation of these results to the Royal Statistical Society in 1956, data have come to light which suggest that fixed assets in glass are higher and in cement very much higher. Stocks in tobacco are also considerably higher if account is taken of bonded stocks. The original estimates were left uncorrected

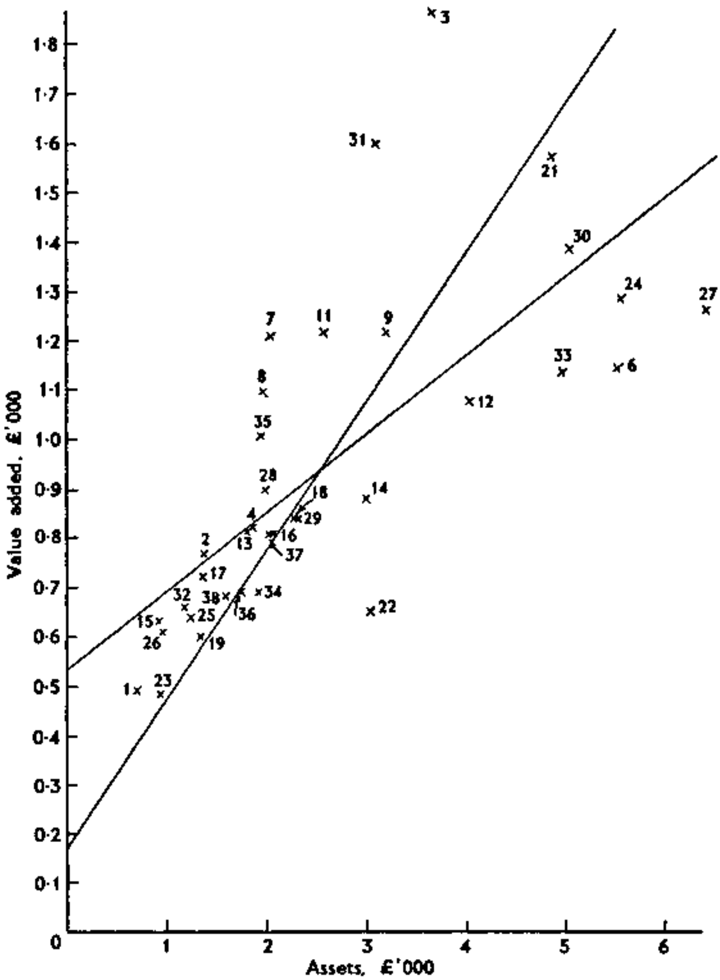

Fig. 5

though the alterations mentioned would improve the fit of the regression lines in Fig. 5.

The coefficient of correlation between the variables shown in the figure is 0·726, which is significant far below the 1 per cent level of

TABLE

VALUE ADDED AND ASSETS PER EMPLOYEE IN DIFFERENT

Number in Figure 5	Industry	Fixed Assets
		£
1	China and earthenware	590
2	Glass	1,140
3	Cement	2,950
4	Other non-metalliferous products	1,580
* 5	Coke ovens	8,450
6	General chemicals, plastics, etc.	5,000
7	Drugs and toilet preparations	1,520
8	Paint and varnish	1,400
9	Soap, etc.	2,560
* 10	Mineral oil refining	12,680
11	Other chemicals	1,820
12	Iron and steel	3,480
13	Foundries, tubes, etc.	1,520
14	Non-ferrous metals	2,370
15	Shipbuilding	860
16	Mechanical engineering, etc.	1,420
17	Electrical engineering, etc.	1,050
18	Motor and aircraft	1,740
19	Other vehicles	950
20	Metal goods n.e.s., precision instruments, etc.	1,050
21	Rayon, etc., production	4,340
22	Other textiles	2,570
23	Leather, fur clothing, etc.	670
24	Grain milling	4,690
25	Bread, etc.	1,170
26	Biscuit	780
27	Sugar and glucose	5,150
28	Cocoa, chocolate, etc.	1,410
29	Other food industries	1,840
30	Drink industries	3,800
31	Tobacco	1,550
32	Manufactures of wood and cork	860
33	Paper and pulp	4,460
34	Cardboard and paper products	1,630
35	Newspaper printing	1,820
36	Printing	1,490
37	Rubber	1,660
38	Other manufacturing	1,310
	All manufacturing industries	1,740

* Nos. 5 and 10 are not shown in Figure 5.

2

MANUFACTURING INDUSTRIES, UNITED KINGDOM, 1954

Stocks, etc.	Total Assets	Value Added	Wages and Salaries	Gross Profits	Ratio of Fixed Assets to Value Added	Ratio of Total Assets to Value Added	Gross Profits to Total Assets
£	£	£	£	£			%
110	700	490	350	140	1·2	1·4	20
220	1,360	770	470	300	1·5	1·8	22
680	3,630	1870	620	1250	1·6	1·9	34
250	1,830	820	500	320	1·9	2·2	17
340	8,790	1100	520	580	7·7	8·0	7
510	5,510	1150	560	590	4·3	4·8	11
500	2,020	1210	420	790	1·3	1·7	39
550	1,950	1100	500	600	1·3	1·8	31
620	3,180	1220	490	730	2·1	2·6	23
2040	14,720	1600	560	1040	7·9	9·2	7
730	2,550	1220	500	720	1·5	2·1	28
530	4,010	1080	540	540	3·2	3 7	13
280	1,800	810	500	310	1·9	2·2	17
600	2,970	880	530	350	2·7	3·4	12
70	930	630	490	140	1·4	1·5	15
630	2,050	810	500	310	1·8	2·5	15
420	1,470	730	460	270	1·4	2·0	18
530	2,270	840	540	300	2·1	2·7	13
390	1,340	600	470	130	1·6	2·2	10
310	1,360	720	430	290	1·5	1·9	21
500	4,840	1580	510	1070	2·7	3·1	22
450	3,020	650	360	290	4·0	4·6	10
260	930	480	320	160	1·4	1·9	17
850	5,540	1290	470	820	3·6	4·3	15
60	1,230	640	370	270	1·8	1·9	22
180	960	610	320	290	1·3	1·6	30
1260	6,410	1270	550	720	4·1	5·1	11
580	1,990	900	370	530	1·6	2·2	27
450	2,290	840	390	450	2·2	2·7	20
1220	5,020	1390	430	960	2·7	3·6	19
1520	3,070	1600	400	1160	1·0	1·9	38
310	1,170	660	440	220	1·3	1·8	19
490	4,950	1140	500	640	3·9	4·3	13
280	1,910	690	390	300	2·4	2·8	16
110	1,930	1010	590	420	1·8	1·9	22
250	1,740	690	450	240	2·2	2·5	14
390	2,050	810	460	350	2·0	2·5	17
280	1,590	680	400	280	1·9	2·3	18
440	2,180	780	450	330	2·2	2·8	15

probability. Hence it is reasonable to describe the relationship between value added per employee per annum (v) and assets per employee (k) by a straight line.

The two elementary regression lines, which are shown in the chart, are

$$v = 532 + 0{\cdot}161k$$

and

$$v = 165 + 0{\cdot}306k$$

(where v and k are in £).

In order to obtain a unique line, which is less confusing when it comes to interpretation, the 'true' regression of v on k was estimated by taking account of the (subjectively determined) error in both variables. The result is

$$v = 458 + 0{\cdot}19k.$$

Now there are theoretical reasons why the observed relationship should be linear. Under competitive conditions wages for the same type of labour in different industries tend to be the same and the rate of profit on capital tends to be the same. Writing w for annual wages per employee and i for the annual rate of profit on capital,

$$v = w + ik.$$

From the previous equation $w =$ £458 (which is not far from the actual average wage in manufacturing) and $i = 19$ per cent.

These results may have to be modified somewhat. Average wages in different industries appear to be correlated with the level of capital-intensity (k); the correlation coefficient of 0·388 is significant at a level of probability just below 1 per cent. With each £100 increase in assets per employee, average wages per annum rise by about £2. If the increase in average wages with capital-intensity is due to a different distribution of the product between capital and labour, all is well with the above estimate of $i = 19$ per cent. If, however, this phenomenon is simply the reflection of differences in skill, in so far as higher skill is associated with a higher level of capital, it is better to estimate the rate of profit from a correlation of gross profits and assets. The data shown in the table then give an alternative $i = 16$ per cent.

Finally, a tentative estimate is made for *net* return on the *net* value of capital. I have made rough estimates to allow for depreciation and certain overhead expenses which are included in gross profits. I have assumed that the *net* value of capital is uniformly two-thirds of the gross value. The best estimate of the net rate of profit then comes to 20 per cent with a margin of error of 2 to 3 per cent. This is, of course, net profit before tax.

This conclusion is important, though not surprising, in so far as it demonstrates that the rate of profit which enters into economic decisions is far above the rates of 4 or 6 per cent which are often mentioned in theoretical literature.

The capital-output ratio, which is also much discussed in economic literature, is k/v in our notation. It is obvious from the relationship surveyed that k/v is increasing with the level of capital intensity. The *marginal* capital-output ratio is constant, as it is simply the reciprocal of the rate of profit. In our case the marginal capital-output coefficient is about 5, compared with an average capital-output coefficient of about 3.

It must be emphasized that this conclusion, which is based on cross-section data, is at variance with conclusions based on time-series. In the latter, marginal and average coefficients are about the same. The reason is that cross-section data refer to static conditions while time-series data embody the effects of technical progress. The two marginal coefficients are not comparable.

It is evident that the Cobb-Douglas function should not have been used by Douglas and his disciples to describe cross-section data. I have argued here that cross-section data are described by a linear and not by a logarithmic relationship. We may visualize a Cobb-Douglas type of production function for each homogeneous industry. In each function only one point is *ex post* measurable, and these points lie on a straight line. This line is the envelope of all production functions.[1]

IIIb. MORTALITY

The literature on the mortality of capital is confined to specific items, the most popular being ships. Kurtz brought together a number of studies [2] but they relate typically to small and standardized assets, mainly in the public utility field, such as telegraph poles and electric bulbs. Whether conclusions derived from these studies can be accepted for the wider field remains to be seen. In the wider field it is obsolescence rather than wear-and-tear which is the dominant cause of mortality — homicide to make room for a new favourite, rather than natural death.

In my enquiry no distinction was made between assets of various types, except that buildings were separated from plant and machinery (excluding vehicles). My main interest was to study the distribution

[1] This argument is more fully developed in my paper, 'Du capital envisagé comme une variable économique', *Centre National de la Recherche Scientifique, Cahiers du Séminaire d'Econométrie* (Paris, 1959).

[2] E. B. Kurtz, *Life Expectancy of Physical Property* (New York, 1930).

of capital according to length of life in different industries. It is clear that some types of asset are more long-lived than others. But assets of a given type will not all have the same life experience, just as human beings of a given class die at different ages. The distribution of assets according to length of life is therefore the result partly of assets being of different type and partly of stochastic elements ; all we observe is that a certain proportion of assets dies at a certain age.

With capital, neither the unit to be observed nor the length of life are as obvious as, say, with a human population. It is not obvious whether the unit is a complex plant or only a small component of it. As regards life, neither the date of birth nor the date of death are always uniquely definable. In the event, the analysis was based on business records, mainly plant registers. These contain entries for the acquisition of capital assets. Length of life is measured simply by the date of entry in and the date of deletion from the register. Sometimes capital expenditure is incurred before the date of entry and sometimes the asset is relegated to stand-by before the date of deletion. The common unit is the money cost of capital.

Two questions were put to firms. (1) The percentage of assets of different dates of birth surviving in 1957 or thereabouts, i.e. the original cost of assets acquired in year x and still on the plant register as percentage of total capital expenditure in year x. (2) The percentage of assets of different dates of birth dying in 1957 or thereabouts, i.e. the original cost of assets acquired in year x and deleted from the plant register in 1957 as percentage of total capital expenditure in year x.

From answers to the first question survival rates at different ages can be directly obtained. From answers to the second question mortality rates at different ages can be obtained and, by cumulating them, survival rates can be obtained. These methods are similar to those used in constructing human life tables and the reason for asking two questions is that the records kept by firms sometimes make possible answers to one or the other question but not to both. It is to be noted that the answer to neither of the two questions involves comparisons of price levels.

About 300 manufacturing firms were selected at random, but useful answers to the above questions were received from only 69, of which 57 were included in the provisional statistical analysis summarized here.[1] This relatively small number of firms is not the

[1] I was helped, in this analysis, by E. Kleiman, now at the Falk Foundation, Jerusalem.

result of a high refusal rate but rather of an inability to answer questions because of lack of records. The refusal rate can be put at about one-third to one-half of the total number of firms approached, while the others gave various explanations of the inadequacy of records. There is ample evidence of a long-term improvement in business records relating to capital assets, especially since 1948, but it will be many years before such records will be ample for research of this type.

But even a small number of observations can be subjected to rigorous analysis to test various theoretical hypotheses. From the present analysis, which was confined to plant and machinery, three conclusions emerged. (1) That the survival curve can be described by a straight line. (2) That important differences exist between survival curves relating to different industries. And (3) that survival rates constructed from current mortality rates are higher than survival rates directly obtained. In the calculations, simple arithmetic averages were taken for survival rates relating to different firms.

First, it was found that a straight line describes the data as well or better than any other simple curve such as the exponential, the logarithmic, or the quadratic. In other words, mortality appears to be constant instead of varying with age. The straight-line description is of course attractive because of its arithmetic convenience. However, the data are not necessarily inconsistent with the logistic curve which is often used for describing biological populations. The straight lines appear to start not from 100 per cent at birth but rather from 100 per cent at 3 or 5 years of age. Moreover, very little information was obtained for higher ages and it is not impossible that beyond the age of, say, 50 years the survival curve would become asymptotic to the age-axis.

Second, the differences between the regression coefficients (i.e. the mortality rates) relating to different industries were found to be significant by Bartlett's homogeneity test.[1] Table 3 shows for each industrial group the square of the coefficient of correlation (R^2) between survival rates and age ; the slope of the regression line (the mortality rate, $-b$) ; and the estimated age at which all assets disappear ($-a/b$). The first estimate is based on survival rates directly given and the second on survival rates derived from current mortality rates. Although some of the coefficients are based on a very small number of observations and are subject to wide error, all the differences between industries are unlikely to have arisen by chance.

[1] Cf. L. H. C. Tippet, *The Methods of Statistics* (Fourth edition, London, 1952), pp. 156-7.

Third, the tests also show that the differences between the first and second sets of estimates of $-b$ are systematic, and that the current mortality rates tend to be lower than those obtained from survival rates. This is consistent with the hypothesis that the rate of scrapping varies with the trade cycle and in boom years scrapping is postponed. This would affect the second estimate much more than the first, which embodies the experience of several trade cycles.

The data are most reliable in the engineering industries, and Bartlett's test of homogeneity showed that the three groups (mechanical engineering, vehicles and electrical engineering) can be aggregated. The survival curves (first and second estimate) for engineering as a whole are shown in Fig. 6.

TABLE 3

COEFFICIENTS OF SURVIVAL CURVES

Industry	First estimate			Second estimate		
	R^2	$-b$ %	$-a/b$ years	R^2	$-b$ %	$-a/b$ years
General engineering	·548	2·43	43	·259	1·76	60
Vehicles	·491	2·09	51	·332	2·03	54
Metals	·586	2·03	53	·900	1·77	59
Chemicals	·580	1·71	64	·885	2·27	40
Food, drink, and tobacco	·458	1·56	61	·163	1·20	69
Textiles	·120	1·40	69	·549	1·11	95
Electrical engineering	·215	1·37	54	·541	0·44	213
Miscellaneous	·254	1·22	82	·284	0·57	174
All firms	·394	1·77	57	·307	1·22	82

The data obtained on the whole describe the first 30 or 40 years of life while it is known that a substantial proportion of assets in particular industries have a much longer life. A supplementary enquiry directed to long-lived assets would be desirable. But even the data here obtained appear to indicate that the life of plant in the engineering industries is longer than generally suspected. The much-dramatized items of equipment — such as some special tools in the motor-car trade — form a relatively small proportion of total assets. It is also surprising that the wastage of assets begins almost at birth. It is of course possible that some short-lived assets are included in the statistics collected here, even though for accounting purposes they are charged against revenue.

One general difficulty with empirical work of this kind, which I have not mentioned so far, is that assets are bought and sold second-hand. In such cases the continuity of records is broken and it is not feasible to trace the history of individual assets through life. As regards my sample, a small number of engineering firms appear to use new or fairly new plant only and to sell plant to other firms after a few years' use. Also, British firms with overseas subsidiaries tend to ship second-hand assets abroad and, equally, British subsidiaries of American firms appear to acquire some assets second-hand from the parent company.

The only statistical solution I could think of is to regard a change in ownership as a death and a re-birth. I have therefore also asked

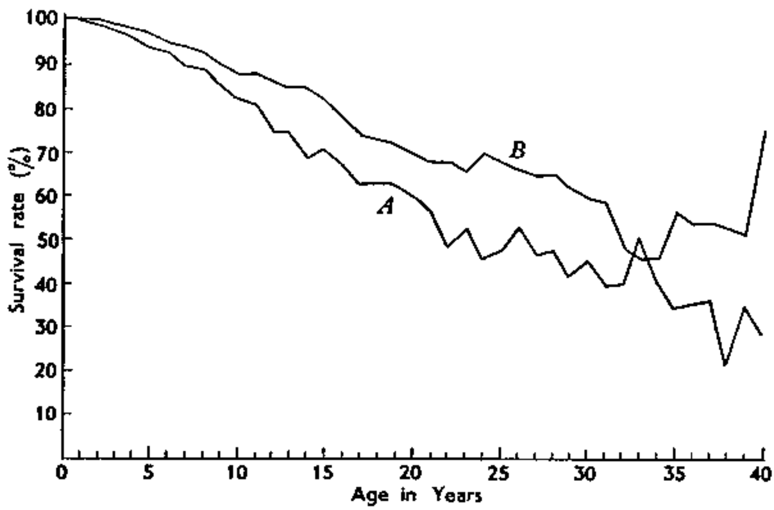

FIG. 6

firms to state the price obtained (whether sold for further use or for scrap) of assets disposed. I have re-inflated these residual values to make them comparable with prices ruling in the year of purchase. On the whole the proportion of residual value (in constant prices) remaining after lengthy use appears to be negligible, and in some cases it is negative (that is, disposal costs exceed receipts). But on disposals in the early years of life residual values are often fairly large. I propose to deal with this problem by saying that only a part of the asset is truly 'fixed' (and depreciating) while another part is of the nature of circulating capital. This point, however, is not yet taken account of in the estimates given above.

IIIc. DEPRECIATION

The great majority of firms in my sample follow the straight-line depreciation method in their own accounts (although they may follow the reducing balance method for tax purposes). The depreciation rates adopted invariably imply a shorter life than those obtained from historical data. It is generally admitted that depreciation, as regards the length of life assumed, is strongly conservative and there is little suggestion that assets currently installed are likely to be less durable than those installed in the past. Company depreciation policies are not suitable to deduce from them the shape of a realistic depreciation curve (i.e. a curve showing the fall in value with age).

The shape of a realistic depreciation curve can be computed *either* from explicit valuations of second-hand assets *or* from valuations implied in data on services rendered throughout life.

The first method has been successfully used in isolated cases where there are good second-hand quotations, such as motor cars. Cramer, for instance,[1] found the exponential curve a satisfactory description of second-hand motor-car prices.

The little information which I obtained suggests that for manufacturing assets the second-hand value is surprisingly high so long as an asset is continuing in use; after a decision is taken to scrap, the asset is run down through lack of maintenance though in some industries maintenance has to be kept up to the end. Buildings are generally kept, through repair and modernization, in a condition which makes them comparable to new buildings of the same type. I found that firms which calculate written-down replacement cost do not diminish the value placed on plant below a figure, varying from firm to firm, between 40 and 75 per cent of replacement cost new. The tendency to adopt the lower of these limits is perhaps more prevalent in the light industries, where marketing conditions depend on less predictable factors so that the expected effective life of equipment is uncertain. All this implies a depreciation curve which is convex rather than concave to the origin.

The direct questioning of business firms on the service yielded by assets is inconclusive because of the difficulties in phrasing satisfactory questions. It seems that most assets which are in use are maintained by considerable expenditure; the data collected indicate maintenance expenditures which are of the same magnitude

[1] J. S. Cramer, 'The Depreciation and Mortality of Motor Cars', *Journal of the Royal Statistical Society*, vol. 121, Part I, 1958, pp. 18-59.

as annual capital expenditure. Even though some repairs are capitalized, it is almost unavoidable that maintenance expenditure should improve the efficiency of plant. In many industries I had the impression that plant and labour form an organic unit and in the course of time gradual but perceptible improvements in efficiency take place. Most of these improvements involve expenditure but only of relatively small amounts. Technical progress is in a sense comparable to mountaineering. One climbs higher and higher on a given mountain until the peak is reached. Then one can see an even higher peak on another mountain but before attempting the second peak, one has to come down from the first, that is, scrap the plant.

In most industries which are capital-intensive the efficiency of plant tends to increase rather than decrease with life. Increased costs of maintenance can rarely be encountered, since plant is generally scrapped before this occurs ; the danger to be avoided is not so much an increase in the actual cost of maintenance but rather the loss of working time caused by breakdowns.

Any improvement in the technical efficiency of the plant may, however, be offset by economic factors, generally a relative fall in the price of the output. In fact it is the dominance of economic over the technical factors which makes it so difficult to obtain quantitative results.

Clearer thinking in this field might be possible by an explicit recognition of the fact that the use to which an asset is put may change during its life. In later life an asset may be put to an inferior use ; second-hand sales usually reflect this transition, but more often an asset is relegated to inferior use within the firm.

Quantitative measurement might become easier if we were to take each use as a separate life. Depreciation is likely to be faster in the superior than in the inferior use and two straight lines might describe depreciation better than a curve. The existence of an inferior use guarantees a relatively high residual value to the superior user and shortens the period for which he has to plan.[1] From the point of view of the inferior user, the existence of a superior use provides cheaper assets. There are industries which could be called inferior ; the furniture industry, for instance, thrives in old buildings but cannot pay its way in new ones.

[1] In other words replacement takes place more frequently.

IV. CONSISTENCY WITH THE PERPETUAL INVENTORY METHOD

Let us write $y(t)$ for the volume of investment in year t and $F(x)$ for the survival rate of assets at age x. $F(x)$ is a monotonic decreasing function between $F(0)=1$ and $F(a)=0$. Let us write $K(t)$ for the gross amount of capital in year t. Then, assuming an invariable survival function,

$$K(t)=\int_0^a y(t-x).F(x).dx.$$

Let us write $G(\xi)$ for the depreciated value of an asset as proportion of its new value, where $\xi=\frac{x}{z}$ is expired life as proportion of total expected life. $G(\xi)$ is a monotonic decreasing function between $G(0)=1$ and $G(1)=0$. Let us assume that $G(\xi)$ is identical (e.g. a straight line) for all assets. Let us write $C(t)$ for the net (depreciated) stock of capital in year t. Then

$$C(t)=\int_{z=0}^{a}\int_{x=0}^{z} y(t-x).\left\{-\frac{dF(z)}{dz}\right\}.G\left(\frac{x}{z}\right).dx.dz.$$

Now the perpetual inventory method requires information on $y(t)$, $F(x)$, and $G(\xi)$. In fact, in published estimates, $F(x)$ is obtained not by empirical observation but from income-tax rules and $G(\xi)$ is assumed to be a straight line, $G(\xi)=1-\xi$.

In Redfern's estimates for British industry,[1] plant and machinery is divided into five classes with lengths of life of 45, 30, 22, 17 and 14 years. He assumes that gross investment in recent years has been distributed between these classes in the proportions of 15, 39, 40, 2, and 4 per cent respectively. This gives an average expected life of 28–29 years.

The empirical data described above give also an average life of 28–29 years for plant. The main difference between the assumed and the empirically-observed lives is therefore not in the average but in the distribution around the average ; while it is assumed that plant dies at specific ages (notably 22, 30, and 45 years) in fact death is evenly spread over a range of about 60 years.

My direct estimate of $K(t)$ for British manufacturing in 1955 came to £15·3 billion and the perpetual inventory estimate of Redfern,

[1] P. Redfern, 'Net Investment in Fixed Assets in the U.K., 1938–1953', *Journal of the Royal Statistical Society*, vol. 118, 1955, pp. 141-92.

adjusted to the same year and the same coverage, to £10·1 billion only. Experiments with the empirically determined $F(x)$ suggest that about one-half of the difference, i.e. 25 per cent out of 50 per cent, is due to an under-estimate on account of length of life by the Redfern method.

The rest of the difference is largely due to errors in $y(t)$. One specific error is the omission by Redfern of most of the government-financed capital formation during the last war and this accounts for another 10 per cent of the difference. For the residue, I came to the conclusion that existing estimates of $y(t)$ are defective. It is more than likely that estimates for the earlier years are deficient and it is also possible that current estimates also tend to be on the low side.[1] In general, estimates of capital formation exclude items which by definition are included in a direct estimate of the stock of assets. The degree of under-estimation in $y(t)$ can be of the order of 15 per cent.

I have not discussed in this paper the question of price deflation. It is evident that the perpetual inventory method depends on correct price deflation just as much as a direct comparison of the volume of capital between two points of time.

The growth of capital can be measured as the increment either in the gross stock or in the net stock of assets, as follows :

$$\frac{K(t)}{dt} = y(t) - \int_0^a y(t-x) . \left\{ -\frac{dF(x)}{dx} \right\} . dx$$

$$\frac{C(t)}{dt} = y(t) - \int_{z=0}^a \int_{x=0}^z \frac{1}{z} . y(t-x) \left\{ -\frac{dF(z)}{dz} \right\} . \left\{ -\frac{G\left(\frac{x}{z}\right)}{d\left(\frac{x}{z}\right)} \right\} . dx . dz.$$

In the first formula, ***replacement*** is deducted from gross capital formation, and in the second formula *capital consumption* (*depreciation*). It can be shown that both replacement and depreciation are weighted averages of gross capital formation in past years. The distribution of weights, however, is different : in calculating depreciation, heavier weights are attached to more recent years and hence with a rising trend in capital formation depreciation is larger than replacement.

Table 4 shows the growth of capital in British manufacturing during 1948–1957. The growth of gross capital is calculated from

[1] It has frequently been said that British estimates of capital formation are understated in relation to estimates for most European countries.

my own estimates of $K(t)$ and $F(x)$. The growth of net capital is estimated by using Redfern's assumption for the lengths of life of assets.[1] Hence there are both conceptual and statistical differences making the second set of percentages bigger than the first.

TABLE 4

GROWTH OF FIXED ASSETS IN BRITISH MANUFACTURING

Years	Gross %	Net %
1948	2·0	4·8
1949	2·3	5·4
1950	2·8	6·2
1951	3·1	6·3
1952	2·8	5·3
1953	2·6	4·4
1954	2·6	4·1
1955	3·3	5·7
1956	3·7	6·4
1957	3·7	6·2

[1] But I corrected annual estimates of capital formation for government-financed investment during the war.

Chapter 6

THE CAPITAL-OUTPUT RATIO IN THE UNITED STATES: ITS VARIATION AND STABILITY[1]

BY

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I. CAPITAL COEFFICIENTS

If only a decade ago the demand for capital coefficients greatly exceeded the supply, recent empirical findings have rectified, if not reversed, the imbalance. Produced by Leontief and his associates in the Harvard Economic Research Project's dynamic input-output model on the one side, and by Kuznets and his associates in the National Bureau of Economic Research study of capital formation on the other, American capital coefficients are now reaching the market in such variety and numbers as to make the exercise of their consumers' choice a fascinating, if not an easy, occupation. It seems that the days of the good old general capital coefficient which could be so conveniently divided into the propensity to save to yield the warranted, required, equilibrium, or some other rate of growth are about over. A present-day American fisherman who is happy to have a box of modern flies each for a particular type of water, fish, season and even hour, may yet be nostalgic for the old-fashioned earthworm good for almost any kind of fish at any time. Similarly, the user of these coefficients must be grateful for the vast and useful amount of research done, and at the same time not quite certain how to pick the right coefficient for the right problem. Fortunately, in some instances the trends are so pronounced that the choice does not matter ; but in others it does. Capital coefficients can be defined,

[1] I am very grateful to the National Bureau of Economic Research for allowing me to use their materials, both published and mimeographed ; to Miss Elizabeth Jenks of the Bureau and to Dr. Daniel Creamer (now with the National Industrial Conference Board), for numerous corrections, revisions, and suggestions ; and to Mr. Tsvi Ophir, a graduate student at the Massachusetts Institute of Technology, for his excellent assistance in revising an earlier draft of the paper. None of them are responsible, of course, for any errors or for the conclusions.

aggregated, and disaggregated in so many ways that the fate of a hypothesis may sometimes depend on the particular coefficients used, and what is proved by one set may yet be disproved by another.

A capital coefficient being the ratio between the capital stock and the output produced by it, each in a total or incremental sense, its meaning and significance depend on the nature of its numerator and denominator. The debates about the 'correct' measurement of capital and output have a long history and several papers at this Conference are devoted to it. Let me steer away from this subject and mention just a few questions which the capital coefficients presented in this paper specifically raise.

In defining capital and output I would place the emphasis on the expression 'produced by it', in the sense that the stock of capital should include all capital needed to produce a given output, while the latter should contain all output produced by a given stock of capital. Some applications of this requirement are easy. If residential buildings are included in the stock of capital, all rents, whether paid or imputed, should be included in output. If services derived from consumer durables are not imputed in output, consumer durables should not be included in the stock of capital. Similarly, if military expenditures are included in output, military assets have their place in the stock of capital, however uneasy Professor Kuznets is on this score. But other applications of this rule are not as easy. The inclusion of a public highway in the stock of capital seems perfectly proper. We do not, however, impute its services to consumers as a part of output (though maintenance expenses may partially compensate for this omission). The same holds true for a good deal of publicly-owned capital and its services. As the importance of government is increasing, these distortions can significantly affect the magnitude and behaviour of the capital coefficients.

Let us next examine the composition of capital. Existing estimates of capital include physical capital, that is, buildings, other structures, machinery, equipment, sometimes inventories in the hands of business units ; agricultural land is usually but not always excluded.[1] Non-physical capital, that is the stock of knowledge, is excluded not by design but because of lack of data, an omission to be, let us hope, rectified with time. The exclusion of land can be justified on the 'gift of nature' grounds, though it is hard to tell what part of the existing stock of agricultural land, particularly in an advanced country, is of that origin. It is probably an ever-diminish-

[1] Alvin S. Tostlebe includes agricultural land in his definition of 'Physical Capital', but excludes it from 'Reproducible Capital'. *Capital in Agriculture: Its Formation and Financing since 1870* (Princeton, 1957). He also includes currency and demand deposits of farmers in his 'Total Capital'.

ing fraction. But the exclusion of inventories, to which Professor Kuznets inclines in his forthcoming fundamental work, is harder for me to understand, unless one is specifically interested in so-called durable capital.[1] In many industries inventories form a small fraction of total capital, and their exclusion is of little significance, but in agriculture their relative magnitude is large. One can, of course, compute the ratio of any particular part of capital to output and find the ratio useful for specific problems, as, for instance, in a study of the inputs needed for a given output. In a general historical problem of the type Kuznets deals with, the total stock of capital, including inventories and land improvements (if not the whole value of land), seems to me to be more appropriate.[2]

This leads us to the disaggregation problem from which one cannot escape even on the country-wide level if international investments and trade are involved. Either the stock of capital should include that invested abroad (and output foreign receipts as well) or both should be excluded. This is not difficult. But suppose coal is imported to make steel. Since this coal is not produced at home and no corresponding domestic stock of capital exists, the coal should be excluded from the estimate of output. But it *was* used to make steel. Is there not something peculiar about the resulting capital coefficient ?

In the United States, where exports and imports are relatively small, this is not much of a problem, but in a smaller country it is. And it is quite a problem in any industrial disaggregation. More precisely, should the denominator consist of the output of a given industry or of its output net of inputs from other industries ? The latter alternative seems more logical, and yet the capital stock of a given industry is used to produce its output, not just its value added. The movements of capital coefficients in American manufacturing are reasonably independent of the method chosen, but in agriculture most of the decline in the capital coefficient (see Table 11 below) has been caused by increasing inputs from outside. The ratio of the capital stock to net value added has declined only slightly. Perhaps the ideal solution is to take the gross output of an industry and divide it into the total stock of capital, including capital in all industries supplying it, however difficult a statistical job this is.[3]

[1] Simon Kuznets, *Capital in the American Economy: Its Formation and Financing*. National Bureau of Economic Research (mimeographed).

[2] It is only fair to add that he frequently presents both kinds of estimates.

[3] This is being done by Leontief and his associates in their dynamic input-output study. See Wassily Leontief, 'Factor Proportions and the Structure of American Trade: Further Theoretical and Empirical Analysis', *The Review of Economics and Statistics*, 1956, pp. 386-407.

The depreciation-replacement problem has received so much attention lately that little need be said about it here. On the whole, one is reluctant to define both capital and output in gross terms because a part of output is used to restore the existing stock of capital, and old capital is not as good as new. So something should be deducted from the gross stock of capital in estimating the average coefficient, or from gross investment in computing the marginal one. But should it be depreciation, and if so, in accordance with what method; or replacement, or more correctly, retirement; or some compromise between the two? Professor Kuznets presents several estimates of capital coefficients (see below, Table 5): gross capital stock to gross national product, gross capital stock net of capital retirements to net national product, and net (of depreciation) capital stock to net national product. On the whole, he is most inclined to take the last one and least inclined to the first; most of the industrial capital coefficients presented by the National Bureau group are in net terms. On the other hand, our other great authority on the subject, Professor Leontief, takes the opposite point of view and declares that:

> 'Recent information indicates that the undepreciated coefficients correspond much more closely to the incremental coefficients than do the depreciated ones. Use of the depreciated coefficients implies that capital stocks decrease in efficiency in exact relation to the depreciation charge. Most available evidence indicates that this is not a reliable assumption. Use of the undepreciated coefficients implies that capital stocks have a constant efficiency from the time of purchase until the time they are fully written off, when their usefulness is assumed to be zero. Both methods are dependent upon accounting procedures, both fail to take account of technological change, and both present an index number problem for the reduction of stock of capital accumulated over time.' [1]

It would be broadminded to say that both or all estimates of capital coefficients are correct, depending upon the problem in hand; but on this occasion at least, it is an empty phrase. It does make a difference. Gross capital formation as a fraction of gross national product in the United States has been reasonably constant over the last eighty years or so, starting at 22·9 per cent in 1869–1878, climbing to a peak of 25·5 per cent in 1889–1898, and ending at 19·1 per cent in 1949–1955 (all in 1929 prices). But for net capital formation as a fraction of net national product the corresponding figures are 15·0,

[1] Harvard Economic Research Project, *Estimates of the Capital Stock of American Industries, 1947* (Cambridge, Mass., 1953), pp. 21-2.

15·5 and 5·3 per cent respectively.[1] If it is net capital formation (among other things) that growth depends on, our prospects may be rather bleak. One would prefer some net figures to gross, but working with net investment and net stock of capital in the conventional sense one loses sight of gross investment as a major vehicle of technological progress. It makes a good deal of difference whether the same stock of capital lasts indefinitely or whether now one part of it and now another is replaced by new capital of superior efficiency. Hence gross figures may be more meaningful, with some unknown deduction of a smaller magnitude than conventional depreciation to account for the deterioration of existing capital.

National Bureau publications usually present capital coefficients (and also estimates of capital and of output) both in current and in constant prices, with an emphasis on the latter. This seems to be quite proper, unless one is interested in immediate policy questions where a marginal capital coefficient in current prices may be more significant. Let me not enter the endless argument about deflation methods, and ask instead a question from a related field; in evaluating a country's effort directed to capital formation, should capital and output be expressed in current or constant prices? If, as has been the case in the Soviet Union, technological progress is more rapid in capital goods industries, even 'correct' current prices will, in a sense, understate capital formation as a fraction of output, because most expenditures on technological progress are not treated as a part of capital formation. Constant prices would undoubtedly show a larger, and probably an increasing capital formation as a fraction of total output, but are constant prices, presumably based on technological relations of some base year, relevant for the present?[2]

Finally, a word about average and marginal coefficients. For many purposes, particularly in the field of policy, the marginal coefficient is certainly more significant. But it is harder to come by and, unless the under-utilization of capital is carefully accounted for, the marginal coefficient is highly unstable and makes little sense.

II. CAPITAL COEFFICIENTS IN THE U.S.A.

My assignment, however, calls not for complaining about the meaning of capital coefficients but for a report on their variation and

[1] Kuznets, *op. cit.*, Table III-7, p. III-52. The increasing difference between the gross and net fractions is caused by the retardation of the rate of growth of investment and the decline in its estimated longevity.

[2] On that see my *Essays in the Theory of Economic Growth* (New York, 1957), pp. 236-40.

stability in the United States. I am hardly the best man for this job because Professor Kuznets and his associates at the National Bureau have been doing exactly this, and could report on the results of their work much better than I. Most of Kuznets' summary volume and several other monographs were made available to me in mimeographed form. They contain all that I could possibly say about capital coefficients and much more. What is presented in this section is several tables taken from these reports, with a few minor comments of my own.

Table 5 shows the behaviour of the average capital coefficients in the United States for the period 1869–1955 and defined in several ways. None of these estimates has been corrected for the frequent under-utilization of capital, particularly important during the 1930s. Hence, capital coefficients for the period 1929–1938 are overstated. Because of this under-utilization, the averages given for a longer period (the lower half of the table) may be more significant.

Table 5 and Table 6, giving the behaviour of the marginal coefficients convey two basic impressions: (1) the coefficients were rising from the beginning of the period until the 1920s, and falling thereafter; (2) on the whole, and with the exception of the 1930s, the average coefficients were quite stable. The same general picture is conveyed by capital coefficients for the private non-agricultural sector of the American economy for the period 1909–1949, implicit in a study by Robert M. Solow.[1]

Leaving the actual behaviour of the coefficients for a later discussion (Part III), let me comment on their stability. By its very nature, the average capital coefficient is a sluggish animal (except for periods of serious capital under-utilization) because the stock of capital does not change rapidly. Suppose we start with a net capital stock of 300, an annual output of 100 and thus with a capital coefficient of 3, while the fraction of net output invested is 10 per cent. A rate of growth of output of 3·3 per cent per year will leave the capital coefficient unchanged. A rate of growth of output of 2 per cent will raise it to 3·9 at the end of 30 years; a 1 per cent rate of growth will increase it to 4·8, while complete absence of growth will double the coefficient after 30 years. On the other hand, a 4 per cent rate of growth will reduce it to 2·7. In general, the capital coefficient will tend to approach the ratio between the fraction of output invested and the rate of growth of output. Both the rate of growth of output in the United States and the fraction invested have been declining. Hence their ratio has been more stable than either

[1] 'Technical Change and the Aggregate Production Function', *The Review of Economics and Statistics*, 1957, pp. 312-20.

TABLE 5

RATIO OF CAPITAL STOCK TO ANNUAL NATIONAL PRODUCT, UNITED STATES, 1869–1955
BASED ON TOTALS IN 1929 PRICES

	Date of Capital Stock	Period of National Product Flow	Ratio, Gross Capital Stock to Gross National Product		Ratio, Gross Capital Stock Net of Retirements to Net National Product		Ratio, Net Capital Stock to Net National Product	
			Kuznets Concept (Variant III) (1)	Department of Commerce Concept (2)	Kuznets Concept (Variant III) (3)	Department of Commerce Concept (4)	Kuznets Concept (Variant III) (5)	Department of Commerce Concept (6)
	Total Stock and Product (including Military)							
1	1869 and 1879	1869–1878	5·2	5·3	4·6	4·6	3·5	3·5
2	1879 and 1889	1879–1888	4·5	4·5	3·9	3·8	2·9	2·9
3	1889 and 1899	1889–1898	5·3	5·2	4·6	4·5	3·5	3·4
4	1899 and 1909	1899–1908	5·4	5·3	4·6	4·5	3·4	3·4
5	1909 and 1919	1909–1918	6·0	5·9	5·2	5·0	3·7	3·6
6	1919 and 1929	1919–1928	6·2	6·0	5·2	5·0	3·6	3·5
7	1929 and 1939	1929–1938	7·6	7·3	6·0	5·7	4·1	3·9
8	1939 and 1949	1939–1948	6·5	5·4	4·6	3·8	3·1	2·5
9	1946 and 1955	1946–1955	6·0	5·4	4·0	3·6	2·8	2·5
	Arithmetic Means of Ratios							
10	Lines 1 and 2	(1869–1888)	4·8	4·9	4·2	4·2	3·2	3·2
11	Lines 3 and 4	(1889–1908)	5·4	5·2	4·6	4·5	3·4	3·4
12	Lines 5 and 6	(1909–1928)	6·1	6·0	5·2	5·0	3·6	3·6
13	Lines 7, 8, 9 (weighted)	(1929–1955)	6·8	6·1	5·0	4·5	3·4	3·0
14	Lines 8 and 9 (weighted)	(1939–1955)	6·3	5·4	4·4	3·7	2·9	2·5

Source: Simon Kuznets, *Capital in the American Economy: Its Formation and Financing*, National Bureau of Economic Research (mimeographed), Table III-5, pp. III-35-36. The figures for 1946–1955 were kindly supplied by Daniel Creamer

TABLE 6

RATIO OF CHANGES IN CAPITAL STOCK (CAPITAL FORMATION) TO CHANGES IN ANNUAL NATIONAL PRODUCT, UNITED STATES, 1869–1955, BASED ON TOTALS IN 1929 PRICES

	Intervals over which changes are compared (dates are for end of year, unless otherwise noted)	Ratio, Gross Capital Stock to Gross National Product		Ratio, Gross Capital Stock Net of Capital Retirements to Net National Product		Ratio, Net Capital Stock to Net National Product	
		Kuznets Concept (Variant III) (1)	Department of Commerce Concept (2)	Kuznets Concept (Variant III) (3)	Department of Commerce Concept (4)	Kuznets Concept (Variant III) (5)	Department of Commerce Concept (6)
	Total (including Military)						
1	1873–1883	3·6	3·6	3·0	3·0	2·3	2·3
2	1883–1893	7·5	7·2	6·7	6·4	5·1	4·9
3	1893–1903	5·6	5·4	4·6	4·5	3·4	3·2
4	1903–1913	7·7	7·4	6·8	6·4	4·6	4·3
5	1913–1923	6·7	6·3	5·3	5·0	3·4	3·2
6	1923–1933	30·2	25·6	20·1	16·2	11·8	9·6
7	1933–1943	4·1	2·8	1·8	1·1	1·1	0·7
8	1943–1952 (mid-year)	4·8	5·3	2·1	2·4	1·5	1·8
	Longer Intervals						
9	1873–1893	5·4	5·2	4·6	4·5	3·5	3·4
10	1893–1913	6·7	6·4	5·7	5·4	3·9	3·8
11	1903–1923	7·1	6·7	5·9	5·5	3·9	3·7
12	1923–1952 (mid-year)	5·8	4·9	2·9	2·4	1·9	1·5

The interval for capital stock is from the mid-year dates shown in Table 5. The decadal flow figures for national product are taken as centered at the mid-point of the decade. The dates for capital stock and for product flow are the same, and hence the resulting intervals coincide.

Source: Simon Kuznets, *Capital in the American Economy: Its Formation and Financing*, National Bureau of Economic Research (mimeographed), Table III-6, pp. III-41-42.

variable. This does not necessarily presuppose any specific causal relationship between these three variables. It is possible that the capital coefficient and the fraction of output invested have yielded a certain rate of growth of output. But it is also possible that other factors besides capital have been mainly responsible for the existing rate of growth of output, and the given capital coefficient has simply resulted from the interaction of the other variables. Thus the relative stability of the capital coefficient is not a sufficient indication of the rôle of capital formation in economic growth.

III. EXPERIENCE IN PARTICULAR INDUSTRIES

Before any hypotheses regarding the specific behaviour of the over-all capital coefficients are suggested, let us examine the behaviour of capital coefficients in particular industries. We have two studies by Leontief and his group : a 68-industry classification for 1939 and a 192 classification for 1947 — certainly a wealth of data.[1] Both reveal a remarkable variability among industrial capital coefficients : in 1939 the highest, in Home Renting, was 7·1, almost exactly one hundred times as large as the smallest — 0·07, in Clothing. In 1947 the highest coefficient was again in Real Estate and Rentals — 8·02 — while the smallest was found in Banking, Finance and Insurance — 0·03. Other industries with high coefficients in 1939 were Petroleum and Natural Gas, Communications, Steam Railroads ; in 1947, Telephone and Telegraph, Non-profit Institutions, etc.[2] These are gross (of depreciation) coefficients, but a comparison of net ones would hardly change the general pattern of diversity. Since the usefulness of any average depends on the dispersion among the components, one cannot avoid concluding that the over-all capital coefficient is not too dependable a tool, however much this conclusion will upset simple growth models constructed by others and myself.

For the examination of the behaviour of industrial capital coefficients over longer periods we turn back to the work of the National

[1] Wassily Leontief, *Studies in the Structure of the American Economy* (New York, 1953), pp. 220-1 ; 'Factor Proportions and the Structure of American Trade : Further Theoretical and Empirical Analysis', *The Review of Economics and Statistics*, 1956, pp. 386-407.

[2] Because of different classifications, the 1939 and 1947 capital coefficients are not directly comparable. The several examples given in the text are merely to illustrate the great dispersion of capital coefficients between industries. The ranking of industries by the sizes of their capital coefficients depends, of course, on the method of classification used.

These coefficients, as well as those obtained from the National Bureau Studies, are *direct*, in the sense that capital of other industries supplying a given one is not taken into account.

Bureau. It provides us with data on coefficients in Manufacturing, Mining, Agriculture, Housing and Regulated Industries (Public Utilities), all in net terms.

Manufacturing

Table 7, taken from a monograph by Daniel Creamer, shows the behaviour of capital coefficients in manufacturing since 1880.[1]

TABLE 7

RATIOS OF CAPITAL TO OUTPUT IN REPORTED, 1929, AND CURRENT VALUES AND OF CAPITAL TO VALUE ADDED (1929 PRICES) ALL MANUFACTURING, SELECTED YEARS, 1880–1953

Benchmark Years	Ratios of			
	Capital (Book value) to Output (in current prices) (1)	Capital (in 1929 prices) to Output (in 1929 prices) (2)	Capital (in current prices) to Output (in current prices) (3)	Capital (in 1929 prices) to Value Added (in 1929 prices) (4)
1880	0·528	0·547	0·489	1·506
1890	0·679	0·730	0·670	1·651
1900 Comparable with preceding years	0·748	0·803	0·795	1·878
1900 Comparable with following years	0·743	0·794	0·790	1·882
1904	0·815	0·891		2·093
1909	0·851	0·967	0·900	2·309
1914	0·894	1·008		2·460
1919	0·688	1·022	0·873	2·555
1929	0·829	0·885	0·867	2·020
1937	0·744	0·741	0·787	1·809
1948	0·532	0·609	0·621	1·550
1953	0·546	0·622	n.a.	n.a.

n.a. = not available.

Source: Daniel Creamer, *Capital and Output Trends in Manufacturing Industries*, 1880–1948, National Bureau of Economic Research, Occasional Paper 41 (New York, 1954), p. 43; revised and extended in *Capital in Manufacturing and Mining: Its Formation and Financing*, by Daniel Creamer, Sergei Dobrovolsky and srael Borenstein, NBER (mimeographed Feb. 1958), Table II (revised), p. III-4.

[1] Daniel Creamer, *Capital and Output Trends in Manufacturing Industries*, 1880–1948, National Bureau of Economic Research, Occasional Paper 41 (New York, 1954), p. 43. As indicated on the several tables below, the figures taken from the occasional paper were revised by Creamer and his associates.

Several variants are presented: book value of capital and output in current prices, both in 1929 prices, both in current prices, and finally capital and value added, both in 1929 prices. Almost irrespective of the method chosen, the same general picture emerges: capital coefficients were rising from the beginning of the period concerned until about 1919 and have been declining ever since. Creamer carefully considers a number of circumstances which might have spuriously produced this pattern, such as changes in accounting methods (capital as compared with current expenses, depreciation practices), errors in reporting, mergers, and treatment of intangible

TABLE 8

RATIOS OF FIXED AND WORKING CAPITAL TO OUTPUT, ALL MANUFACTURING, SELECTED YEARS, 1890–1951
(Based on 1929 Prices)

Benchmark Years	Ratio of	
	Working Capital to Output	Fixed Capital to Output
1890	0·366	0·364
1900	0·387	0·416
1904	0·420	0·471
1929	0·452	0·433
1937	0·395	0·346
1948	0·324	0·285
1953	0·333	0·289

Source: Daniel Creamer, *Capital and Output Trends in Manufacturing Industries*, 1880–1948, National Bureau of Economic Research, Occasional Paper 41 (New York, 1954), p. 49, revised and extended in *Capital in Manufacturing and Mining: Its Formation and Financing*, by Daniel Creamer, Sergei Dobrovolsky and Israel Borenstein, NBER (mimeographed Feb. 1958), Table 20, p. IV-3.

assets, and concludes that they could not have affected the general nature of his findings. The fall in the capital coefficients between 1929 and 1948 is confirmed by gross (of depreciation) estimates as well. The gross coefficients were 1·199, 0·998, and 0·856 in 1929, 1937, and 1948 respectively.[1] It is most interesting that capital coefficients, however defined, fell between 1929 and 1937 in spite of the greater degree of capital under-utilization in the latter year.

The division of total capital into working and fixed, with the corresponding capital coefficients, is given in Table 8. The working capital-output ratio reaches a peak in 1929; the fixed ratio reaches

[1] *Ibid.* p. 48.

its peak in 1904 but the general pattern of an early rise and a subsequent fall is confirmed once more.

The same conclusion emerges from a subdivision of manufacturing into thirty-nine minor industries, with the peaks in most capital and consumer goods industries reached by the respective capital coefficients in the first quarter of this century.[1] Thus, the changing industrial composition of manufacturing was not responsible for this pattern : only some 17 per cent of the rise in the capital coefficient for all manufacturing between 1880 and 1919 can be attributed to this cause.[2] But it is worth pointing out that the dispersion of capital coefficients among the minor subdivisions of manufacturing has been declining since 1900. By 1919, the coefficient of variation had fallen by nearly two-fifths as compared with 1880, and the decline continued after 1919 as well.[3]

It may be best to conclude this section with a statement of Creamer's general hypothesis regarding the behaviour of capital coefficients in manufacturing :

> 'On this evidence we can say that manufacturing has developed along the following course : In the earlier decades an increasing fraction of a dollar of capital was used to produce a dollar of output ; in more recent decades a decreasing fraction of a dollar of capital has been sufficient to produce a dollar of output. This is consistent with the interpretation that in the earlier decades capital innovations on balance probably served more to replace other factor inputs than to increase output. More recently the balance has been in the other direction — capital innovations serve more to increase the efficiency of capital, hence to increase output, than to replace other factor inputs.' [4]

Mining

Tables 9 and 10, taken from a monograph by Israel Borenstein, present the behaviour of capital coefficients defined in several ways since 1870 in mining as a whole and in its principal subdivisions. Their general pattern, with a few exceptions, is similar to that in manufacturing : a rise up to 1919 or 1929, followed by a fall, and

[1] Daniel Creamer, *Capital and Output Trends in Manufacturing Industries*, 1880–1948, National Bureau of Economic Research, Occasional Paper 41 (New York, 1954), p. 54.

[2] *Ibid.* p. 57.

[3] *Ibid.* pp. 68-9. The decline in the capital coefficients since 1929 (or 1919) is particularly striking when we note that according to Creamer in most subdivisions of manufacturing (except for food and kindred products, beverages, and tobacco products), or more exactly in 19 out of 22 industry groups, the capital coefficient seems to be directly related to the size of the firm (*ibid.* p. 62), and the size of firms has been increasing. Yet the data presented by Creamer are somewhat sketchy and require further study.

[4] *Ibid.* p. 44.

TABLE 9

CAPITAL-PRODUCT RATIOS IN MINING, BASED ON REPORTED VALUES AND VALUES IN 1929 PRICES, SELECTED YEARS, 1870–1953

Total Mining	1870	1880	1890	1909	1919	1929	1940	1948	1953
Values in 1929 prices, ratio of :									
1. Capital to product	0·72	1·16	1·36	1·80	2·30	2·14	1·59	1·34	1·26
2. Plant to product	0·61	1·02	1·19	1·52	2·00	1·57	1·10	0·92	0·84
3. Working capital to product	0·11	0·14	0·17	0·28	0·30	0·57	0·49	0·42	0·42
Reported values, ratio of :									
4. Total capital to product	1·39	2·21	2·55	2·77	2·23	2·88	2·18	1·08	1·16
5. Land to product	0·83	1·34	1·46	1·22	0·84	0·89	0·50	0·12	0·10
6. Capital to product	0·56	0·87	1·09	1·55	1·39	1·99	1·68	0·96	1·06
7. Plant to product	0·43	0·74	0·92	1·22	1·05	1·41	1·20	0·59	0·69
8. Working capital to product	0·13	0·13	0·17	0·33	0·34	0·58	0·48	0·37	0·37

Meaning of Concepts :

Plant = depreciated net value of structure and equipment.
Working capital = inventories + cash + receivables.
Land = net value of surface land and mineral resources owned by the mining establishment, excluding leased land.
Capital = plant + working capital.
Total capital = capital + land.
(Borenstein, *op. cit.* p. 16.)

Source : Israel Borenstein, *Capital and Output Trends in Mining Industries, 1870–1948*, National Bureau of Economic Research, Occasional Paper 45 (New York, 1954), p. 34 ; revised and extended in *Capital in Manufacturing and Mining: Its Formation and Financing*, by Daniel Creamer, Sergei Dobrovolsky and Israel Borenstein, NBER (mimeographed Feb. 1958), Table 23, p. IV-12.

TABLE 10

CAPITAL-PRODUCT RATIOS, BY MAJOR MINING INDUSTRIES, SELECTED YEARS, 1870–1953

	1870	1880	1890	1909	1919	1929	1940	1948	1953
(a) *Based on values in* 1929 *prices, excluding land*									
Total mining	0·72	1·16	1·36	1·80	2·30	2·14	1·59	1·34	1·26
Metals, total	1·29	2·30	2·73	2·50	2·16	1·71	1·24	1·00	1·39
Iron	0·71	1·22	1·53	2·21	1·95	n.a.	n.a.	n.a.	n.a.
Copper	0·76	1·74	0·82	1·47	2·46	n.a.	n.a.	n.a.	n.a.
Lead and zinc	0·77	0·74	0·75	1·49	1·27	n.a.	n.a.	n.a.	n.a.
Precious metals	2·55	3·15	5·41	5·44	3·35	n.a.	n.a.	n.a.	n.a.
Anthracite	0·41	0·63	0·50	0·51	0·53	0·83	0·55	0·43	0·64
Bituminous Coal	0·91	0·70	0·69	1·06	1·25	1·06	0·88	0·88	1·17
Petroleum and natural gas	1·75	2·06	3·78	5·05	5·86	3·58	2·25	1·79	1·41
Other non-metals	n.c.	1·04	1·16	1·28	1·19	1·17	0·94	0·56	0·61
(b) *Based on reported values*									
Total mining	0·56	0·87	1·09	1·55	1·39	1·99	1·68	0·96	1·06
Metals, total	0·49	0·80	1·23	1·71	1·67	1·59	1·11	0·90	1·06
Iron	0·37	0·58	0·94	1·53	1·30	n.a.	n.a.	n.a.	n.a.
Copper	0·35	0·74	0·67	1·16	2·26	n.a.	n.a.	n.a.	n.a.
Lead and zinc	0·90	0·75	0·78	1·11	1·15	n.a.	n.a.	n.a.	n.a.
Precious metals	0·54	0·87	1·49	2·86	1·92	n.a.	n.a.	n.a.	n.a.
Anthracite	0·63	1·17	0·80	0·75	0·51	0·77	0·63	0·38	0·57
Bituminous coal	0·66	0·57	0·62	0·93	0·71	0·99	0·73	0·45	0·71
Petroleum and natural gas	0·46	1·55	2·49	3·65	2·51	3·31	2·74	1·33	1·25
Other non-metals	0·45	0·58	0·70	0·99	0·93	1·10	1·02	0·63	0·70

n.a. = not available.
n.c. = not comparable.
'Capital' as defined in Table 5 is the numerator.

Source: Israel Borenstein, *Capital and Output Trends in Mining Industries, 1870–1948*, National Bureau of Economic Research, Occasional Paper 45 (New York, 1954), pp. 34-7; revised and extended in *Capital in Manufacturing and Mining: Its Formation and Financing*, by Daniel Creamer, Sergei Dobrovolsky and Israel Borenstein, NBER (mimeographed Feb. 1958), Tables 23, 24, pp. IV-12-15.

sometimes a slight increase after 1948. This is true of mining taken as a whole and of most of its subdivisions, both in current and in 1929 prices. Inter-industrial shifts in mining have only tended to increase the capital coefficient, and thus accounted for a part of the increase between 1870 and 1919. After that, the capital coefficient in mining as a whole fell in spite of inter-industrial shifts. 'If the 1919 ratios for each industry had prevailed in 1948, almost 3 times as much capital would have been used in mining as actually was used in that year. . . . Even if the relatively low industry ratios of 1890 had been maintained through 1948, the volume of capital in that year would have been twice as high as it really was.'[1]

Borenstein explains the particular behaviour of capital coefficients in terms of the rates of growth of mining and of its subdivisions:

> 'The average annual percentage rate of growth in output of aggregate mining between 1870 and 1919 was 5·4 per cent, and was accompanied by a 7·9 per cent annual increase in capital, but between 1919 and 1948 the annual percentage increase in output was only 2·9 and was accompanied by a 1·1 per cent growth in capital. While a roughly similar pattern is found for each mining industry, the relationship between the rate of an industry's growth and the movement of the capital-product ratios appears more complex.'[2]

More exactly, Borenstein finds that from 1880 to 1919 the increases in capital-product ratios in the several mining subdivisions were positively correlated with their respective rates of growth, while after 1919 the respective correlations were inverse. He suggests that:

> 'Exceedingly high and exceedingly low rates of growth seem to have been associated with less efficient use of the input factors, while more moderate rates of growth have been associated with higher efficiency in the use of the input factors in a technical sense. This suggested relationship appears reasonable only if the border line between excessively high or low rates of growth and optimal rates is considered flexible and is assumed to vary with the given stage of economic and technological development.'[3]

Agriculture

The basic data computed by Tostlebe is presented in Table 11. In Tostlebe's original table, land was included as a part of 'Physical

[1] Israel Borenstein, *Capital and Output Trends in Mining Industries, 1870–1948*, National Bureau of Economic Research, Occasional Paper 45 (New York, 1954), p. 52. [2] *Ibid.* p. 8. [3] *Ibid.* pp. 56-8.

TABLE 11

RELATION OF REPRODUCIBLE FARM CAPITAL TO FARM INCOME IN 1910–1914 PRICES, UNITED STATES, CENSUS YEARS, 1870–1950

(dollars in millions)

Year	Reproducible Farm Capital	Five-Year Average Gross Farm Income		Ratio of Reproducible Farm Capital to Average Gross Farm Income		Five-Year Average Net Farm Income	Ratio of Reproducible Farm Capital to Average Net Farm Income
		Strauss-Bean	BAE	Strauss-Bean	BAE		
	(1)	(2)		(3)		(4)	(5)
	$	$	$			$	
1870	6,131	2392		2·56			
1880	8,704	3542		2·46			
1890	10,770	4400		2·45			
1900	12,287	5748		2·14			
1910	15,152	6313	6,708	2·40	2·26	5330	2·84
1920	17,911	6973	7,471	2·57	2·40	6020	2·98
1925	16,852		8,021		2·10	6310	2·67
1930	16,703		8,660		1·93	6735	2·48
1935	14,088		8,015		1·76	6036	2·33
1940	15,362		9,623		1·60	7267	2·11
1945	17,665		11,348		1·56	7733	2·28
1950	19,172		11,778		1·63	7606	2·52

Source: Alvin S. Tostlebe, *Capital in Agriculture: Its Formation and Financing since 1870* (Princeton, 1957), pp. 66, 101, 113.

Capital'. Following economic tradition, I excluded land from the numerator of the capital-output ratio, though, to be frank, I am not at all sure which procedure gives a more meaningful result. Between 1870 and about 1920 the capital coefficient given in column 3 was more or less constant; after 1920 it was clearly declining.[1] Much of this decline was caused by increasing inputs taken by agriculture from other industries: the ratio of capital to net farm income declines less rapidly, as shown in column 5 of the table. But even this ratio does decline, presumably due to technological progress which has been quite impressive.

Table 11 refers to American agriculture as a whole. Regional differences in capital coefficients have been large, ranging in 1950 from 2·7 (the numerator including land) in the Delta states and 2·8 in the South-east and the North-east to 5·5 in the Cornbelt and the Mountain states and 7·7 in the Great Plains. Similar differences have persisted throughout the whole period. (There has been a slight and frequently interrupted convergence of regional coefficients.) Yet, in all regions we find roughly the same trend as in agriculture as a whole: the coefficient declines in all regions between 1920 and 1950, and in 8 out of 10, between 1870 and 1920.[2]

Thus, the changes in the relative importance of the several regions have accounted for a rather small part of the decline of the total coefficient. It is curious that until 1920 inter-regional changes retarded the fall in the capital coefficient; after that date they have contributed somewhat to its decline.

Two reasons for the observed behaviour of the coefficient are given by Tostlebe. First, there was an increased use of intermediate products on the farm. This fact exaggerates the decline of the ratio of capital to gross income, and indeed the ratio to net income declined much more moderately, as shown in Table 11. Secondly, after 1920, there were many and far-reaching technological advances, including improvement in varieties, fertilizers, insecticides, and farming methods. It is interesting to note that this argument about increased efficiency of production methods since 1920 closely parallels that of Creamer with respect to industry.

Housing

As shown in Table 12, taken from a study by Grebler, Blank and Winnick, at present this industry has the highest capital coefficient of any major industrial division. In the decade 1939–1948, the

[1] If land is included, the capital coefficient falls slowly between 1870 and 1920 and rapidly thereafter.

[2] Tostlebe, *op. cit.* Table 22, pp. 108-9.

TABLE 12

CAPITAL-OUTPUT RATIOS, NON-FARM RESIDENTIAL REAL ESTATE, SELECTED PERIODS, 1889–1948

(dollars in millions)

Decade	Current Dollars			1929 Dollars		
	Average Annual Value of Residential Real Estate (including Land) (1)	Average Annual Residential Rent (2)	Capital-Output Ratio (3)	Average Annual Value of Residential Capital (excluding Land) (4)	Average Annual Residential Rent (5)	Capital-Output Ratio (6)
	$	$		$	$	
1889–1898	16,917	1,130	15·0	28,594	2,043	14·0
1899–1908	27,147	2,160	12·6	39,845	3,680	10·8
1909–1919	46,134	3,661	12·6	52,444	5,451	9·6
1921–1929	90,285	{7,873 {8,768	{11·5 {10·3	68,559	{7,655 {8,471	{9·0 {8·1
1929–1938	91,214	8,731	10·4	78,868	11,096	7·1
1939–1948	138,249	11,823	11·7	80,175	15,157	5·3

Source: Leo Grebler, David M. Blank and Louis Winnick, *Capital Formation in Residential Real Estate, Trends and Prospects* (Princeton, 1956), p. 407.

coefficient in current prices (including land) was 11·7; in constant 1929 prices (excluding land) — 5·3. Both coefficients declined markedly over time, the former from 15·0 and the latter from 14·0 in 1889–1898. Since housing absorbs a high fraction of current investment and of the stock of capital, its high capital coefficient is of considerable significance: a country investing heavily in housing can expect a lower rate of growth of its output, at least at the start. On the other hand, houses depreciate more slowly than other forms of capital and thus require smaller replacement expenditures. Hence, they will be a smaller drain on capital later on.

Capital coefficients in housing are probably less reliable than those in other industries because so much of the output consists of imputed rent — as much as two-thirds of the total in 1950. Imputed rents are a rather arbitrary number, and are actually often based on an assumed capital-output ratio or capitalization factor.[1] Falling interest rates, however, should give a larger capitalized value of the capital stock and hence a higher capital coefficient. Instead, we are confronted with a falling one.

A possible explanation may be sought in the declining ratio of net to gross rents, which reflects the increased provision of certain services (water, electricity, refrigeration, furniture, and taxes representing municipal services) by the landlord. The cost of these services is included in gross rent (the denominator), but it leads to no corresponding increase in the estimate of capital in housing (the numerator) as it is defined here.[2]

Public Utilities (Regulated Industries)

This category includes Steam Railroads, Electric Light and Power, Telephones, Street and Electric Railways, Local Bus Lines, All Other Transportation and Utilities. I have left this industry to the very end because of the most spectacular behaviour of its capital coefficients. As shown in Table 13, adapted from a study by Melville J. Ulmer, the total capital coefficient in this industry fell from 15·29 in 1880 to 1·63 in 1950, with a few minor zigzags. This type of decline has been more or less true of all its components, with the exception of local bus lines. Thus in Steam Railroads the coefficient started in 1880 at 15·95 and, with a few interruptions, fell to 2·66 in 1950. In Electric Light and Power it started at 4·42 in 1887, rose rapidly to a peak of 18·40 in 1893, and then fell gradually

[1] Leo Grebler, David M. Blank and Louis Winnick, *Capital Forma ion in Residential Real Estate, Trends and Prospects* (Princeton, 1956), p. 408.
[2] *Ibid.* p. 406.

to 1·30 in 1950. Telephones have behaved in a somewhat similar fashion, and so have Street and Electric Railways. In Local Bus Lines the coefficient has risen almost steadily from 0·16 in 1922 to 1·66 in 1950, while the coefficient in the All Other category has behaved like that of the industry taken as a whole.[1]

TABLE 13

CAPITAL-PRODUCT RATIOS IN REGULATED INDUSTRIES, 1929 DOLLARS

(Selected Years)

Year	All Regulated Industries	Steam Rail-roads	Electric Light and Power	Tele-phones	Street and Electric Railways	Local Bus Lines	All Other Transporta-tion and Utilities
1880	15·29	15·95					
1887	11·08	11·61	4·42				
1890	9·50	9·84	12·06	4·99	3·33		12·64
1893	9·33	9·61	18·40	4·20	4·48		11·53
1900	6·57	6·43	12·48	4·12	6·85		7·14
1910	4·55	4·35	10·47	2·54	5·77		4·27
1920	3·27	3·17	4·51	1·58	4·01		3·44
1922	3·65	3·91	4·50	1·36	3·74	0·16	3·49
1930	3·53	4·23	3·64	1·88	3·39	0·71	2·93
1940	2·76	4·10	2·39	1·84	3·49	0·89	1·70
1950	1·63	2·66	1·30	1·85	2·28	1·66	0·94

Source: Melville J. Ulmer, *Capital in Transportation, Communications and Public Utilities: Its Formation and Financing* (Princeton, 1960), Tables I-1, p. 458; I-13, p. 472; I-16, p. 476; I-20, p. 482; I-22, p. 486; I-27, p. 494; I-29, p. 496.

Two suggestions may be ventured to explain this remarkable fall in capital coefficients in public utilities. The first is technological progress, which has produced more efficient capital and also allowed its more efficient utilization. This factor has been present in all industries considered here. The second and more special cause arises from the indivisibilities inherent in the very nature of these industries: they require railway lines, power lines or pipe-lines connecting all participating points, however scant the initial demand for the service in a given area may be. A rising demand can be satisfied with relatively small additional capital expenditures.

These attributes of public utilities are important for economic development which is unthinkable without them. Undeveloped

[1] Local bus lines present a special problem because capital invested in them does not include streets. This probably explains the low capital coefficient and deprives the estimate of much of its meaning.

countries do not have to repeat the American experience and start with capital coefficients of the American 1880 vintage. Yet it is very likely that at an early stage of development public utilities will require large capital investments per unit of output. This is perhaps a partial explanation of the 'hump' which must be passed before economic growth becomes assured.

IV. CONCLUSIONS

It would not be unreasonable to expect that a paper like this, having examined the behaviour of several industrial capital coefficients, would now be able to offer some explanation of the movements of the over-all coefficient for the whole economy. Unfortunately, this turns out to be quite a difficult job, and for more than one reason. First, we have the difference in definition. The over-all ratio, being the ratio of the stock of capital to net or gross product (or income), can be expressed as a weighted average of industrial coefficients defined as ratios of the respective capital stocks to the respective products originating in each industry, with the products originating serving as weights. Most of the industrial coefficients presented above are, however, ratios of respective stocks of capital to *outputs*, not to products originating. While output and income originating move together more often than not, one need not be a satisfactory substitution for the other in long-range comparisons.

Secondly, the available data are scattered over time, and the years chosen by the several investigators do not coincide with sufficient frequency to yield a clear picture. This is particularly true of our housing data as compared with the other industries. The relative unreliability of housing estimates has already been mentioned (p. 113). For both reasons, housing is taken out of the immediately following discussion.

We are left with manufacturing, mining, agriculture and public utilities, the income originating in them comprising, roughly speaking and depending on the year chosen, about one-half of net national product. Table 14, taken from Kuznets, shows the behaviour of two sets of the combined average coefficients for these four industries. In both cases the net stock of durable capital (exclusive of inventories) is in the numerator, while the denominator consists of (1) output, and (2) net product originating, all in 1929 prices. The first method produces a sharp fall in the combined coefficient from 1880 to 1900, little change from 1900 to 1922, and another and even sharper fall from 1922 to 1948. Similar results are obtained from the second method, except for a slight rise in the coefficient between 1900 and 1922. My very rough attempt to compute a combined coefficient for these four industries as a ratio of the net stock of capital, inclusive of inventories, to output, also in 1929 prices, gave similar results : a fall from 1890 to 1909 (or 1910), a plateau between 1909 and 1919, and a sharp fall between 1919 and 1947–1950.

TABLE 14

RATIOS OF NET STOCK OF DURABLE CAPITAL TO OUTPUT AND TO NET INCOME ORIGINATING FOR MANUFACTURING, MINING, AGRICULTURE AND PUBLIC UTILITIES COMBINED FOR SELECTED YEARS

	Ratio of Capital Stock to Output	Ratio of Capital Stock to Net Product
June 1, 1880	1·33	2·98
June 1, 1900	1·07 *	2·65 *
	1·09 †	2·62 †
December 31, 1922	1·06	2·73
December 31, 1948	0·58	1·56

* Comparable with entry for 1880.
† Comparable with entries for later years.

Source : Kuznets, *op. cit.* Table IV-18, pp. IV-108-110.

Thus the trend of the combined coefficient for these four industries has almost always (remembering the few years considered and the limitation of the data) been downward. It may help to explain the decline of the over-all capital coefficient for the whole economy since 1919 or so, but not the preceding rise. The explanation of the latter will have to wait until we learn more about the rest of the economy.

A theorist always itches to present a bold aggregate hypothesis which could explain the behaviour of the over-all capital coefficient in its own right, so to speak, and not merely as a weighted average of the coefficients in the several industries. Sometimes such aggregate hypotheses make sense. There is a much more systematic relation, for instance, between total investment and total product (or income) than between investment and product originating in each particular industry in a given period of time. In the present case, I can only try to develop slightly further a hypothesis already mentioned above on pp. 100-3, a hypothesis which, if true, will hardly enhance the importance of the aggregate capital coefficient in economic development.

The capital coefficient, as the ratio between the stock of capital and product, will approach as a limit the ratio between the fraction of product invested (the average propensity to save, let us say) and the product's rate of growth. If we join the company of several recent investigators (Abramovitz, Kendrick and Solow) who have found, each in his own way, that by far the largest fraction of the *per capita* rate of growth of income in the United States should be attributed to technological progress rather than to capital accumulation,[1] then the over-all capital coefficient will emerge as a relatively passive result of the interaction between the propensity to save and the rate of technological progress. The propensity to save, both in the short and in the long run, has been investigated time and again. The results have not been definitive, but a good deal of knowledge has been accumulated. About technological progress we know remarkably little, even if always eager to learn, and not only for the sake of capital coefficients. Yet much more knowledge about this remarkable process is required to deal with our hypothesis. At this stage it seems best to leave the issue completely open and to invite further work. International comparative studies, among many others, would be particularly welcome.

[1] See Moses Abramovitz, *Resource and Output Trends in the United States since 1870*, National Bureau of Economic Research, Occasional Paper 52 (New York, 1956), also in *American Economic Review, Papers and Proceedings* (May 1956), pp. 5-23; John W. Kendrick, *Productivity Trends: Capital and Labour*, National Bureau of Economic Research, Occasional Paper 53 (New York, 1956); Robert M. Solow, 'Technical Change and the Aggregate Production Function', *The Review of Economics and Statistics*, 1957, pp. 312-20.

Chapter 7

LONG-TERM GROWTH AND CAPITAL FORMATION IN GERMANY[1]

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IT is my task at this Conference to report on the results of empirical investigations concerned with the relationship between capital formation and economic growth. Rather than venture into new fields of the theory of growth, I am going to confine myself to presenting some new data and the results of a few tests of existing theory. In these studies I have tried to cover rather long periods as far as the statistical data allow. Many of the past analyses of growth suffer from the fact that the time-span investigated was only one or two decades. Here I have tried to use statistical data starting from the middle of the last century. Obviously, such a long-term study has to be restricted to only some of the great number of interesting problems which should be considered in connection with the long-term growth of an economy. In particular, I shall try to show how far the growth of real capital and the changes in its composition can be used to explain the growth of the German economy, but I shall have to leave out other factors, such as the growth of the labour force. This paper will contain no more than the presentation of a number of data and their interpretation as the preliminary result of a comprehensive study of German economic growth.[2] Future

[1] See also W. G. Hoffman and J. H. Müller, *Das deutsche Volkseinkommen, 1851–1957* (Tübingen, 1959). Since the publication of this book an attempt has been made to estimate the German national income by other methods. Preliminary estimates on the basis of calculations on consumption and savings seem to indicate that the average rate of growth of the German national income from the middle of the last century until 1913 was the same as the results of the book suggested, but that the rates of growth within individual periods were somewhat different. Consequently, any analysis of national income, capital-output ratio and investment quota for individual periods within the whole time span may require some correction.

[2] The statistical data on German capital formation are preliminary results of a study undertaken by Dr. F. Grumbach in the Institut für Industriewirtschaftliche Forschung an der Universität Münster.

investigations will have to concentrate on the breakdown of the aggregate data into sectors and branches and on a detailed analysis of these sub-groups.

We are mainly concerned here with the long-term changes of such macro-economic variables as the capital stock, total investment and total saving in relation to the growth of national income. In particular, we have tried to find out what use can be made, in explaining the growth of the German economy, of the well-known Harrod-Domar condition for equilibrium growth, viz. that the rate of growth of production equals the inverse of the marginal capital coefficient times the share of net investment or net saving in national income. We shall therefore start with a presentation of the figures of changes in German national income from 1851 to 1956 and then discuss changes in the capital coefficient and in the shares of total investment and total saving in national income for the same period. In studying these figures it should be observed that the data for the three periods 1851–1913, 1925–1939 and 1950–1956 do *not* refer to the same territory.

I. THE GROWTH OF NATIONAL INCOME

A short time ago, new and comprehensive estimates of national income became available for the whole period starting in 1851.[1] These estimates use data on tax assessment bases as their starting-point ; they are presented in the form of (three-year) moving averages of the total income of the resident population of Germany. The average rate of growth of real income (in 1913 prices) for the period from 1851 to 1913 is 2·6 per cent *per annum*. Since this whole period exhibits long-term price stability, the average rate of growth of undeflated income is the same.

As Table 15 shows, there were considerable deviations within shorter periods in the rates of growth of real income as compared with the rates of growth of money income.[2] After a period of moderate increase in real income from 1851 to 1875, when the rise was only 1·7 per cent *per annum*, real income rose particularly fast, i.e. by 3·1 per cent *per annum*, from 1876 to 1895, a period characterized by a protracted decline in the average price level. From 1896

[1] W. G. Hoffmann and J. H. Müller, *op. cit.*

[2] In considering the exceptionally high rates of growth during the short periods after the two World Wars one should bear in mind that the long-term rate of growth for Germany, say for the period 1910–1956, is much smaller than during the periods before World War I (see Figure 7). The extremely high rates of growth are due to the fact that the real *per capita* income had reached abnormally low levels when the recoveries started.

to 1913 the price level shows an upward trend, while real income is seen to rise by 2·6 per cent *per annum*. Although the data for the first sub-period (1851–1875) have to be used with great caution, we are safe in stating that the rate of growth was considerably lower during this than during later periods.

In the development of *per capita* income, we may distinguish the same three sub-periods. Between 1876 and 1895 growth was about twice as fast as in the preceding quarter of a century, while during the period of rising prices before World War I the growth of *per capita* income was again more moderate. The extremely high rates of growth observed in the periods after the two World Wars are due to the rapid reconstruction starting from an extremely low level of production.

TABLE 15

ANNUAL RATE OF GROWTH OF THE GERMAN NATIONAL INCOME

Period	Total		Per Head of Population	
	Current Prices	1913 Prices	Current Prices	1913 Prices
1851–1875	2·3	1·7	1·5	0·9
1876–1895	2·3	3·1	1·2	2·1
1896–1913	4·2	2·6	2·7	1·5
1851–1913	2·6	2·6	1·6	1·7
1925–1939	3·6	4·1	2·8	3·3
1950–1956	12·1	8·5	10·9	7·8

For a critical interpretation it may be useful to make a few international comparisons. According to Tables 15 and 16, the rates of growth of *per capita* national income were the same in the United Kingdom and Germany during the period 1876 to 1895. During the subsequent period both countries exhibit a decline in the rate of growth, which is more marked in the United Kingdom than in Germany. During the same period 1896–1913 the rate of growth of real *per capita* income in the United States was on the same relatively high level of 2·1 per cent *per annum* that had been observed in the United Kingdom and Germany during the previous sub-period. In view of the fact that after the two World Wars both the British and, even more, the American economy started out from a relatively much higher level than the German economy, it is not surprising that the rates of growth of real *per capita* income should also have been much lower in the former two countries than in Germany.

TABLE 16

RATES OF GROWTH OF PER CAPITA REAL INCOME IN THE UNITED STATES AND THE UNITED KINGDOM [1]

Period	U.S.A. 1929 Prices	U.K. 1912/13 Prices
1876–1895		2·1
1896–1913	2·1	0·8
1925–1939	0·9	1·8
1950–1956	2·1	2·5

II. THE AGGREGATE CAPITAL COEFFICIENT

A. The Growth of the Capital Stock

(1) *The Methods Used in Estimating the Capital Stock.* It is hardly necessary to mention that calculations on the German capital stock going as far back as 1850 can only be rough estimates.[2] The data available relate to buildings and equipment in agriculture, mining, manufacturing, handicraft industries, trade, banking, and insurance companies.[3] Furthermore, we have been able to use data on agricultural inventories and total capital invested in railways and in road construction. It has *not* been possible to include land values and household inventories (durable consumer goods).

In most of the original statistics each piece of capital equipment is valued, throughout its whole lifetime, at its original purchase price. This raises two problems: first, the problem of deflating the series; and second, the problem of depreciation. To get round the first difficulty, all series have been expressed in 1913 prices. This was done by deflating each year's increment of the capital stock by the same year's index of the price level of capital goods. In addition, the total capital stock of the initial year, 1850, has had to be re-calculated in 1913 prices. Special price indices have been constructed for the various series. For example, the price index of building

[1] Sources: R. W. Goldsmith, D. S. Brady and H. Mendershausen, *A Study of Saving in the United States*, vol. iii (Princeton 1956), pp. 427 and 429; U.S. Bureau of Census, *Historical Statistics of the United States, 1789–1945* (Washington, D.C., 1949), p. 26; *Statistical Abstract of the United States*, 1956, pp. 292 ff.; A. R. Prest, 'National Income of the United Kingdom, 1870–1946', *The Economic Journal*, 1948, pp. 31 ff.; Central Statistical Office, *Annual Abstract of Statistics*, No. 94, 1957 (London), pp. 7 and 245 ff.

[2] A full description of the methods used can only be given in a later publication.

[3] All the non-agricultural private sectors will in the remainder of this chapter be referred to as the 'industrial sector'.

construction was calculated on the basis of data on wage rates in building activity and the price index of building materials. The main difficulty lies in the fact that for most manufactured products no price indices are available.

The consideration of *depreciation* charges poses special problems. As indicated, equipment is valued during the whole lifetime at the price at which it had originally been purchased. Thus capital depreciation is not allowed for, and the data are rather an indicator of productive capacity. Our estimate of the capital stock coincides with the value of the capital stock as obtained by taking the value of gross investment and deducting depreciation charges only if the actual obsolescence of the capital goods is the same function of time as the depreciation charges. This condition is not fulfilled in reality, since the number of machines that become worn out annually is much smaller during the first years following the purchase of such machines than in later years when they reach the age of their life expectancy. Usually the amount of annual obsolesence conforms to some kind of normal distribution, while total depreciation charges are normally a linear function of time. For this reason, capital stock is overvalued in our study when compared to that of others.

However, the degree to which the capital stock is over-estimated is reduced, if the rate of growth of the capital stock is high or increases with time, since this increases the share of the relatively new units of capital. This is valid for Germany (Table 17). If depreciation charges were to be allowed for adequately, our capital values would have to be reduced by approximately 25 per cent. This figure is estimated from different sources. As for the comparison with the United Kingdom, it should be remembered that the capital stock of Germany has on the average a lower age than the British capital stock and that therefore the difference between depreciated and undepreciated capital stock is relatively smaller in Germany than in the United Kingdom.[1] A somewhat higher percentage would have to be deducted for agriculture and a somewhat lower one for industry. Since the rate of growth of the capital stock is not much influenced by the deduction of a nearly constant percentage, we have used the original data rather than deduct such a constant proportion. To be precise, the deflator would also have to take account of changes in the shares of agriculture and industry in the total economy. It has to be stressed at this point, therefore, that the absolute level of the capital coefficient and of the share of investment in national income is consistently over-estimated. Strictly speaking,

[1] K. Maywald, 'Fire Insurance and the Capital Coefficient in Great Britain', *Economic History Review*, second series, vol. ix, No. 1, 1956, p. 89.

what we get in this way is a 'capacity coefficient' rather than a 'capital coefficient'.

(2) *The Data on the Capital Stock.* The results of our estimates for the period from 1851 to 1956 are plotted in Fig. 7.[1] As was

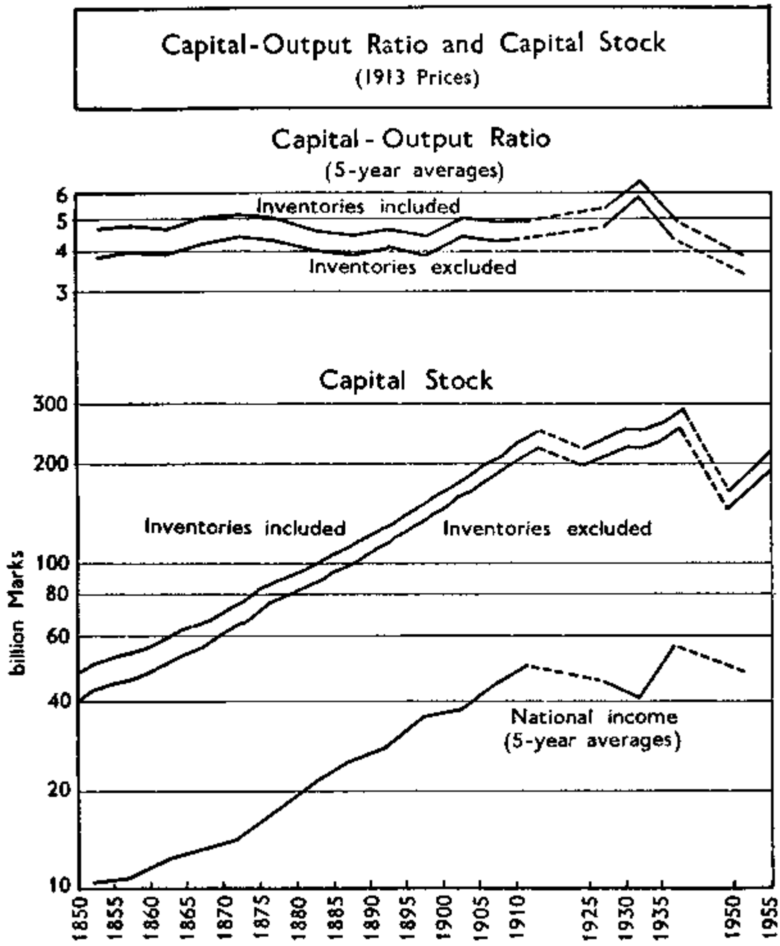

FIG. 7

[1] For the period 1913–1955 the Deutsches Institut für Konjunkturforschung (see: Ferdinand Grünig, 'Versuch einer Volksvermögensrechnung der Bundesrepublik Deutschland', *Deutsches Institut für Konjunkturforschung, Sonderhefte, Neue Folge Nr.* 41, *Reihe B: Vorträge* (Berlin, 1958)) has published estimates of the German capital stock, which, however, refer to the area of the Federal Republic only. Comparisons with the estimates in this paper are therefore confined to the period 1950–1955. On the average the data on the capital stock given here are about 14 per cent lower than the corresponding figures of Grünig. These differences will have to be explained in the course of further investigation.

to be expected, the value of the capital stock at 1913 prices rose strongly during the whole period. Territorial changes are not allowed for, so that each figure refers to the area which at that particular moment belonged to the German Reich or the Federal Republic. This is one among several reasons why the data start on a lower level after each of the two World Wars. Within each of the three periods 1851–1913, 1925–1938 and 1950–1956, a definite upward movement is observable. During the period from 1851 to 1913 the rate of growth varied. Table 17 shows that at 3·4 per cent *per annum* the average rate of growth was highest from 1896 to 1913. The rate of growth was somewhat lower (2·7 per cent) during the sub-period (1876 to 1895) after the foundation of the German Reich, after an even more moderate rate of growth of 2·3 per cent during

TABLE 17

RATES OF GROWTH OF THE CAPITAL STOCK (AT CONSTANT PRICES) IN GERMANY, THE UNITED STATES AND THE UNITED KINGDOM [1]

Period	Germany	U.S.A.	U.K.
1851–1875	2·3	—	—
1876–1895	2·7	—	2·2
1896–1913	3·4	3·4	2·4
1851–1913	2·7	—	—
1925–1939	1·9	1·2	1·3
1950–1955	4·6	—	—

the earlier sub-period. During the inter-war period the growth of the German production apparatus appears to have been slower, partly as a result of the consequences of the First World War and of the Great Depression. By contrast, the massive destruction of plant and equipment during and partly after World War II necessitated a huge amount of subsequent investment, first to secure minimum levels of living conditions, and later to keep in step with the growth of neighbouring economies. This explains the exceptionally high figure of 4·6 as the annual percentage rate of growth of the capital stock from 1950 to 1955. None of these figures would have altered significantly if inventories had been left out, since their share in the total stock was not large.

For comparison, Table 17 also supplies data on the rate of growth of the capital stock in the United States and the United

[1] Sources: E. H. Phelps Brown and B. Weber, 'Accumulation, Productivity and Distribution in the British Economy, 1870–1938', *The Economic Journal*, 1953, pp. 263 ff.—R. W. Goldsmith, D. S. Brady and H. Mendershausen, *op. cit.* p. 20.

Kingdom. In all periods the capital stock grew more rapidly in Germany than in the United Kingdom — though the figures offered below are of course only partly comparable. One might argue that the relatively slower growth in the United Kingdom was partly due to the fact that the British capital stock had already attained relatively higher values than the German at the beginning of the periods considered and that therefore the United Kingdom could afford a smaller rate of growth. But the difference may also be due to the fact that particularly before the First World War the United Kingdom was a big exporter of capital, a circumstance which would necessarily reduce the rate of domestic accumulation. From 1896 to 1913 the growth of capital in America too was more rapid than in the United Kingdom, the figure for the USA. coinciding with the estimate for Germany during the same period.

B. *The Composition of the Capital Stock*

(1) *Breakdown according to Sectors of the Economy.* For an analysis of the growth of the aggregate capital stock, the total capital stock can be broken up according to sectors and according to kinds of investment. In analyzing first the share of various sectors of the economy in the total capital stock one finds that the share of the industrial sector steadily increased from about 15 per cent in 1850 to about 44 per cent in 1955. The most rapid growth of capital in this sector occurred between 1880 and 1900. By contrast, the share of the capital stock of agriculture, which at the beginning surpassed that of all other sectors, fell continuously. The capital stock of the agricultural and industrial sectors taken together accounted for about two-thirds of the total in 1850 ; at present their combined share is about 58 per cent. The share of the public sector has fluctuated during the past, but no definite trend can be observed for the periods under review. In 1850, non-agricultural dwellings represented 15 per cent of total capital. This percentage had risen to nearly 30 per cent by 1930, but has since levelled-off at a little over 20 per cent.

(2) *Breakdown according to Kinds of Capital.* If the total capital stock is analyzed according to different kinds of investment it is seen that most of the newly formed capital has always been absorbed by new buildings, whether in the form of residential construction or for productive purposes. As Table 18 indicates, the share of all buildings in the total capital stock was always more than 50 per cent prior to World War II, but had fallen slightly by 1955. For the United Kingdom the corresponding figure [1] for the period 1870–1913 is

[1] Calculated from E. H. Phelps Brown and H. Weber, *op. cit.* p. 286, Table II columns 3 and 4.

about 45 per cent. In comparing the figures for the United Kingdom and Germany, however, one has to bear in mind the fact that the statistical methods for estimating the figures are not quite the same in these two countries. The combined share of equipment and inventories in agriculture and in the industrial sector in Germany was only about one-third during the whole period.

Since capital invested in buildings has always been of overwhelming importance, it is worth considering sector-wise. Agricultural buildings, in 1850 the most important sub-item, grew least in the process of industrialization, a fact that is easily explained by the relative decline of German agriculture as compared with the development of the other sectors of the economy. The capital stock represented by public buildings rose steadily, and roughly in proportion

TABLE 18

PERCENTAGE SHARE OF DIFFERENT KINDS OF CAPITAL IN THE TOTAL CAPITAL STOCK OF GERMANY

Kinds of Capital	1850	1880	1913	1939	1955
Buildings	54	55	52	53	48
Equipment	13	13	22	19	26
Inventories	20	13	12	13	12
Railways	3	11	9	8	6
Roads	11	8	6	8	8

to the total capital stock. The most rapid rate of growth is exhibited by both non-agricultural (private) plant and equipment and non-agricultural dwellings, and it is not by chance that the two latter rates of growth are very similar. It appears that the expansion of the productive capacity of the industrial sector and of actual production led to an equally rapid growth of housing facilities for people engaged in this sector.

(3) *Breakdown according to Kinds of Capital in Agriculture and Industry.* A more detailed analysis of the capital stock of agriculture and of the industrial sector shows that in both sectors the growth of equipment was more pronounced than the growth of buildings. As regards agriculture, one has to take into account the fact that the data on buildings include agricultural dwellings, which make up about 40-50 per cent of all buildings in this sector. In the industrial sector, equipment always represents a larger share of total capital than do buildings, but the long-term rate of growth for equipment is very much the same as that for buildings. Only after World War II

did investment in equipment have priority, since large amounts of equipment had been damaged or had become obsolete.

C. The Aggregate Capital Coefficient: Interpretation

After the discussion of the development of the capital stock and of output these two variables will now be related to one another in order to obtain the capital-output ratio. The variations of this coefficient are shown in Fig. 7. At first sight, the capital-output ratio during the period 1850 to 1913 exhibits a relatively high long-term stability, with no marked trend in either direction. As has been shown, the influence of inventories is relatively small. Therefore the inclusion of inventories does not in any way alter the results but only raises the absolute level of the capital coefficient. If inventories are excluded the limits between which the coefficient oscillates are 3·8 and 4·5; by inclusion of inventories these two values rise by 0·7, to 4·5 and 5·2 respectively. Since these estimates are based on a measure of the productive capacity rather than of the capital stock, the latter figures indicate the ratio between capacity and annual output. To obtain the capital-output ratio we would have to reduce the figure of 4·5 by approximately 25 per cent, which would put the average capital-output ratio for Germany from 1850 to 1913 at 3·6.[1]

Both on the absolute level and on the relative constancy of the capital-output ratio, our results show remarkable agreement with the figures obtained in studies of long-term economic development in other countries. As an example we may cite the capital-output ratio calculated for the United Kingdom from 1870 to 1938. Despite the fact that the methods used were different from ours, the results show a coefficient varying between 3·3 and 3·9.[2] If it is correct that the capital-output ratio exhibits a long-term stability, then it would follow that there is no difference between the average and the marginal capital-output ratios.

Since the data on the German national income are three-year moving averages, they do not allow for any short-run analyses. But we can make the following comments about somewhat longer periods. From 1850 to the 'seventies the capital coefficient showed a clearly visible upward trend, whereas after this period it first declined and then rose again until 1913. These observations agree with experience in the United Kingdom where the capital-output ratio exhibited the

[1] The question to what extent the capital-output ratio may have been influenced by long-term changes in the degree of utilization of the capital stock — e.g., as a consequence of growing monopoly — cannot be answered here.

[2] E. H. Phelps Brown and H. Weber, *op. cit.* p. 266.

same long-term cycles. Although the fluctuations are not very large, they may be relevant for an interpretation of economic development during that time.

In trying to explain these variations we first compared the changes in the capital coefficient with the long-term interest rate. The two series exhibit roughly parallel movements; for instance during the period with the highest rate of growth of national income (1876–1895), both the interest rate and the capital-output ratio declined. The same positive correlation is shown for other periods. The inter-war period is better not included in this analysis because of the surplus capacity during the Great Depression. Preliminary analysis shows that the ratio between gross profit and national income also has a high positive correlation with the capital coefficient.

The correlation between the capital coefficient and the interest rate may be of some economic significance. If there is a functional relationship between the two variables — an increased demand for capital per unit of output leading to a rise in the interest rate and vice versa — then any long-term stability of the capital coefficient should lead us to expect that the interest rate also would show no upward or downward trend in the very long run. This holds only under a *ceteris paribus* assumption as to the supply of capital. Actually, however, no such assumption is justified, since one very important factor, the ratio of savings to income, rose strongly during the period 1850–1913. A plausible explanation seems to be that the upward shift in the supply curve of savings was just offset by a rise in the marginal productivity of capital schedule. Looking at the long-term movement of the interest rate, it is debatable whether one should interpret its behaviour as long-term stability or as a slight downward trend.

The relationship between capital and output can also be considered the other way round. Then the long-term constancy of the capital coefficient would mean that total output per unit of capital has been constant in the very long run. The productivity of capital would not have changed over time for the economy as a whole. Starting from the identity that the capital coefficient is the mathematical product of capital intensity (i.e. capital per unit of labour rather than per unit of output) times the inverse value of the productivity of labour (i.e. employment per unit of output), we could also state that this mathematical product of capital intensity times the inverse value of the productivity of labour must have been constant through time if the capital coefficient shows long-term stability. The series over time for both the capital intensity and the productivity of labour do in fact move in roughly parallel fashion, which means that the ratio between the two — equal to the product of the capital intensity and the *reciprocal* value of the productivity of labour — has also been constant. Therefore, along with a nearly threefold rise of (physical) capital intensity between 1850 and 1913 the (physical) productivity of labour must also have risen to something like 300 per cent of the initial value. In other words, labour input per unit of output must have declined correspondingly. The technical progress which made this possible must have neutralized the decline in the marginal efficiency of capital which would otherwise have resulted from the large expansion of the capital stock.

If the results for the whole period from 1870 to 1938 are compared with those obtained for the United Kingdom,[1] it is seen that the (real) productivity of labour in the United Kingdom doubled during this time, while in Germany a value two and one-half times as high as in 1870 was attained by 1938. It should be remembered, however, that owing to the consequences of the First World War and to the Great Depression, the level of productivity was relatively low in Germany during the inter-war period.

III. THE SHARE OF INVESTMENT IN NATIONAL INCOME

A. The Rise of Total Investment

(1) *Methods used in Estimating Total Investment.* Owing to the lack of adequate statistical data, total investment has had to be taken as the increment of the capital stock, a fact which has the obvious drawback that even relatively small errors in the estimates of capital stock lead to large errors in the figure for annual investment. Inclusion of inventory investment and of depreciation charges would influence the absolute level of the totals, but would not significantly change the long-term rate of growth of total investment as long as the (long-term) development of inventory does not deviate too much from that of other investment, and as long as the ratio of depreciation to the total capital stock does not vary too much.

In calculating the share of total investment in national income the appropriate values have been compared in terms of current prices. In calculating investment at 1913 prices, the same indices have been used as for the deflation of the capital stock.

(2) *The Estimates.* The estimates of total investment at 1913 prices are shown in Fig. 8. The rate of growth is highest from 1851 to 1875; after a few years of absolute decline in investment, it

[1] E. H. Phelps Brown and H. Weber *op. cit.* p. 266.

remains at a relatively low level; and even in the sub-period before 1913 it is rather smaller than at the beginning. This is brought out by Table 19, in which the same sub-periods have been chosen as for the capital stock and for national income. Thus, over the whole

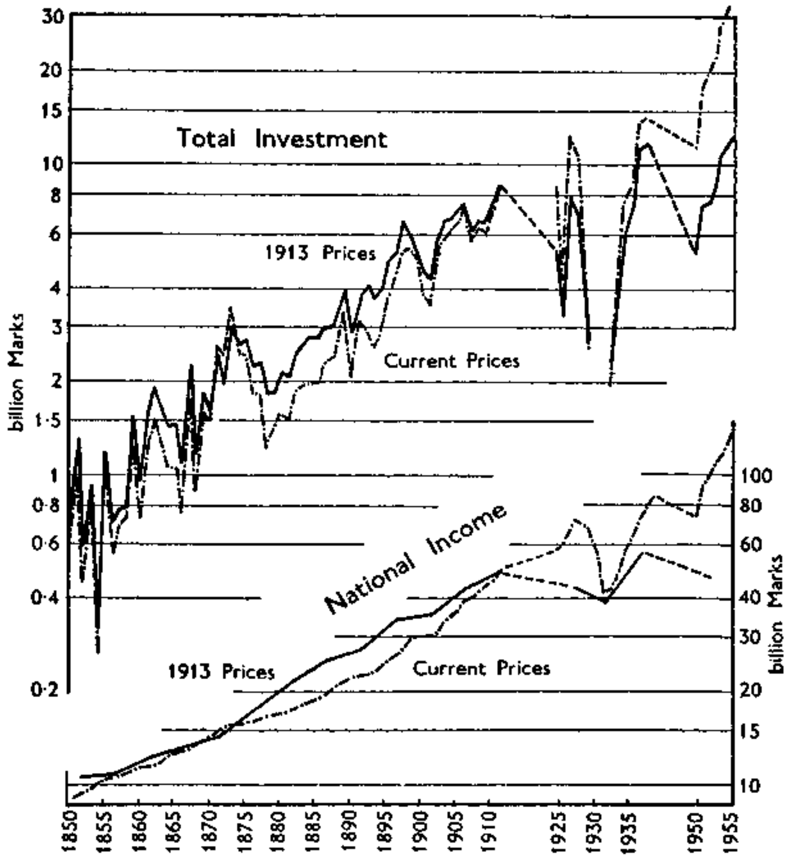

Fig. 8

period from 1851 to 1913, real income rose by an average 2·6 per cent *per annum*, the real stock of capital by 2·7 per cent, which is almost equal to the rise in income, and the volume of investment by 4 per cent *per annum*.[1]

[1] Tibor Barna and others argued during the discussion of this paper (see p. 353) that in the long run the average rates of growth of the capital stock and of the volume of investment should be identical. But in shorter periods — as is the case here — deviations between the two are possible if the initial investment is not the same percentage of the initial capital stock as the rate of growth of investment. Taking the

If, for comparison, total investment is plotted in terms of current prices, one gets the same impression. Total investment in monetary terms has fluctuated very strongly — but it should be added that owing to the lack of reliability of some of the data this result has to be taken with caution.

B. The Composition of Total Investment

(1) *Division according to Sectors of the Economy.* The different sectors of the economy have benefited in varying degrees from the growth of total investment, as has already become evident from the analysis of the capital stock. A study of the sectoral distribution of investment at current prices shows the dominating rôle of the non-agricultural residential construction, whose share rose from 20 per cent to 30 per cent before 1875 and then remained almost constant during the whole period thereafter. Investment in public buildings maintained an almost constant share of 10 per cent from 1850 to 1913, and even in the Federal Republic its share is not significantly higher. These data on buildings do not consider the monetary flows that financed their construction and therefore leave public aid for private residential construction out of the picture.

If investment in private residential construction and in public buildings is added together, a figure of 30 per cent of total investment for the years 1851–1875 is obtained; this share slowly rises to 40 per cent and remains at that level up to the present, if the large fluctuations that occurred during this century are neglected. Investment in agriculture for productive purposes made up two-thirds of the total in the agricultural sector, the other third being investment

figures for Germany we get the following result: initial investment (1913 prices) around 1850 is 0·75 billion Marks, equal to 1·5 per cent of the initial capital. In 1913 investment is about 8·0 billion Marks, equal to 3·2 per cent of the capital stock, which means that investment has grown faster than capital. As is shown in Table 17 the rate of growth of capital rises from 2·3 (1851/75) to 3·4 per cent (1896/1913), i.e. it approaches 4 per cent asymptotically. Paul Samuelson points out that the mathematical expression would be:

$$I_{(t)} = I_0(1{\cdot}04)^t \text{ implies}$$

$$K_{(t)} = K_0 - \frac{I_0}{0{\cdot}04} + \frac{I_0}{0{\cdot}04} \cdot 1,04^t$$

where I_0 = initial investment

K_0 = initial capital

and $0{\cdot}04$ = rate of growth of investment.

If $$K_0 - \frac{I_0}{0{\cdot}04} = 0, \text{ then } \frac{\Delta K}{K} = 0{\cdot}04.$$

If $K_0 > \frac{I_0}{0{\cdot}04}$, then the rate of growth of the capital stock will be smaller than the rate of growth of investment, but asymptotically approaching it.

in agricultural housing. While up to 1880 railways absorbed 15 to 20 per cent of the total, their share later declined and then stabilized at 6 to 7 per cent. The share absorbed by industry shows a large increase at the expense of investment in agriculture and railways. From 1896 to 1913 investment in industry declined, however, from 53 per cent to 43 per cent of the total. In the inter-war period a larger share of investment went into residential construction and public buildings ; the share of industry declined to 30 per cent — a level that had first been reached as early as the 1870s. In the Federal Republic the huge reconstruction boom caused the share absorbed by industry to rise to 54 per cent during 1950–1955 — an unprecedented record for Germany.

(2) *Breakdown into Kinds of Investment.* As is to be expected from our earlier analysis of the capital stock, investment in construction was quantitatively most important, investment in equipment and inventories taking the second place. Within total construction, residential construction accounted for 50 to 60 per cent of the total, a rising share of the rest being absorbed by industrial plant. A declining share went into agriculture, while the share of public building was more or less constant.

The long-term growth of investment in building construction exhibits wide fluctuations, which are reflected in the large variations of *total* investment. These fluctuations occurred both in the real series and in the monetary series, and they were particularly violent up to the year 1895. During this early period they were accompanied by large variations in building costs. However, the turning-points of the two series do not always coincide exactly. The cycles in construction prices (brick prices) had a length of fifteen years.

During the period from 1851 to 1913 the average rate of growth of investment in equipment and inventories and of total investment in railways was higher than that of total investment in building construction, as is shown by Table 19. Especially from 1875 to 1880, the share of these kinds of investment in total investment rose rapidly at the expense of building construction. During the inter-war period the rate of growth of both of these series was small. After the Second World War it was investment in equipment that rose most rapidly.

If the whole period from 1851 to 1913 is considered, it is seen that the rate of growth of real investment in equipment has been constant in the long run, in spite of large short-run fluctuations. Even during the period of intensive railway construction, the growth of investment in equipment did not slow down. Apparently, the expansion of the railway system was effected not at the expense of other kinds of investment, but at the expense of consumption, a fact which led to a rise of the share of total investment in national income during the period of intensive railway construction.

A comparison of the long-term development and the cyclical changes of real investment in industrial equipment and railway construction with the variations in pig iron consumption indicates parallel changes in the two series. The data yield a long-term rate of growth of 4·8 per cent *per annum* for the former series and a similar rate of growth of 4·9 per cent for pig iron consumption. This correlation may be taken as an indirect confirmation of the correctness of our data on these kinds of investment — though it has to be admitted that pig iron was not the only material which played an important rôle in the production of the investment goods. We have not investigated whether a similar quantitative relationship holds for the inter-war period, because of the big effect of the Great Depression on these economic variables.

TABLE 19

RATES OF GROWTH OF TOTAL INVESTMENT IN GERMANY AT 1913 PRICES

Period	Total Investment		Investment in :	
	including	excluding	Building Construction	Production Equipment and Railways
	Inventories			
1851–1875	5·7	5·4	5·2	5·8
1876–1895	4·1	3·4	3·0	3·9
1896–1913	4·1	4·2	4·4	4·2
1851–1913	4·0	3·8	3·7	4·3
1925–1939	0·8	1·8	1·3	1·3
1950–1956	18·9	19·3	10·4	27·1

(3) *Division of Total Investment according to both Sectors and Kinds of Investment.* If total investment is broken up into kinds of investment absorbed by agriculture and industry, one finds that, in *agriculture*, investment in buildings always exceeded investment in equipment during the period from 1851 to 1913. The annual changes in these two components of total agricultural investment exhibit great similarities, though the turning-points of the two series may vary by a year — a difference that should not be taken to be economically significant, because it may be due to some deficiency

of the method applied. Allowing for a possible error of one year, it may be said that the upper turning-points for agricultural investment occurred in 1863, 1873, 1886 and 1904. During the inter-war period, by contrast, investment in agricultural equipment at times exceeded investment in building construction. In the first years after the Second World War, however, building construction had again attained the dominant place.

With total investment in *industry*, investment in production equipment is quantitatively more important than investment in buildings, for the reasons given above. The amplitude of the cyclical changes in investment in equipment is considerably greater in this sector than in building construction or in agricultural investment. The annual variations of the two kinds of industrial investment show much less similarity than do the respective agricultural series. The upper turning-points in industrial investment were: 1863, 1870/71, 1886–88, 1898, 1906/07 and 1912 for equipment; and 1862, 1875/76, 1899 and 1906 for building construction.

C. The Ratio of Investment to Income

Before reporting the data on the ratio of net investment to income, the following points should be mentioned. Since investment is the increment of the capital stock, the same arguments hold concerning depreciation charges, etc., as were discussed in connection with the capital stock on page 122. Furthermore, the fact that national income has been estimated in terms of moving averages becomes important now. During boom years, the income estimates are somewhat too low, and during depression years they are too high. This means that the ratio of investment to income is *over*-stated for boom years and *under*-stated for depression years, so that the cyclical fluctuations in this ratio are somewhat exaggerated. This fact is conceptually significant but has been neglected in preparing the data since it would probably not alter the percentage share of investment by more than one point.

The ratio of net investment to national income rose continuously from 1850 to 1913, from about 6 to 8 per cent in the middle of the last century, to more than twice this percentage at the onset of the First World War. The rate of growth of the share of net investment was 0·13 per cent *per annum* if inventories are excluded and 0·15 per cent if inventories are included. During the inter-war period the share of investment in national income was considerably smaller; its maximum in the boom year of 1928 did not exceed 14 per cent. The share has been exceedingly high since the Second World War, reaching 22 per cent in 1955 — the highest rate for the whole of the last 100 years of German economic history.

A study of annual changes discloses an exceptionally high rate of growth of the share of investment for the years after the foundation of the Reich. This growth was followed by a sharp decline about 1880, which reduced the share from about 18 per cent in 1875 to about 8 per cent in 1880 — about the value first reached around 1860. Thereupon it gained strongly up to 1898, but after that date rose only moderately. Most of these fluctuations are concealed if, as in Table 20, the averages of the three sub-periods are taken for comparative purposes. Table 20 shows that from 1896 to 1913, a period of rising prices, the share of investment was particularly great. Conversely, during the period 1876 to 1895, the decline of prices

TABLE 20

AVERAGE RATIO OF INVESTMENT TO INCOME IN GERMANY, THE UNITED STATES AND THE UNITED KINGDOM [1]

Period	Germany	U.S.A.	U.K.
1851–1875	10·2	—	—
1876–1895	11·4	—	9·5
1896–1913	16·5	16·0	11·8
1851–1913	13·2	—	—
1925–1939	10·7	8·1	6·4
1950–1956	20·8	—	—

was accompanied by a reduction in the ratio. During the latter period the share of investment was approximately the same as during the preceding period from 1851 to 1875 when the level of prices had been relatively high, but did not rise markedly as was the case in the period immediately before World War I (1896–1913).

For an evaluation of both the long-term and the cyclical development in Germany, it may be useful once more to compare the data with those of the United Kingdom and the United States. As already mentioned, the share of investment in income for Germany must be reduced by about 25 per cent to allow for depreciation. On the other hand German figures should be increased about 2 per cent for the share of capital exports in national income because British and American figures include capital exports. After these modifications

[1] Sources: R. W. Goldsmith, *A Study of Savings in the United States*, vol. ii, Princeton, 1955, p. 199. R. W. Goldsmith, D. S. Brady and H. Mendershausen, *op. cit.* p. 26. E. H. Phelps Brown, *op. cit.* pp. 286-7.

the investment ratio for Germany would be 10·5 for 1876–1895 and 14·4 for 1896–1913. This means that the ratios for the United Kingdom and Germany are similar during 1876–1895 and that the ratio for Germany is higher during 1896–1913. The figures for the rate of growth of national income are the same for the United Kingdom and Germany during 1876–1895 (3·1 per cent per annum). During 1896–1913 the United Kingdom has a lower investment ratio and also a lower rate of growth of national income (1·5 per cent per annum) than Germany (2·6 per cent per annum) which may be explained by different capital-output ratios during this period.

These international comparisons are also interesting from a cyclical point of view, because the dates of a number of turning-points in the shares of investment in total income coincide in the three countries. Striking examples of this are the high share of investment in 1906 in all three countries, the turning-points for the United Kingdom and Germany in 1899 — at different absolute levels. In the year 1889, once more, the turning-points in the share of investment in national income coincide in the United Kingdom and Germany, this time at the same absolute levels. The decline in the share of British investment during the 1870's also indicates that the decline in German investment had its counterparts in other countries.

IV. THE AGGREGATE SHARE OF SAVING IN INCOME

A. The Rate of Growth of Saving

(1) *Methods Used in Estimating Saving.* As indicated above, no adequate data for the earlier periods are available from which total saving might be calculated. Using *ex post* concepts for both saving and investment, we have therefore taken total investment as a substitute for total saving. We have also tried to obtain as much independent information as possible on the composition of saving and on the behaviour of individual sources of saving. Actually, the only part of saving which can be estimated is that deposited with banks, insurance companies, etc. Since the shares of saving so deposited by the government, by firms and by households as a percentage of total saving cannot be ascertained, it is not possible to obtain a picture of the rate of growth of total saving or of any relative measure of the development of saving, let alone the absolute level of total saving.

For these purposes, the available balance-sheets and other statistical data of the monetary sector, i.e. private banks, savings banks, co-operative banks, state banks, insurance companies and the stock exchanges have been analyzed. We have tried to cover all the savings deposits in these institutions — without being able to check, however, whether these sums were actually invested in the productive apparatus of the economy. Since we are concerned with net saving, we could not take the total amounts of saving, but only changes in total liabilities within the calendar years.

(2) *The Results.* As Table 21 indicates, the share of saving that could be identified by our method is very small at the beginning of the period under investigation. The data show that the importance of the banking institutions and of the insurance companies has increased over time. This means that to an ever larger extent savings have been channelled through savings institutes (in the widest sense of the term), at the expense of non-institutional saving. This is important because it shows that the rate of growth of saving channelled through savings institutions has been greater than that of total saving. Thus any study that tries to estimate total saving on the basis of the data of the monetary institutions alone is bound to overestimate the rate of growth of total saving. The share of saving invested in government securities exhibits no definite trend up to 1913. Assuming that the government used these funds to finance public buildings, one would expect that the share of these buildings in total investment had also been constant. This is to some extent confirmed by the facts. The share of public investment in total investment (including investment in railways) was an average of 22·6 per cent from 1855 to 1865 and of 19·0 per cent from 1900 to 1910. If railways are excluded, the shares are 8·3 and 10·8 per cent respectively. A short-run comparison of the data on saving discloses a positive correlation between the individual series. The upper turning-points of the series occurred in 1889, 1898 and 1906.

B. The Composition of Total Saving

It would be desirable to trace the savings deposited with the above-mentioned institutions according to origin, to find out to what extent they were household, business or public savings. The available data do not allow for such a procedure. Data on the origin of savings have been published only since the Second World War. They are given in Table 22 and indicate that public and corporate saving by far exceed the savings of private households in 1950. By 1957 the savings of private households had partly caught up with other savings.

TABLE 21

DISTRIBUTION OF SAVINGS BY INSTITUTIONS (PER CENT)

	1851/60	1861/70	1871/80	1881/90	1891/1900	1901/13	1925/38	1950/55
Banks Total	10·90	14·42	20·03	28·55	31·83	35·81	47·00	44·86
Savings Banks	2·64	3·25	6·38	8·34	8·28	10·92	11·87	10·15
Credit Co-operatives	0·11	1·55	1·62	0·84	2·08	4·61	4·61	
Commercial Banks	3·52	4·39	1·44	7·68	8·61	11·26	9·03	11·00
Mortgage Banks	—	1·95	7·34	7·61	10·66	7·55		
Public Banks	0·71	2·39	3·25	3·30	1·71	1·47		
Central Banks	3·92	0·89	0·00	−0·78	0·49	−0·00		
Insurance								
Social Insurances	—	—	—	0·40	1·30	0·67		
Life Insurance	0·51	0·79	1·15	2·44	3·77	3·86		
Joint Stock Companies				9·98	12·48	8·42	8·11	
Other Incorporated Business (*G.m.b.H.*)	—	—	—	—	1·89	2·73	0·31	
Public Loans	12·37	11·31	10·87	25·39	9·90	12·87		
Reich	—	2·25	0·01	4·85	2·69	2·97		
Länder	11·32	8·24	9·49	18·41	4·23	6·35		
Local Authorities	1·05	0·82	1·37	2·13	2·98	3·55		
Total				66·76	61·23	64·36		
Total Savings (100 per cent) Mill. Marks *	750	1,187	2,088	2,165	3,707	6,169	6,471	21,728

* Annual averages.

C. The Ratio of Saving to Income: Interpretation

Owing to the inadequacy of the data the approximately constant rate of growth of total saving cannot be explained by a detailed analysis of its component parts. Therefore another line of interpretation will be attempted. We shall investigate whether there is any relationship between the total of savings and the distribution of income. If, on the basis of the tax assessment statistics for Prussia and other German *Länder*, the distribution of income is measured by Paretian α's, it is seen that the ratio of investment to income shows a high degree of negative correlation with the corresponding α's for the whole period from 1850 to 1913 and even for the inter-war period.

TABLE 22

SOURCES OF SAVING IN THE FEDERAL REPUBLIC, 1950 AND 1957 [1]
(PER CENT OF THE TOTAL)

Period	Private	Public	Undistributed Profits	Net Transfers from other Countries
	Households			
1950	17·7	28·4	40·8	13·1
1957	30·6	41·2	29·0	0·8

Though the data do not allow us to be dogmatic on annual changes of the two series, we may safely state that the relatively high rate of investment as compared with income after 1870 was accompanied by a rise in the inequality of income distribution, that the long-term stability of the ratio of investment to income from 1896 to 1913 had its counterpart in stable values for α, and that the relatively low rate of investment during the inter-war period corresponds to a high value of α. The relationship between the distribution of income and the ratio of investment to income is so evident as to suggest that the long-term changes in savings were partly a function of the income distribution. On the other hand, income distribution is often also influenced by the rate of investment, as is in particular suggested by the recent West German experiences.

[1] Sources: *Monatsberichte der Deutschen Bundesbank*, June 1958, p. 75. The item 'Undistributed Profits' includes the surplus of public enterprises, Federal Railways and Post Office. The item 'Net Transfers from Other Countries' includes foreign aid and German restitution payments.

V. INTERPRETATION OF THE RESULTS OF THE STUDY

It was pointed out in the beginning that it was one of the aims of this study to prepare statistical data which would allow us to analyze in quantitative terms the relationship between capital formation and economic growth during the course of German economic development. Now theory tells us that, if certain conditions are fulfilled, with a constant marginal capital coefficient the rate of growth of income is a function of the rate of net investment. The data of this study suggest strongly that in the very long run, i.e. between 1851 and 1913, the average capital coefficient was approximately constant at 4·8 : 1, which means that the marginal capital coefficient also cannot have varied very much. If this coefficient of 4·8 — which, however, is a 'capacity coefficient' rather than a 'capital coefficient' — is correct, then an average rate of growth of income of 2·6 per cent per annum would yield a value of 12·5 per cent for the share of investment in national income enabling such growth to take place. Actually, the share of investment was 13·2 per cent. If depreciation charges are also taken into account, the capital coefficient assumes a value of 3·6, and this gives us a rate of net investment of 9·4 per cent which would have resulted in a rate of growth of real income of 2·6 per cent. The estimate of 9·4 per cent compares with an actual value of 9·9 per cent.

Apart from the shortcomings of the statistical material, which did not allow a correct calculation of depreciation, the discrepancy is mainly due to the fact that we have used long-term averages. The results of such a procedure are the better, the smaller are the deviations of the annual data from the averages. As was pointed out, however, the fluctuations in the ratio of investment to income were large. In view of the deficiencies of the data, the discrepancy between the actual and the estimated average rates of investment is certainly not very large and does not cast any doubt on the validity of our conclusions.

Chapter 8

CAPITAL INTENSITY AND THE COMBINATION OF FACTORS OF PRODUCTION[1]

BY

ALAIN BARRÈRE

I. INTRODUCTION

THE entrepreneur has to combine factors of production on the basis of a rational calculation, certain elements of which can be determined only in advance. His forward-looking decision fixes the amount of capital to be combined with labour, and so determines the capital intensity of the firm.

Capital intensity for the whole economy is subject to so many uncertainties[2] that it is preferable to study it at the level of the firm, where it can be described, for a given output, as the expected ratio between the value of the capital used and the total expenses of production. Capital intensity is closely linked with the whole cost structure. Since the latter depends upon the way in which the entrepreneur combines the factors of production in producing any given output, the determination of capital intensity in effect depends upon the factor combination.

To study the way in which the factor combination is determined, means considering decisions about how much capital is to be used in the firm. In this paper the main emphasis will be on the first point and the question of capital intensity will be regarded as a corollary depending on the long-run combination of factors.

The combination of factors of production is not fixed accidentally but depends on several elements which enter into the entrepreneur's calculation. Since it involves the building-up of a production process, this is a long-period problem. The creation of capital assets and the establishment of conditions for bringing such assets into use

[1] Translated from French by Elizabeth Henderson.

[2] Cf. Alain Barrère, 'L'Analyse des relations entre le capital et la production', *Revue d'Economie Politique*, 1955, pp. 332-408.

may require more or less time, but it will always need more than can properly be described as the short period.

Our first task is to define the elements of the entrepreneur's calculation.

A. The Elements of the Entrepreneur's Calculation

The entrepreneur decides on the combination of factors of production on the basis of certain data over which he has no control. Some of these data are known to him, while others relate to the future and he forecasts them. Both are elements of his calculation. This blend of known facts and forecasts opens the door wide to uncertainty and risk.

(1) One of the known facts is the state of technique. Its future can be assumed to be one of more or less constant improvement, thanks to the steady spread of improvements in production methods. There is an incentive for the entrepreneur to introduce new and more productive factor combinations and, in so doing, he can choose between several solutions of equal technical merit.

Another known fact is the labour supply, both in terms of quantity (the current volume of employment) and of quality (the skill of labour in operating the technical equipment of the entrepreneur's choice). The entrepreneur also has to consider the wage rate, which may vary with the number and ability of the workers he intends to employ with his capital. With the reservation that the entrepreneur has some freedom of choice, the general wage level has to be taken as a datum which takes its place in the entrepreneur's calculation just like the state of technique, which also allows him a choice between alternative solutions.

The price level is a further initial fact which the entrepreneur must take into account for two reasons. It determines the prices he has to pay for his equipment and raw materials, and also the prices he can expect to charge for his products. Here, known elements are mixed with forecasts. In calculating his capital cost, the entrepreneur has to base his calculations mainly on the existing price level, just as he reckons with current wages. But when he estimates his selling price, he must forecast the price level at the time when his products reach the market. In long-period analysis, it is the long-run price trend which is relevant, and for simplicity's sake temporary disturbances may be ignored. In determining the factor combination at any particular moment the entrepreneur must thus start out with a given price level and take a long view of subsequent price movements.

Finally the current level of the rate of interest is a more certain datum, leaving less room for uncertainty. Moreover, the entrepreneur's future interest payments are fixed at the level ruling at the moment when he borrows.

Two more elements enter into the entrepreneur's calculation: the amount of disposable funds and the technical relation between capital and labour.

(2) The amount of disposable funds is that available not to the economy as a whole but to his own firm. One therefore has to consider not only the volume of saving and banking policy, which determine the general terms on which capital can be borrowed by all entrepreneurs, but also the attitude of banks towards the particular category to which the firm belongs. This attitude will not be the same towards large and small firms; it varies with planned production, with the personal credit-worthiness of the entrepreneur, and so on. It follows that the amount of capital to which the entrepreneur has access is subject to two limitations: on the global level it is limited by certain factors connected with the general conditions of the economy, and on the level of his own firm it is limited by factors peculiar to the latter. Each firm is in its own particular position for borrowing, quite apart from any considerations regarding the interest payments it can afford.

(3) The technical relation between capital and labour springs from the fact that the use of any particular type of capital in the industry to which the firm belongs requires a certain amount, and just that amount, of labour. To operate, supervise, maintain or use a given type of machinery or equipment requires precisely one, three or five workers, or more. It is not an economic but a technical requirement. Capital and labour are technically complementary in the sense that any particular type of technical capital, that is any particular machine or piece of equipment, requires a strictly determined amount of labour, which is a datum for the entrepreneur. This complementarity underlies the technical structure of the process of production. To obtain a desired volume of output the entrepreneur has a choice of several technical processes, but each of them entails given proportions of capital and labour as determined by technical requirements.

B. The Entrepreneur's Choices

An entrepreneur wishing to employ factors of production has to make a fundamental choice between various possible combinations. He may incline towards the combination which he thinks technically best, but may be deterred by economic considerations such as the production cost of the new process, that is, the cost of the equipment; the amount and cost of the labour required; the volume of output at which he aims; and so on.

Thus, with a given state of technique, the entrepreneur has a choice of several ways of producing, and if he is rational he must choose the economically most advantageous solution.

In order to simplify, we shall assume that the most advanced techniques require most capital and least labour. There are of course exceptions, but the proposition seems plausible as a general rule, since in most cases technical progress leads to the appearance of more costly equipment operated at a lower labour cost.

Armed with these data, the entrepreneur can make the calculations which determine his choice of a combination of factors of production, and hence the capital intensity of his firm. To see how this combination comes about, it will be convenient to proceed along two lines: (*a*) an appraisal of alternative technical combinations; (*b*) economic choice in the light of profit maximization.

II. AN APPRAISAL OF ALTERNATIVE TECHNICAL COMBINATIONS

The relationship of technical complementarity between capital and labour is largely given, but it deserves some special observations. To clarify its meaning, we must distinguish it from a concept which is all too easily taken for granted, namely the principle of substitution. This will enable us to see how the principle of technical complementarity works and how it is expressed by the indifference curve of technical combinations.

A. Technical Complementarity between Capital and Labour

The combination of capital and labour is not determined by substitution at the margin of the productivity or cost of capital and labour respectively. This principle derives from short-period analysis and is not applicable to a combination destined to establish a long-period production process. This is not a question of extending an existing process based on an existing combination of factors of production, but of the reasons which may cause one production process to be replaced by another.

If we assume, as we properly may in this context, that the use of a larger amount of capital in a new combination implies the use of more advanced equipment, the productivity of the labour employed with that capital must rise and the marginal productivities of both capital and labour change. In the new combination it is the principle of technical complementarity which is relevant.

Two considerations force themselves on the entrepreneur. The first is that the more costly equipment needs less labour ; the second is that each type of equipment, representing some particular application of technical progress, requires a particular amount of labour. Consequently, an increase in the employment of capital entails a decrease in the employment of labour.[1] This process cannot normally be expected to lead to any opportunity for substitution at the margin, because each type of equipment requires, for technical reasons, a certain amount of labour, and just that amount. In other words, when it comes to building technical progress into a new factor combination, capital and labour are complements rather than substitutes.

One should also note that capital is not increased by the addition of one unit after another, but by a transition from one technique to another, more advanced, technique requiring more capital and less labour. Hence, capital intensity does not increase in a continuous fashion but in steps. Each type of equipment calls for definite amounts of labour and capital, determined technically rather than economically. Leontief's technical coefficients are an illustration of this principle.

The relation between capital and labour, then, is not a matter of substitution at the margin, resting on considerations of cost and productivity, but is ruled by a principle of complementarity of a technical nature.

The problem is not one of substitution at the margin by equating costs or productivities, but one of comparing the total cost of every combination which is technically possible and capable of producing the desired product.

The economic problem then becomes that of choosing between the different combinations which are technically possible, and this economic choice works on the basis of total cost. For example, an entrepreneur wishing to produce a given volume of output can choose between three types of equipment, each of which is technically more advanced than his present equipment and requires more initial capital but less labour to run it. The first type of equipment requires an amount of capital C_1 and is operated by an amount of labour L_1 ; the second requires C_2 and L_2, the third C_3 and L_3. The three possible combinations are $C_1 L_1$, $C_2 L_2$ and $C_3 L_3$, where we assume that $C_1 < C_2 < C_3$ and $L_1 > L_2 > L_3$.

[1] With some exceptions, as has been said before.

We may note at once that if the entrepreneur wants to diminish the amount of labour L_1, he cannot simply increase the amount of capital in the combination $C_1 L_1$. The relationship of technical complementarity forces him to choose another combination, either $C_2 L_2$ or $C_3 L_3$. In other words, because of technical complementarity he has a choice between three combinations, but no possibility of substitution at the margin. He cannot modify a combination, but is forced to change to a quite different one. The concept of marginal changes gives way to the concept of structural changes, exemplified by the adoption of a new factor combination. Attention is focussed on the transition from one production process to another with greater capital intensity.

The choice between these technical processes is essentially an economic choice, depending upon the entrepreneur's economic calculation. Each combination has a total cost, composed of capital cost and labour cost. By his calculation, the entrepreneur has to determine what is economically the most advantageous technical combination.

B. The Indifference Curve of Technical Combinations

Application of the principle of substitution leads to the notion of indifference with respect to the use of factors of production at the margin. Application of the principle of technical complementarity gives a choice between a number of technical combinations yielding the same product, such that, in order to obtain the same output, it is a matter of indifference which of them is selected. The indispensable thing is to determine the economically most advantageous one. We have to construct an indifference curve of technical combinations yielding a given product.

Let C_1, C_2 and C_3 be capital expenditure, that is to say the value of equipment at replacement cost. Let L_1, L_2 and L_3 be the respective labour costs which, together with other production expenses, make up the operating cost of each type of equipment. If we reduce the whole of operating cost to labour cost, total operating cost is proportional to the amount of labour employed. This is not completely accurate, but it would not be difficult to get rid of this simplification.

By choosing any one of the combinations $C_1 L_1$, $C_2 L_2$ or $C_3 L_3$, the entrepreneur would in each case obtain the same output but the total costs would differ, because the amounts of capital and labour vary inversely. What combination will he choose?

From the point of view of output he is indifferent, since the three combinations produce the same amount. If the necessary factors are freely available, it makes no difference to the entrepreneur which combination he chooses. But this is not so from the point of view of cost, since each different combination requires different amounts of capital and labour and so entails a different total cost. The entrepreneur has to determine which of them costs least.

To this end our entrepreneur moves along the indifference curve

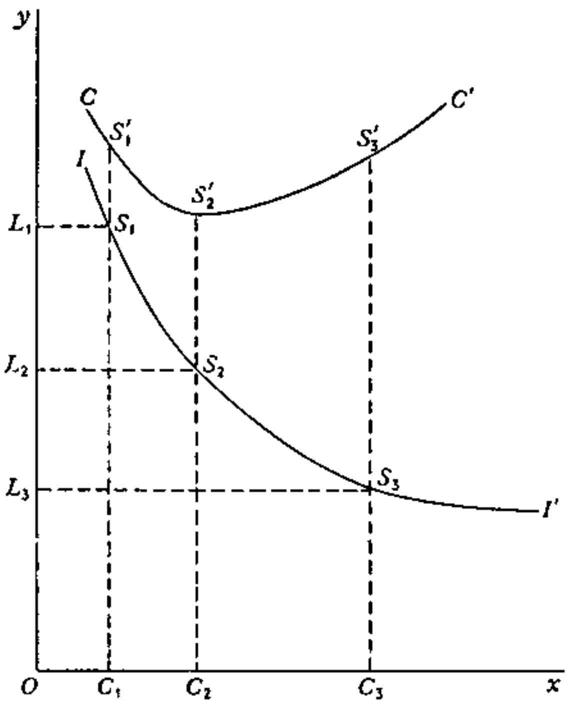

FIG. 9

of technical combinations. At each point of this curve he disposes of a factor combination yielding the same product but having different cost. We must draw the indifference curve of technical combinations and the curve showing their total cost.

On the *x*-axis in Fig. 9 we mark the amounts of capital C_1, C_2, C_3 ... C_n, representing the money value of equipment at replacement cost. On the *y*-axis are the corresponding operating costs. The curve *II'*, which traverses the intersection points of the respective co-ordinates, is the indifference curve of technical combinations. At any point on this curve the entrepreneur can obtain the same volume of output. In reality there are only a few such points and they are probably spaced out rather widely ; but we may, without changing the argument, assume that there are enough of them to define a continuous curve.

Each combination has its own cost and is economically more or less advantageous than the others. The indifference curve of technical combinations must therefore be considered in the light of the curve of their total cost. This can be drawn into the same diagram, by marking on the ordinate the total cost of each combination, that is to say its capital cost plus its operating cost. We trace the co-ordinates C_1 and (C_1+L_1), C_2 and (C_2+L_2), C_3 and (C_3+L_3) and through the intersection points draw the curve CC' which represents the total cost of the various combinations. The two curves together provide the following information.

The curve II' shows the technically possible combinations. The curve CC' indicates the total cost of these combinations. The total cost can be read off the diagram if we prolong each ordinate upwards to CC'. If the entrepreneur chooses the third combination, the ordinate starting at C_3 places him at S_3 on the indifference curve and at S'_3 on the total cost curve.

At first sight it would seem that all that the entrepreneur has to do is to choose the combination with the lowest total cost, that is C_2 L_2 which places him at S_2 and S'_2, since he obtains the same output in every case. But this does not necessarily follow. At this stage of the demonstration we do not as yet know what kind of profit the entrepreneur wants to achieve and how it is determined. It depends not only on the cost of the combination, but also on the productivity and yield of the factors employed and thereby in part on the precise amounts used. In particular, the return on capital as a function of its quantity will play a decisive rôle.

III. THE ECONOMIC CHOICE IN THE LIGHT OF PROFIT MAXIMIZATION

The question of what profit the entrepreneur wants to maximize is a highly controversial one.[1] There is no need to go into it here ; we merely note that in our case the entrepreneur may wish to maximize either the current value of total profit, or the ratio between the current value of total profit and the cost of his capital equipment. We do not concern ourselves with maximum total profit as given by

[1] Cf. especially F. A. Lutz, 'Théorie du capital et théorie de la production', *Economie Appliquée*, 1948, p. 8.

the equality of marginal cost and marginal revenue. For these the principle of substitution between capital and labour does indeed come into play, and it is not applicable to our case because it neglects the time which must pass in altering the length of the production process as well as technical differences between the various types of capital. Nor is it possible, with the principle of substitution, to take account of the fact that the entrepreneur has to compare present cost with future returns, that is, returns at the end of the production process. The argument is legitimate in a static situation but not in a dynamic one, where all the above-mentioned elements come in, and more particularly the fact that cost is a matter of the present and returns a matter of the future. This fact is all the more important in the long period, because of the lengthening of processes of production which follow upon the use of more and more advanced equipment.

A. The Nature of the Profit to be Maximized

We have indicated two kinds of profit maximization which the entrepreneur may aim at. Let us examine their meaning.

If the entrepreneur desires to maximize the current value of total profit, he has to consider a series of annual revenues spread over the life of his capital equipment. Their future value is a problem of forecasting. To work out the total profit he can expect he must confront the present value of his anticipated future earnings with the current total costs of production. In other words, the entrepreneur has to calculate the present value of the total profit he expects from the capital under consideration. Then he can make up his mind whether it is worth while creating that capital, on which he will have to pay interest at the current rate. How can he make such a calculation ?

The present value of the anticipated sales receipts is obtained by discounting the latter at the going rate of interest. He then deducts current production cost and so obtains the current value of total profit. He must finally find out what factor combination maximizes that value.

If the entrepreneur desires to maximize the ratio between the current value of total profit and the cost of equipment, he has to consider not the over-all size of total profits, but to calculate them as a percentage of the value of his capital. In other words, he relates the current value of total profit, defined as above, to the cost of the equipment used, and tries to maximize the percentage so obtained. This will give him the highest average rate of profit, in present value, per unit of capital. The merit of this kind of profit maximization lies in its enabling the entrepreneur to see at a glance how far he can go in increasing the amount of capital in a combination. All he has to do is to compare the present value of the average profit rate with the rate of interest. He will rule out combinations requiring so much capital as to cause the average profit rate to decline, even if they yield a higher total profit.

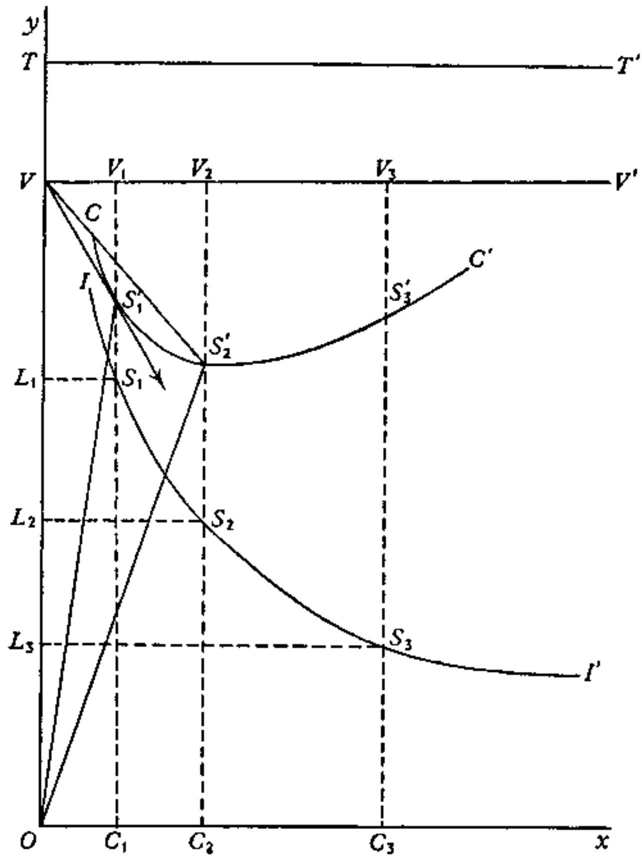

Fig. 10

The choice between the two types of profit maximization thus has a bearing on the choice of the technical combination, which now ceases to depend on attaining the lowest total cost. We therefore have to examine the elements of that further choice.

A graphic demonstration of the two types of profit maximization can easily be carried out on our original diagram, as in Fig. 10.

We draw a straight line TT' parallel to the abscissa, at a height which can plausibly represent the anticipated future value of output, that is, the anticipated sum of annual receipts. Having calculated the current value of future output by discounting the latter at the current rate of interest, we represent it by another parallel VV' below TT'. We can now show the entrepreneur's two profit choices on the diagram.

Maximization of the current value of total profit is determined as follows. The ordinates starting at the points C_1, C_2, C_3 are prolonged upwards to VV', where we mark the points V_1, V_2, V_3. The respective distances between the straight line VV' and the curve CC' indicate the current value of total profit for each combination, that is, the current value of total output less total production cost. The distance is greatest between V_2 and S'_2, and therefore the current value of total profit is maximized by the combination C_2 L_2, which has the lowest total production cost.

Now let us look for the combination which maximizes the ratio between the current value of total profit and the cost of equipment. We draw a tangent through V to the total cost curve CC'. This tangent is VS'_1 and the point S'_1 indicates where, on the cost curve, the ratio between total profit and amount of capital is maximized.[1] Thus it is the combination C_1 L_1 which maximizes the ratio between the current value of total profit and the cost of equipment.

B. The Entrepreneur's Calculation

We are now in a position to examine the reasons which may lead the entrepreneur to prefer to maximize either total profit or the average profit rate and so to choose one or the other of the possible combinations. To this end, we must look at the results using each of these combinations for profit and for the quantities of factors of production.

(1) *The Consequences of the Feasible Combinations.* If the entrepreneur wishes to maximize the current value of total profit, he will choose the combination C_2 L_2 which has the lowest total cost but requires more capital ($VV_2 > VV_1$). If he wishes to maximize the current value of the average rate of profit per unit of capital, he will choose the combination C_1 L_1, which needs less capital. With the combination C_2 L_2 total profit is higher but average profit per unit of capital lower; with the combination C_1 L_1 total profit is lower but average profit per unit of capital higher.

[1] Because $\frac{V_1 S'_1}{V_1 V}$ is the largest tangent.

Both combinations require a known amount of capital, but the amount differs in the two cases. Hence the factor combination is different; it employs more capital in one case and less in the other, accordingly the production process is longer or shorter and the capital structure of production deeper or less deep. It may be measured by the intensity of capital.

(2) *The Degree of Capital Intensity.* Capital intensity is the anticipated ratio between the cost of equipment C and total production cost $C+L$. It can be seen in the diagram if we trace straight lines from origin to the points S'_1 and S'_2 which represent the total production cost of the two combinations. We see that the combination C_1 L_1 has a lower capital intensity (less capital and higher total cost) than the combination C_2 L_2 (more capital and lower total cost).

We can now look at the consequences of each combination in the light of capital intensity. When capital intensity is lower, that is, when less capital is used, the capital structure of production is less deep. Total profit is relatively modest but average profit per unit of capital is higher. When capital intensity is higher, the capital structure of production is deeper, total profit is high but average profit per unit of capital is relatively modest.

All these things are obviously relevant to the decision which the entrepreneur is now in a position to make with respect to the kind of profit he wants.

(3) *The Entrepreneur's Profit Choice.* The preceding discussion furnishes the entrepreneur with the elements of his choice between maximization of the current value of total profit and maximization of the ratio between the current value of total profit and the cost of capital equipment. This choice is a matter neither of accident nor of some conventional attitude, but depends on the relative ease with which the entrepreneur can get capital and labour. Assuming rational behaviour on the part of the entrepreneur and looking at the problem from the point of view of capital supply, the choice is determined as follows.

If the entrepreneur encounters difficulties in getting capital, he will seek his return by maximizing the ratio between the current value of total profit and the value of his assets. This will give him the largest average profit per unit of capital. The corresponding factor combination gives rise to a production process of low capital intensity.

If the entrepreneur has easy access to capital, he will aim at maximizing total profit, that is to say the sum of money values. The corresponding factor combination gives rise to a production process of high capital intensity. Labour requirements, of course, move in the opposite direction in the two cases.

In these circumstances, the entrepreneur's position on the capital market is of decisive importance. We have already made it clear that this position depends not only on the aggregate quantity of disposable funds in the economy as a whole, but also on the particular attitude of the capital market and the banks to the firm under consideration. Things like the entrepreneur's personal credit-worthiness, the financial situation of the firm, or the degree to which credit policy is restrictive or selective, may play an important part. Generally speaking, if we assume, as seems reasonable, that a large firm is better placed in this respect than a small one, it follows that the better equipped a firm is to begin with, the more opportunities and reasons it will have for acquiring yet better equipment and so deepening the capital structure of its production process.

Thus in the long period the productive apparatus changes by virtue of the decisions of entrepreneurs seeking their profit in one of two principal forms. The choices leading to a progressive deepening of the capital structure are determined by elements some of which are internal to the firm and others external. But there seems to be a presumption that a deep capital structure tends to generate a yet deeper one.

IV. FACTOR COMBINATIONS AND EXPANDING OUTPUT

We have so far argued on the assumption that the desired volume of output is fixed. Now we have to consider the entrepreneur's choice in terms of factor combination and profit maximization, as well as the development of capital intensity, in the light of opportunities for expanding production.

Let us suppose that market research leads our entrepreneur to conclude that there is some unsatisfied demand and that he has a chance of increasing his sales. Obviously, it cannot be to his advantage to produce and sell just any arbitrary volume of output which may be absorbed by demand.

The entrepreneur has to reckon with certain data, such as market price, wage level, capital cost, factor supply, etc. On the basis of these he has to determine the volume of sales which seems best able to secure him the profit he seeks.

To this end he must now weigh two groups of questions: one, as before, concerns the choice of factor combination, the other the most favourable volume of output.

A. The Complexity of Real Situations

The choice of factor combinations becomes much more complicated in this case. When the volume of output is fixed, the entrepreneur moves along only one indifference curve offering him several combinations and he can choose between any two of them in the way described. But when production can increase, two types of choice are open to him. He may enlarge an existing production process, let us say, for the sake of simplicity, expand his firm on the basis of the existing factor combination or set up a new firm on the same basis. Then the problem is simple.

But it may be possible to use more advanced techniques and so to create a production process of higher productivity; then it may be better for the entrepreneur to proceed to new combinations incorporating the latest technical advances. Instead of buying another machine like those he has already, he could buy a more modern and technically more efficient one. It will no doubt cost him more (on the assumption we made earlier), but may also increase his profits. In that case the entrepreneur modifies his production process not by widening but by deepening the capital structure. He has to choose between several sets of factor combinations, all entailing higher total cost and all capable of larger output.

The procedure we have described enables him to determine, for each volume of output, the two points at which he can maximize the current value of total profit and the current value of the average profit rate per unit of capital respectively. But this twofold determination is much more complicated than in the case of given output.

With rising output, the entrepreneur's supply grows, and it becomes more difficult for him to estimate the future as well as the current value of his anticipated receipts. There are two reasons for this.

The first is that uncertainty and risk increase in so far as the introduction of new technical processes lengthens the production period, that is to say the lapse of time between the putting to work of the capital good and the marketing of its product.

The second reason is that the response of demand is much more difficult to foresee. Demand may not grow exactly in step with supply. What is more, the entrepreneur must expect his future sales receipts to increase less than proportionately with his output, since growing supply may depress the market price. The elasticity of demand enters into the calculation. According as demand is more or less elastic, the entrepreneur's profit expectations lag somewhat irregularly behind his expansion plans. The determination of his profit, in whichever form, becomes very hazardous.

We are led to the conclusion that while the determination of the factor combination in any given set remains fairly easy, the choice of the set of combinations is a very risky matter. It depends on the accuracy of demand forecasts and on the reliability of market research.

On the theoretical level, the problem looks like becoming intractable, because precise curves can be drawn only on the basis of specific knowledge of the total cost of each combination belonging to each set, as well as of the reaction of demand to each increase of supply.

In these circumstances valid generalizations can be made only by simplifying the assumptions to the point of depriving the explanation of all but a certain limited illustrative value. Thereafter every particular case requires particular treatment. It would seem very difficult, at any rate within the framework of this paper, to construct the various indifference curves reflecting different assumptions with respect (1) to the shape of the indifference curves and the total cost curves, and (2) to the reaction of demand and expected receipts to each volume of output.

If we nevertheless take our demonstration a step further, we do so only for purposes of illustration and do not conceal its limitations.

B. A Simple Model

We must introduce a number of simplifying assumptions in order to make our illustration valid even within the limits indicated. As regards the set of combinations, we shall retain our previous simplifications and assume, in addition, that the combinations yielding more output are technically more advanced and require progressively more capital and less labour. We neglect major innovations disrupting the regular course of technical development, because their effect on the shape of the curves cannot be predicted in abstract terms.

As regards the volume of demand, we assume that demand grows to the extent foreseen in the determination of the supply obtained by production with various combinations, but that the value of sales receipts grows less than proportionately with the rise in supply, owing to a consequential price fall.

On these arbitrary assumptions we can construct a model which, we repeat, is not intended to reflect the general case but an arbitrarily simplified one. The various sets of combinations *a*, *b*, *c* can each be exemplified by an indifference curve and a total cost curve. With these combinations the entrepreneur can obtain volumes of output, the value of which is represented by the annual sales receipts T_a, T_b, T_c. By discounting these latter, we reduce them to their present value V_a, V_b, V_c (see Fig. 11). The differences between the present

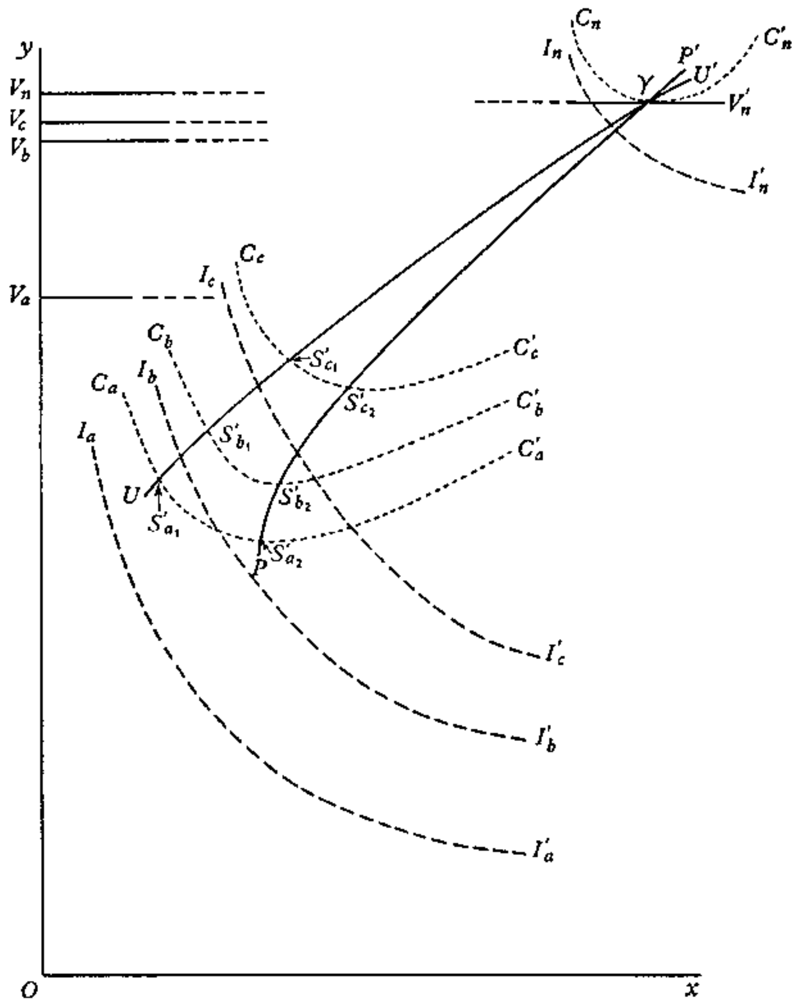

FIG. 11

values are not proportional to the differences between the outputs produced by the various factor combinations, since the selling price falls as supply increases.

Proceeding as before, we obtain for each set of combinations the points S'_{a1}, S'_{b1}, S'_{c1}, where the average rate of profit per unit of capital is maximized, and the points S'_{a2}, S'_{b2}, S'_{c2}, where the present value of total profit is maximized. If we connect these two sets of points, we get two curves which tend to meet. The curve UU' is the curve along which moves the entrepreneur who increases his output with a view to maximizing the average rate of profit per unit of capital; the entrepreneur seeking maximum present value of total profit moves along the curve PP'.

An important observation can be made: since the two curves tend to meet, the two types of profit maximization approximate each other more and more closely and merge in a critical combination. For that combination the present value of sales receipts equals total cost. The points S'_{n1} and S'_{n2} coincide at the point where the straight line $V_nV'_n$, expressing the current value of the output produced by the combination n is tangent to the lowest point of total cost curve. The point γ so determined is the critical point which shows the limit of extension of the firm in given current circumstances (with given rate of interest, wages, price level, volume of demand, etc.). At the point γ the current value of total profit becomes zero, because the present value of sales receipts equals current total cost. Similarly, the average rate of return on capital during the useful life of the equipment equals the rate of interest. The rate of interest having served to reduce the future value of sales receipts to their present value has, in this critical case, equated it with cost. Furthermore, the different degrees of capital intensity tend to become the same under the influence of the progressive approximation between the two types of profit maximization. At point γ there is only one possible degree of capital intensity; the firm has fully adapted its productive apparatus to maximum output. In other words, the firm has reached the maximum intensity of its capital structure in the sense that it employs the greatest possible quantity of capital to obtain maximum production, given the basic data and the general market situation (price level expected, etc.).

There is a presumption that the capitalistic firm will never reach this degree of capital intensity, because the entrepreneur will wish to keep a high average rate of profit per unit of capital. But there is a type of firm which may reach this point and stay there. It is the public enterprise which, in the absence of a capitalistic profit motive, pays less regard to the average rate of profit per unit of capital. Consequently, we may conclude that the public enterprise tends towards the highest degree of capital intensity, as we have defined it. The ultra-modern and very costly equipment of the French National Railways is a case in point.

V. CONCLUSION

The question of the combination of factors of production has been discussed here principally as a function of the entrepreneur's disposable funds and of his particular profit motive. We have considered capital supply as the determining factor. The conditions of labour supply and wages probably play a secondary part, but would certainly have to be considered in a fuller investigation. We would then see that the combination of factors of production and the resulting capital intensity of the production process are linked to the more general problem of the division of human effort between the production of capital goods and their use. From that point of view capital is seen as the derived factor, enabling human labour to become more productive. The determination of the capital intensity of the process of production is ultimately one of the solutions to the general problem of strategy in the war against scarcity and hard physical effort.

Chapter 9

AN ANALYSIS OF A MARKET FOR INVESTMENT GOODS[1]

BY

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If prices become more favourable, users of capital goods may want to increase their *stock* of this factor of production. In a theory of investment the problem is, however, to explain a *stream*, the growth of capital per unit of time. We have to explain, therefore, not only how much more capital the users want to take into use, but also how rapidly they do it (or how rapidly new capital goods are produced).[2] In the first model below I assume that when, for example, changes in the price-situation make capital users want more capital, the immediate result will be an increase in the price of capital goods. Such a price-increase will then induce the producers of these goods to speed up production. In the second model below I operate not only with a price of capital goods ready for delivery today, but a schedule of prices, depending on the time of delivery. The users and the producers of capital goods will, through their bargaining, decide both the amount of new capital and the time of delivery, which together determine the level of investment. Within both models I have studied how a change in the rate of interest will affect investment and the substitution of capital for labour.

I. MODEL 1

We consider a sector which produces a certain type of homogeneous consumer good. Its production-function is:

$$X=\phi(N, K), \tag{1}$$

[1] I am indebted to Mr. Gerhard Stoltz, Oslo, whose suggestions may have helped to make the paper more readable.

[2] This idea has been developed by Professor T. Haavelmo in his lectures at the University of Oslo.

where

X ... is the total number of units of product per unit of time,
N ... total input of labour, measured in number of workers (fixed hours of work),
K ... total input of capital (see below).

We assume that the input of capital consists solely of a certain type of homogeneous machine, for which technological efficiency and maintenance costs are constant (independent of age). X measures net production, exclusive of maintenance costs of the machines. These assumptions make the measurement of capital — as a factor of production — straight-forward; K is number of homogeneous machines. Assumptions like these, which allow us to measure input of capital as a number of homogeneous units, seem reasonable from a *theoretical* point of view.[1]

We assume that our sector can employ as many workers as it wants at the given wage rate, w. Further, we assume that the price of its products, p, is given. We operate with p as a 'net price', after costs of raw materials have been deducted.

One condition for maximum profit is (as usual):

$$\phi_N' = \frac{w}{p}. \tag{2}$$

As for the input of capital, K, we assume that at the margin one more machine gives a discounted return which equals its price:

$$\int_0^\infty \phi_K' p e^{-h\tau} e^{-r\tau} d\tau = q,$$

which gives

$$\phi_K' = \frac{r+h}{p} q. \tag{3}$$

Here r is the rate of interest. (We assume that the producers want to get at least this return on their capital outlay.) The parameter h expresses something about how confidently the producers believe that the price-situation will remain the same. This confidence is here assumed to decline gradually with the distance of time ahead. The calculated income τ units ahead is therefore not $(\phi_K' p)$, but this amount multiplied by a factor $e^{-h\tau}$, where $h > 0$.[2] If, for example,

[1] If we want to describe the input of capital in the real world, and compare our models with reality, we try to construct an index for the input of capital, which we think we can use *as if* this index expresses the number (or changes in the number) of homogeneous units of capital.

[2] See G. L. S. Shackle, 'Interest-rates and the Pace of Investment', *Economic Journal*, 1946, p. 2 and p. 11. Another reason for such a cautious calculation may be anxiety that the machine may become out of date at some unpredictable future time because of new inventions.

we are in a boom-period which is expected to be very short, h may have a very high value. The sum $(r+h)$ we may call the subjective rate of interest.

At any point of time, t_0, the stock of capital in the sector, K_0, is given. (Production of new capital requires time.) At t_0 the level of employment, N_0, is then given by (2). When we assume that r and h ('the state of confidence') are given, the relation (3) determines the price of machines at t_0, q_0. (We assume that our sector is the only one using this particular type of machine.) The price, q, of capital goods is supposed to be flexible. The immediate effect of a shift in r or h will then be such a change in q, that (3) is fulfilled.

Now we turn to the other participants in our market for investment goods, the producers of machines. To simplify, we assume that only the input of labour enters into their production function. From the principle of profit-maximization we then can deduce that the production of machines depends on $\frac{q}{w}$. As an approximation we write:

$$\dot{K}=a_1\frac{q}{w}-a_0, \text{ where } a_1>0 \text{ and } a_0>0. \tag{4}$$

$$\dot{K} \text{ is } \frac{dK}{dt}, \text{ the rate of growth in } K \text{ per unit of time.}$$

We here assume that the production of machines per unit of time equals the growth in the capital users' input of capital. This means that the users of capital immediately take up the newly produced machines. To make them willing to do that, the price q has (*ceteris paribus*) to fall gradually. (Compare the cobweb model. We might say that $K_{t+1}-K_t$ depends on the volume of machine production started at t, which is a function of q_t.) We now have a determined dynamic model (2)-(4), in the 3 endogenous variables K, N and q, with p, w, r and h as constants.

Ia. COMPLEMENTARY FACTORS OF PRODUCTION

We assume that (1) may (approximately) be described by a 'Cobb-Douglas function':

$$X=AN^{\alpha}K^{\beta},$$

where
$$0<\alpha<1$$
$$0<\beta<1$$
$$\alpha+\beta<1.$$

By this type of production function the marginal productivity of the one factor increases when the other factor increases ('complementary factors of production'). The inequality $\alpha+\beta<1$ indicates 'decreasing returns to scale'. (2) and (3) now become:

$$A\alpha K^{\beta}N^{\alpha-1}=\frac{w}{p}. \tag{5}$$

$$A\beta N^{\alpha}K^{\beta-1}=\frac{r+h}{p}q. \tag{6}$$

K_0, the size of capital at the starting-point t_0, is given. Then (5) and (6) determine N_0 and q_0. We assume that q_0 is sufficiently high to make $\dot{K}_0$ positive. From (4), (5) and (6) we get:

$$\dot{K}=\frac{A\beta a_1 p}{(r+h)w}\left(\frac{pA\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}K^{\frac{\alpha+\beta-1}{1-\alpha}}-a_0, \tag{7}$$

or $$\dot{K}=DK^{\frac{\alpha+\beta-1}{1-\alpha}}-a_0,$$

where D and a_0 are positive constants.

We see that $\dot{K}$ will decrease when K increases, (because $\alpha+\beta<1$). As $\dot{K}_0$ was assumed positive, K will rise from t_0 on, but at a decreasing rate, and will approach a stationary value, $\bar{K}$.

$$\bar{K}=\left(\frac{a_0}{D}\right)^{\frac{1-\alpha}{\alpha+\beta-1}}. \tag{8}$$

When K increases towards $\bar{K}$, q decreases towards a stationary value $\bar{q}$, cf. (4).

$$\bar{q}=\frac{a_0 w}{a_1}. \tag{9}$$

At this low price, our type of machine is not produced any longer. From (5) we get:

$$N=\left(\frac{A\alpha p}{w}\right)^{\frac{1}{1-\alpha}}K^{\frac{\beta}{1-\alpha}}. \tag{10}$$

We see that the sector chooses to expand both K and N when one factor-price, here q, decreases (on our assumption of complementary factors of production).

From (10) we get:

$$\frac{\dot{N}}{\dot{K}}=\left(\frac{A\alpha p}{w}\right)^{\frac{1}{1-\alpha}}\left(\frac{\beta}{1-\alpha}\right)K^{\frac{\alpha+\beta-1}{1-\alpha}}.$$

$\dot{N}$ will go down relatively to $\dot{K}$ as K becomes larger (and q decreases).

From (5) and (6) we get:

$$\frac{\bar{N}}{\bar{K}}=\frac{\alpha\bar{q}}{\beta w}(r+h).$$

There is a tendency to use more capital in relation to labour, the lower q becomes in relation to w.[1] There is also a tendency to use relatively more capital the lower r, and h, are. Changes in p, however, have no influence on the relation between $\bar{N}$ and $\bar{K}$. A positive shift in p will immediately give a positive shift in N, and hence also an immediate increase in $\frac{N}{K}$, but afterwards K will increase relatively more strongly than N.

We see that changes in the production-function of the capital-users will affect the relationship of $\dot{N}$ to $\dot{K}$, and of $\bar{N}$ to $\bar{K}$. But changes in the constant A (which affect the two marginal productivities equally) do not affect the relation between $\bar{N}$ and $\bar{K}$.

Effects of Changes in the Rate of Interest

We consider a change in r occurring at a point of time t_1, when capital has a given size K_1. N_1 is then given by (5). This means that the left-hand and so also the right-hand side of (6) will not change at t_1; a change in the price of capital must occur which balances the change in r. The elasticity of q_1 with respect to r is therefore:

$$e_{q_1,\,r}=-\frac{r}{r+h}.$$

If h is zero, a fall in r from, say, 5 to 4 per cent will immediately increase q by 20 per cent. From (4) we see that the elasticity of $\dot{K}$ with respect to q exceeds one. The reduction in r from 5 to 4 per cent will therefore initially induce an increase in the production of machines by more than 20 per cent. But if the price-expectations of the users of machines are pessimistic, if, say, $h=3r$, the immediate increase in the production of capital goods will exceed 5 per cent, not 20. (A higher value of h thus lessens the effect of changes in r.)

From (8) we see that $\bar{K}$ will go up when r shifts downwards (D increases). $\bar{q}$ is, however, independent of r, cf. (9). From (10) we see that when r shifts downwards, $\bar{N}$ will go up. The increase in

[1] If we assume that our sector can immediately import as many machines as it wants to at a fixed constant price, but that it needs time to get more labour (people need time to move), w being an endogenous variable, we get inverted results.

$\bar{N}$ will, however, be (percentage-wise) less than the increase in $\bar{K}$. If in (5) or (10) we take the elasticity with respect to K, we get

$$e_{NK} = \frac{\beta}{1-\alpha}.$$

This elasticity is positive, constant and less than one (because $\alpha+\beta<1$). When no shifts occur in w and p, K will consequently always change relatively more than N. A shift in the rate of interest will therefore affect $\bar{K}$ relatively more than $\bar{N}$. In this sense we can conclude that the input of capital is — in this model — more sensitive to changes in r than the input of labour is.

Constant and Increasing Returns to Scale

By constant returns we mean that $\alpha+\beta=1$, and by increasing returns that $\alpha+\beta>1$. In these cases no certain values of N and K will give the capital users maximum profit, given the values of p, w, q and r. I shall, therefore, not analyse these cases on the above assumptions. (We may start as above by saying that K is given at t_0, and then (5) and (6) will give us N_0 and q_0. But, at least when $\alpha+\beta>1$, we cannot then claim that there is no excess demand for machines at t_0. A user of capital can raise his profit by bidding up q, if he thereby induces other users to sell him machines.)

Ib. ALTERNATIVE FACTORS OF PRODUCTION

We now assume that the production-function of the capital-users, (1), is such that the marginal productivity of one factor decreases when the other factor increases (alternative factors of production). In order to simplify the analysis we write ϕ_N' and ϕ_K' as linear functions of N and K. (2) and (3) then become :

$$b_0 - b_1N - b_2K = \frac{w}{p}, \tag{11}$$

$$c_0 - c_1N - c_2K = \frac{(r+h)}{p}q, \tag{12}$$

where all coefficients are positive, and where $b_2 = c_1$. As above we assume that w, p, r and h are constants.

At a certain point of time, t_0, the stock of capital, K_0, is given. N_0 and q_0 are then determined by (11) and (12). A stability condition is that $b_1c_2 > c_1b_2$. This granted, we can conclude that there is no excess demand for machines at t_0 (at the price q_0).

From (4), (11) and (12) we get:

$$\dot{K} = \frac{pa_1}{(r+h)w}\left(\frac{c_1 b_2}{b_1} - c_2\right)K + \frac{pa_1}{w(r+h)}\left(c_0 + \frac{c_1 w}{pb_1} - \frac{b_0 c_1}{b_1}\right) - a_0, \tag{13}$$

or $\qquad \dot{K} = \delta_1 K + \delta_0.$

The general solution of this differential equation is:

$$K(t) = Ce^{\delta_1 t} - \frac{\delta_0}{\delta_1}. \tag{14}$$

We have $\delta_1 < 0$, as we assumed $b_1 c_2 > c_1 b_2$. C is a constant determined by the value of K_0. If $\dot{K}_0$ is positive, we find that K will increase from t_0 on, but at a decreasing rate, and will approach a stationary value,

$$\bar{K} = -\frac{\delta_0}{\delta_1}.$$

From (4) and (13) we find that q will fall as long as K rises. From (11) we get

$$N(t) = -\frac{b_2}{b_1}K(t) + \frac{1}{b_1}\left(b_0 - \frac{w}{p}\right), \tag{15}$$

and by differentiating (15) with respect to t we get

$$\frac{\dot{N}}{\dot{K}} = -\frac{b_2}{b_1}. \tag{16}$$

In this case, where the factors of production are alternative, we find that an increase in the input of capital is accompanied by a proportional *decrease* in the input of labour. If b_2 and c_1 are negative, we again have the case where the factors of production are complementary. Then, according to (16), $\dot{N}$ and $\dot{K}$ are proportional. (When we assumed a 'Cobb-Douglas function' we found that the *relative* growth in N was proportional to the *relative* growth in K.)

Effects of a Shift in the Rate of Interest

We imagine the 'market-experiment' — that a shift occurs in r at point of time, t_1. If h is zero, an immediate result will be an opposite shift in q of the same relative magnitude (cf. the discussion above). From (4) we find that $e_{\dot{K}q} > 1$. If then r is reduced, say, from 5 to 4 per cent, $\dot{K}$ increases immediately by more than 20 per cent. (But it rises less the more pessimistic price-expectations are.) From (16) we see that if $\dot{K}$ increases by 20 per cent, $\dot{N}$ decreases

20 per cent. A reduction in r will, according to this model, speed up the substitution of capital for labour.

A reduced value of r means, however (cf. 13), that $\dot{K}$ now falls more quickly with increasing K. (The upward shift in $\dot{K}$ of more than 20 per cent is thus not permanent.) Still K now approaches a higher stationary level, while the stationary level for N is reduced. We see that a reduction in h ('more confidence') has the same effects as a reduction in r. And the effect of changes in r is less the greater the value of h is. A positive shift in p will in the first moment increase N, and the relation $\frac{N}{K}$ (cf. 15). Such a shift may also induce a positive shift in q, and it is possible that both $\dot{K}$ and $\dot{N}$ will increase.

II. MODEL 2

We still consider a sector which produces a certain homogeneous consumer good. Its production function is (1). As in Model 1, we assume that the price of its product, p, is given, and that the sector can employ as many workers as it wants at the given wage rate w.

One condition for a maximum profit is (as usual) that the marginal productivity of labour is equal to the real wage rate :

$$\phi_N'(N, K) = \frac{w}{p}. \tag{17}$$

We assume that (17) is always fulfilled.

The input of capital — in (1) — again consists solely of a certain type of homogeneous machine whose technological efficiency and maintenance costs are constant and independent of age. (The price of a machine will then be independent of its age.) [1] The price of a machine ready for delivery today we denote by q.

In this second model we suppose that there are direct negotiations between the users and the producers of capital goods, and that they make contracts about future deliveries. The price of a machine ready for delivery in T years we denote by q_T.

The discounted expected income due to a unit of capital (cf. (3) above) delivered today, and delivered in T years, respectively, is :

$$\int_0^\infty \phi_K' p e^{-(r+h)\tau}\, d\tau = \phi_K' \frac{p}{(r+h)},$$

and
$$\int_T^\infty \phi_K' p e^{-(r+h)\tau}\, d\tau = \phi_K' \frac{p}{(r+h)} e^{-(r+h)T}.$$

[1] Both old and new machines may be traded. We may, for instance, think of ships.

If the capital user considers especially the purchase of machines ready for delivery today, a condition for maximum profit would be :

$$\phi_K' \frac{p}{(r+h)} = q. \tag{18}$$

If r, p and w are constants, the equations (17) and (18) determine K and N as functions of q (cf. Model 1).

If we assume that machines ready for delivery in T years are especially considered, we get :

$$\phi_K' \frac{p}{(r+h)} e^{-(r+h)T} = q_T. \tag{19}$$

In this case (17) and (19) determine K and N (from the point of time T on) as functions of q_T.

We may define the concept 'equivalent price' as that value of q_T for which (17) and (19) give the same values for K and N as (17) – (18).[1] The expression for this 'equivalent price' is obtained by dividing (18) by (19), i.e.

$$q_T = qe^{-(r+h)T}. \tag{20}$$

We see from formula (20) that given the price q for immediate delivery, the equivalent price q_T of a machine delivered in T years will decrease when T increases. If $T=0$, we get of course $q=q$, if T becomes infinite, q_T approaches zero. The form of this schedule depends very much on the value of $(r+h)$. Higher h (more pessimistic price-expectations) or a higher rate of interest means that the capital users must have a bigger cut in the price to be compensated for late delivery.

If the capital users want immediate delivery (T is zero), they will, as already mentioned, choose a value of K, $K=K(q)$, given by (17) and (18). If they are concerned with any other specific value of T, they will choose this same value of K, $K(q)$ (from T on), if q_T is given according to (20). Considering the whole schedule of prices given by (20) the capital users will choose the same value of K, $K(q)$, but they will be indifferent about the time of delivery, at least for the marginal unit of capital. The number of new machines the capital users want to buy, the excess demand of capital, we denote by Q^D. We must have : $Q^D = K(q) - K_0$, where K_0 is the total stock of capital already held or contracted for by the capital users.

If, as in Model 1, we specify the production function of the

[1] From $\tau = T$ and $\tau = 0$ respectively onwards. If we get the same value for K we must also get the same value for N (cf. (17)).

capital users as a Cobb-Douglas function, (17) and (18) are written (5) and (6) respectively. By means of (5) we express N as a function of K, (6) will then give us the K as a function of q. We get :

$$Q^D = \left(\frac{(r+h)}{pA\beta}\right)^{\frac{1-\alpha}{\alpha+\beta-1}} \left(\frac{w}{\alpha pA}\right)^{\frac{\alpha}{\alpha+\beta-1}} q^{\frac{1-\alpha}{\alpha+\beta-1}} - K_0,$$

or
$$Q^D = Bq^{\frac{1-\alpha}{\alpha+\beta-1}} - K_0. \tag{21}$$

Because $B>0$, and we assume $\alpha+\beta<1$, the excess demand for machines, Q^D, will rise when q falls.

We may sum up our description of the demand side of our market for capital goods by the following statement : If a certain schedule of prices, $q_T = f(q)$ given by (20), is announced (cf. Walras' provisional prices 'cried out'), the users of capital will order the amount $Q^D(q)$ of capital goods, given by, e.g., (21). As to the time of delivery they will (because of the form of the q_T-curve) have no special wishes.

We now turn to the supply side of our market for capital goods. We assume that the production function of the producers of machines is :

$$Y = \Psi(NT, T). \tag{22}$$

Here Y is the number of machines produced in a period of T years, and N the number of workers, with fixed hours of work. NT is therefore the total hours worked in the period.

We imagine that the producers of machines also face schedules of prices q_T, given by formula (20). The shorter the time of delivery, the higher is the price they get.[1] We assume (for simplicity) that the producers will deliver *all* machines contracted for today at the same point of time T, and further that they receive their payment today, while they themselves have to pay their workers every day. The discounted net income of a contract may then be written :

$$\pi(N, T) = q_T\Psi(NT, T) - \int_0^T wNe^{-r\tau}d\tau. \tag{23}$$

If producers maximize π, we get, through the conditions $\pi_N' = 0$ and $\pi_T' = 0$ (and using 20), N and T as functions of q (the starting-point of the curve q_T). From (22) we then get the number of machines supplied, Q^S, as a function of q. Alternatively we may

[1] Cf. the famous 'wine-example' of Wicksell. Here the producers of wine get a higher price the longer they wait.

envisage the producers, expecting to be able repeatedly to make the same contract, as maximizing

$$\pi = \sum_{i=0}^{\infty} q_T \Psi(NT, T) e^{-riT} - \int_0^{\infty} wNe^{-r\tau} d\tau \tag{24}$$

$$= q_T \Psi(NT, T) \frac{1}{1 - e^{-rT}} - wN\frac{1}{r}.$$

To get further in our analysis we will work with a specific form of the production function (22). We assume :

$$Y = \Psi(NT, T) = AN^{\gamma}T^{\delta}, \text{ where } 0 < \gamma < 1, \text{ and } \delta > 1. \tag{25}$$

This function implies that the shorter the period of construction (T) for a given amount of machines contracted, the higher will be the required number of man-hours (NT). If T increases one per cent, Y increases more than one per cent (with N constant). (This will, I think, be relevant in many cases.) Technically a long period of construction is thus favourable for the capital-producers. On the other hand, the longer the period of construction, the lower will be the price q_T.

If, in this case, (24) is maximized, we can, in the way suggested above, deduce Q^S as a function of q. Q^S increases with q, the elasticity being a constant :

$$e_{Q^s, q} = \frac{\gamma}{\gamma - 1}. \tag{26}$$

Further, T depends on r and h, but *not* on q. The period of construction thus depends on the *form* of the q_T-schedule (and of 25), but not on the *level* of this schedule. We also get the result (26) when (23) and not (24) is maximized. (In this case the period of construction may be somewhat longer.)

If we imagine that the machine producers receive their payments evenly throughout the period of production, instead of (24) we get $\pi^* = \frac{1}{r}\left(q_T \Psi(NT, T)\frac{1}{T} - wN\right)$. Assuming that π^* is maximized we can find an explicit expression for T ; namely :

$$T = \frac{\delta - 1}{r + h}. \tag{27}$$

We find, as we should expect, that the delivery period will be shorter, the higher is the sum $(r + h)$. (Cf. the discussion about the form of the curve q_T.) Further we find, as we should expect, that a higher value of δ will (*ceteris paribus*) result in a longer period of construction.

Our description of the supply side of our market for capital goods may now be summed up : If a certain schedule of prices, $q_T(q)$ given by (20), is announced, the producers will want to deliver a certain number of machines, $Q^S(q)$, with a certain time of delivery. The latter depends on r and h, and on the parameters of their production function, cf. (27).

$Q^D(q)$ and $Q^S(q)$ intersect, we assume. The value of q in the intersection point we denote $\bar{q}$. Thus if the schedule $q_T(\bar{q})$ is announced, users and producers will agree upon the number of new machines to be ordered. As to the time of delivery, we assume that the users will not object to the value of T proposed by the producers (e.g. given by (27)).[1]

This last-mentioned assumption, that the users will not object to the time of delivery, $\bar{T}$, which the producers propose, may need further justification. It is mentioned that, because of the form of our q_T-curves (20), the users of capital are indifferent as to the time of delivery for the marginal machine they order. But if they wait $\bar{T}$ time units for all units of capital they demand $(K(\bar{q}) - K_0)$, we may say that they are first fully adjusted from that point of time on. Before this time (for $T < \bar{T}$), the level of $\phi_K{}'$ is a higher one, and we might think that the discounted calculated income of a new machine will exceed $q_T(\bar{q})$. We may, nevertheless, assume that the users will not try to bid up the price for shorter deliveries (than $\bar{T}$) in order to increase their stock of capital before $\bar{T}$, because they then simultaneously take a value loss into account. If they consider purchasing a machine delivered before T at a price higher than $q_T(\bar{q})$, they do not believe that they can sell it at $\bar{T}$ for a higher price than $q_T(\bar{q})$.

In this analysis it is assumed that the intersection point of Q^D and Q^S will be realized, and I am not going to examine more concretely which sort of mechanism may bring about such an adjustment. One often does the same in ordinary static market-theory when one assumes that the intersection of the demand and supply curves will be achieved. It should be noted, however, that if one says that the adjustment occurs as though a market administrator found the equilibrium-price by announcing different provisional prices, he would here be announcing price-curves of the type (20).

[1] We may, alternatively, try to carry out our analysis 'the other way round'. We may deduce schedules $q_T(q)$ which make the *producers* indifferent as to the time of delivery (these schedules will depend upon their production function) and assume that the *users*, taking these schedules as data, will propose the time of delivery.

In some respects the two models may give different conclusions. In Model 1 we assumed that the price of a machine ready for immediate delivery, q, was so adjusted that the excess demand for capital would always be zero. At this point, q would (except in a special case) be higher than at the point where Q^D and Q^S intersected. Our second model thus gives us a lower price for immediate delivery than our first model does. The explanation is that in our second model those who offer old machines, ready for immediate delivery, have to compete with producers who offer machines for delivery later on.[1]

Effects of a Shift in the Rate of Interest

If we get a reduction in r, the curve $Q^D(q)$ undergoes a shift upward, cf. (21). (The elasticity of $(Q^D + K_0)$ with respect to r, q being constant, is $\frac{(1-\alpha)}{(\alpha+\beta-1)}\frac{r}{(r+h)}$). This will (*ceteris paribus*) tend to increase the number of machines ordered. The reduction in r will, however, also affect Q^S. Producers will increase N and T, and so cause a downward shift in $Q^S(q)$. The result is an increase in the number of machines contracted for.

The fact that more machines are ordered, tends (*ceteris paribus*) to increase investment — the number of machines produced per unit of time. On the other hand, a longer period of construction will represent a tendency towards decreasing investment. To investigate the net effect we take the elasticity of $\frac{1}{T}\Psi(NT,\, T)$ with respect to r, and get

$$e_{\dot{K},\,r} = \gamma e_{N,\,r} + (\delta - 1)e_{T,\,r}.$$

Because both $e_{N,\,r}$ and $e_{T,\,r}$ are negative, $\dot{K}$ will rise when r is reduced.

If a reduction in r occurs at a point of time t_1, employment will be increased in the machine-producing industry. The users of capital will, however, not change their input of N before the point of time $t_1 + T$. At that point they increase both K and N, if the factors of production are complementary. If p rises at point of time t_1, the capital users will immediately increase N (cf. (17)); thus $\frac{N}{K}$ shifts upward. Also $Q^D(q)$ will shift upward at t_1, causing both q and the

[1] The second model will often, I think, tend to give a relatively more stable time curve for q. But a high value of h may weaken this tendency. For instance, the price of old ships, ready for delivery today, may rise violently with strong increases in p which are expected to be temporary (say, when the Suez canal is expected to be blocked for a limited time).

number of machines ordered to increase. T, which is independent of q, and of p, remains unaffected. Investment will thus increase.

This analysis is partial. The assumptions that w, p and r are constants may therefore be justified. When considering a whole country, we still have to explain not only how much more capital entrepreneurs will want to take into use, but also how rapidly they do so. In this respect the analyses above are relevant, and may perhaps give useful suggestions. But in more global analyses we cannot, of course, discuss the effects of *partial* shifts in r, p, etc., in the same easy way.

Chapter 10

CAPITAL ACCUMULATION AND ECONOMIC GROWTH[1, 2]

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I. INTRODUCTION

A THEORETICAL model consists of certain hypotheses concerning the causal inter-relationship between various magnitudes or forces and the sequence in which they react on each other. We all agree that the basic requirement of any model is that it should be capable of explaining the characteristic features of the economic process as we find them in reality. It is no good starting off a model with the kind of abstraction which initially excludes the influence of forces which are mainly responsible for the behaviour of the economic variables under investigation; and upon finding that the theory leads to results contrary to what we observe in reality, attributing this contrary movement to the compensating (or more than compensating) influence of residual factors that have been assumed away in the model. In dealing with capital accumulation and economic growth, we are only too apt to begin by assuming a 'given state of knowledge' (that is to say, absence of technical progress) and the absence of 'uncertainty', and content ourselves with saying that these two factors — technical progress and uncertainty — must have been responsible for the difference between theoretical expectation and the recorded facts of experience. The interpretative value of this kind of theory must of necessity be extremely small.

Any theory must necessarily be based on abstractions; but the type of abstraction chosen cannot be decided in a vacuum: it must be appropriate to the characteristic features of the economic process

[1] Editor's footnote: Mr. Kaldor's paper as printed here represents an extended written version of an address delivered by him orally to the conference in accordance with prior arrangement made with the I.E.A. In the subsequent discussion the members of the Round Table did not have the present text in their hands.

[2] The author is indebted to Mr. L. Pasinetti and Mr. F. H. Hahn for assistance in setting out the models in algebraic form.

as recorded by experience. Hence the theorist, in choosing a particular theoretical approach, ought to start off with a summary of the facts which he regards as relevant to his problem. Since facts, as recorded by statisticians, are always subject to numerous snags and qualifications, and for that reason are incapable of being accurately summarized, the theorist, in my view, should be free to start off with a 'stylized' view of the facts — i.e. concentrate on broad tendencies, ignoring individual detail, and proceed on the 'as if' method, i.e. construct a hypothesis that could account for these 'stylized' facts, without necessarily committing himself on the historical accuracy, or sufficiency, of the facts or tendencies thus summarized.

As regards the process of economic change and development in capitalist societies, I suggest the following 'stylized facts' as a starting-point for the construction of theoretical models :

(1) The continued growth in the aggregate volume of production and in the productivity of labour at a steady trend rate ; no recorded tendency for a *falling* rate of growth of productivity.

(2) A continued increase in the amount of capital per worker, whatever statistical measure of 'capital' is chosen in this connection.

(3) A steady rate of profit on capital, at least in the 'developed' capitalist societies ; this rate of profit being substantially higher than the 'pure' long-term rate of interest as shown by the yield of gilt-edged bonds. According to Phelps Brown and Weber [1] the rate of profit in the United Kingdom was remarkably steady around 10½ per cent in the period 1870–1914, the annual variations being within 9½-11½ per cent. A similar long-period steadiness, according to some authorities, has shown itself in the United States.

(4) Steady capital-output ratios over long periods ; at least there are no clear long-term trends, either rising or falling, if differences in the degree of utilization of capacity are allowed for. This implies, or reflects, the near-identity in the percentage rates of growth of production and of the capital stock — i.e. that for the economy as a whole, and over longer periods, income and capital tend to grow at the same rate.

(5) A high correlation between the share of profits in income and the share of investment in output ; a steady share of profits (and of wages) in societies and/or in periods in which the investment coefficient (the share of investment in output) is constant. For example, Phelps Brown and Weber found long-term steadiness in the investment coefficient, the profit share and the share of wages in the U.K., combined with a high degree of correlation in the (appreci-

[1] *Economic Journal*, 1953, pp. 263-88.

able) short period fluctuations of these magnitudes.[1] The steadiness in the *share* of wages implies, of course, a rate of increase in real wages that is proportionate to the rate of growth of (average) productivity.

(6) Finally, there are appreciable differences in the *rate* of growth of labour productivity and of total output in different societies, the range of variation (in the fast-growing economies) being of the order of 2-5 per cent. These are associated with corresponding variations in the investment coefficient, and in the profit share, but the above propositions concerning the constancy of relative shares and of the capital-output ratio are applicable to countries with differing rates of growth.

None of these 'facts' can be plausibly 'explained' by the theoretical constructions of neo-classical theory. On the basis of the marginal productivity theory, and the capital theory of Böhm-Bawerk and followers, one would expect a continued *fall* in the rate of profit with capital accumulation, and not a steady rate of profit. (In this respect classical and neo-classical theory, arguing on different grounds, come to the same conclusion — Adam Smith, Ricardo, Marx, alike with Böhm-Bawerk and Wicksell, predicted a steady fall in the rate of profit with economic progress.) Similarly, on the basis of the neo-classical approach, one expects diminishing returns to capital accumulation which implies a steady *rise* in the capital-output ratio *pari passu* with the rise in the capital-labour ratio; and a diminishing rate of growth in the productivity of labour at any given ratio of investment to output (or savings to income). Finally, the fluctuations in the *share* of profits that are associated with fluctuations in the rate of investment cannot be accounted for at all on the basis of the marginal productivity theory — if we assume, as I believe we must, that the fluctuations in the level of investment are the causal factor, and the fluctuations in the share of profits consequential, rather than the other way round.

My purpose here is to present a model of income distribution and capital accumulation which is capable of explaining at least some of these 'stylized' facts. It differs from the prevailing approach to problems of capital accumulation in that it has more affinities with the classical approach of Ricardo and Marx, and also with the general equilibrium model of von Neumann, than with the neo-classical models of Böhm-Bawerk and Wicksell; or with the theories which start off with the Cobb-Douglas type of production function. It differs from the classical models in that it embodies the basic ideas of the Keynesian theory of income generation, and it takes the well-known 'dynamic equation' of Harrod and Domar as its starting-point.

[1] *Op. cit.* Fig. 7.

II. THE CHARACTERISTIC FEATURES OF THE CLASSICAL APPROACH

The peculiarity of classical models as against the neo-classical theories is that they treat capital and labour as if they were complementary factors rather than competitive or substitute factors. Of course Ricardo was well aware that the use of capital is not only complementary to labour but also a substitute to labour — hence the famous 'Ricardo effect'.[1] This demonstrates that with a rise in wages more machinery will tend to be employed per unit of labour, *because* the price of machinery will fall relatively to labour with any rise in the share of the produce going to labour — but he did not accord this substitution-aspect any major rôle in his distribution or growth theory. As far as his distribution theory is concerned he treated the amount of capital per unit of labour as something given for each industry (and similarly, the distribution of labour between different industries as given by the 'structural requirements' of the system). He solved the problem of distribution between wages and profits (after deduction of the share of rent which is determined quite independently of this division) by assuming that the amount going to one of these two factors, labour, is determined by its supply price, whereas the share of the other is residual — the share of profits is simply the difference between output per man (after deduction of rent) and wages per man, the latter being treated as constant, governed by the 'natural price' of labour at which alone the working population *can remain* stationary.

Since profits were assumed to be largely saved and invested, whilst wages are consumed, the share of profits in income also determines the share of investment in total production, and the rate of accumulation of capital. The rate of accumulation of capital in turn determines the rate of increase in the employment of labour (since employment was assumed to increase at the same rate as capital, there was no scope for any consequential change in the amount of capital per unit of labour) without enquiring very closely where this additional labour comes from. The model is consistent with the assumption that there is an unlimited labour reserve, say, in the form of surplus population in an under-developed country (the assumption favoured by Marx) or with assuming that the rate of increase in population is itself governed by the rate of growth in the demand for labour (the assumption favoured by Ricardo).

[1] *Principles*, ch. i, sec. v.

Von Neumann's general equilibrium model,[1] though on a very different level of sophistication, explicitly allowing for a choice of processes in the production of each commodity, and abstracting from diminishing returns due to the scarcity of natural resources to which Ricardo accorded such a major rôle, is really a variant of the classical approach of Ricardo and Marx. Von Neumann similarly assumes that labour can be expanded in unlimited quantities at a real wage determined by the cost of subsistence of the labourers, and that profits are entirely saved and re-invested. These two assumptions enable him to treat the economic problem as a completely circular process, where the outputs of productive processes are simultaneously the inputs of the productive processes of the following period ; this is achieved by treating not labour, but the commodities consumed by labour, as the inputs of the productive processes, and by treating the surviving durable equipment as part of the outputs, as well as of the inputs, of the processes of unit length. Von Neumann is concerned to show that on these assumptions an equilibrium of balanced growth always exists, characterized by the equi-proportionate expansion in the production of *all* commodities with positive prices : and that this rate of expansion (under perfect competition and constant returns to scale for each process) will be the maximum attainable under the given 'technical possibilities' (the real wage forming one of the given 'technical possibilities'), and will be equal to the rate of profit (= rate of interest) earned in each of the processes actually used.[2]

The celebrated Harrod-Domar equation can be applied to the Ricardian model and the von Neumann model as well as to other models.[3] Though it can be interpreted in many ways (according to which of the factors one treats as a dependent and which as an independent variable) it is fundamentally a formula for translating the

[1] *Review of Economic Studies*, 1945–1946 ; originally prepared for a Princeton mathematical seminar in 1932.

[2] Von Neumann was only concerned with demonstrating the *existence* of such an equilibrium solution. Later Solow and Samuelson (*Econometrica*, 1953) have shown that on certain further assumptions this solution will be stable both 'in the large' and 'in the small' — i.e. the balanced growth equilibrium will be gradually approached from any given set of initial conditions ; and it will restore itself if it is disturbed for any reason.

[3] In von Neumann's formulation, where the surviving equipment at the end of each period is treated as a part of the output, v is $1/1+g$, when Y is defined as the gross output of the period (since then K_t and Y_{t-1} are identical) whilst s is unity if Y is defined as the *net* output (since the wage bill forms part of the commodities consumed in the process of production) so that the net-output/capital ratio is equal to g, the rate of growth of the capital stock. It is possible, however, within the framework of the model, to define Y in the usual way as being the sum of profits and wages — in which case the output-capital ratio (in a state of balanced growth) is identical with the net rate of expansion of the system multiplied by the ratio of Y (thus defined) to net output (i.e. the ratio by which the sum of wages and profits exceeds profits). Given a fixed real wage, and the possibility of expanding the rate of employment at the rate dictated by the requirements of a

share of savings (and investment) in income (s) into the resulting growth rate of capital (G_K), given the capital-output ratio, v $\left(\equiv\frac{K}{Y}\right)$[1]

$$G_K = \frac{s}{v}, \tag{1}$$

which can also be written

$$s = \frac{I}{Y} = G_K v. \tag{1a}$$

It further follows that when $s = \frac{P}{Y}$, i.e. all profits are saved and all wages are consumed,

$$\frac{P}{Y} = G_K \frac{K}{Y}.$$

But since

$$\frac{P}{K} = \frac{P}{Y} \cdot \frac{Y}{K}$$

$$\frac{P}{K} = G_K. \tag{2}$$

the rate of profit on capital is the same as the rate of growth of capital.

As far as Ricardo and von Neumann are concerned, this is really the end of the story, for they do not introduce any limit to the *speed* with which additional labour can be introduced into the system, so that the rate of growth of employment, and hence of income, is fully determined by the rate of growth of capital. Supposing, however, that even if the supply of labour can be increased to an indefinite extent *ultimately*, there is a maximum to the rate of increase of population and/or of employment per unit of time, determined by biological or institutional factors. Writing L for the quantity of employment, this gives us another equation

$$G_n = l, \text{ where } l = \frac{1}{L} \cdot \frac{dL}{dt}. \tag{3}$$

The Ricardo-Marx-von Neumann model clearly does not work when $G_K > G_n$ since in that case the rate of growth of production cannot be determined by G_K alone.

In a progressive economy the labour potential increases, however, not only on account of the rise in numbers, but also on account of

balanced-growth economy, the ratio of wages to profits is itself determined by the relative input-intensities of labour and non-wage commodities when (at the given wage and with the given range of available processes) the rate of expansion of the system is maximized.

[1] Time subscripts are omitted, except in the formal presentation of the models.

the rise in the productivity of labour due to technical progress. Hence, allowing for technical progress,

$$G_n = l + t, \text{ where } t = \frac{1}{Y/L} \cdot \frac{d(Y/L)}{dt}, \tag{3a}$$

which is Harrod's formula for the 'natural' rate of growth.

Harrod realized that balanced-growth equilibrium is only conceivable when his 'warranted rate of growth' equals the 'natural rate',

$$G_K = G_n,$$

in other words $$\frac{s}{v} = l + t.$$

Since he assumed, however, that s, v, l and t are all independently given and invariant in relation to each other, such an equality, on his theory, could only be the result of a fortunate accident. Moreover, he thought that any discrepancy between $\frac{s}{v}$ and $(l+t)$ must set up cumulative forces of disequilibrium, so that a moving equilibrium of steady growth, even if momentarily attained, is necessarily unstable.

The problem takes on an entirely different aspect, however, once we recognize (as we must) that these variables are not mutually invariant, but that there are certain inter-relationships between them. Thus, as will be shown, the proportion of income saved s, is by no means independent of $(l+t)$; nor is the rate of increase in productivity, t, independent of the rate of capital accumulation, $\frac{s}{v}$.[1]

III. THE NATURE OF GROWTH EQUILIBRIA

In order to exhibit the rôle of these various factors it is best to start from a model based on a number of artificial assumptions which together produce the simplest solution to the problem of growth-equilibrium. We shall afterwards remove these assumptions one by one (with the exception of the first assumption listed below) in the reverse order in which they are presented here. The six critical assumptions of our 'basic model' are:

(1) Constant returns to scale in any particular process of production; natural environment does not impose any limitation to

[1] In the above equation, in deference to the generally accepted use of symbols, we have denoted the rate of growth of labour by l and the rate of growth of output per man by t. In the rest of this paper, however, we shall denote the *maximum* rate of population growth by λ, and the rate of growth of productivity by G_o; reserving the letter t to denote time.

expansion (i.e. there are two factors of production, Capital and Labour (K and L), and two kinds of income, Profits and Wages (P and W)).

(2) The absence of technical progress — i.e. the function relating the output of various commodities to the input-coefficients of production remains unchanged over time.

(3) General rule of competition: the prices of commodities in relation to the prime costs of production settle at the point where the market is cleared. Capital earns the same rate of profit, and labour the same rate of wages, in all employments.

(4) All profits are saved and all wages are consumed; the division of output between equipment goods (or 'input goods') and wage goods (consumption goods) is the same as the division of income between Profits and Wages.

(5) There is strict complementarity between Capital and Labour (or commodity-inputs and labour-inputs) in the production of both equipment goods and wage goods; there is therefore a single kind of 'equipment good' for the production of each wage good, and the different kinds of wage goods are also complementary in consumption.

(6) There is an unlimited supply of labour at a constant wage in terms of wage goods.[1]

Under these assumptions the rate of growth of the capital stock, G_K, will govern the rate of growth of the economy, G_Y; and G_K in turn depends on the proportion of output saved, s, and the capital-output ratio, v. The proportion of output saved is determined by the condition that the wage rate cannot fall below a certain minimum, determined by the cost of subsistence,

$$w \equiv w_{\min} \tag{4}$$

so that the excess of output per head over the subsistence wage alone determines the share of profits. Output per head (O), the capital-output ratio (v), and hence capital per head, are given technical constants; and in addition the total amount of capital at some arbitrary point of time, $t=0$, is taken as given.

These assumptions yield a model which can be formally stated as follows. Using our previously introduced notation[2] and denoting

[1] These six assumptions are identical (except for (5)) with those underlying Neumann's model; they are substantially the same as those implicit in Ricardo's theory (except for (1)); and Marx's theory (except of course in its 'dynamic' aspect, assumptions (2) and possibly (5)).

[2] This notation may be summarized as follows:

$$G_K = \frac{dK}{dt}\frac{1}{K} \qquad G_Y = \frac{dY}{dt}\frac{1}{Y} \qquad v = \frac{K}{Y} \qquad O = \frac{Y}{L}$$

and the symbols K, Y, L, w and s represent the stock of capital, output (or income), labour employed, wage per worker, and the proportion of income saved respectively.

output per worker by O, we obtain a system of six relationships, of which four represent assumptions, one is a definitional identity and one equation the equilibrium condition.

$$\left.\begin{aligned} O(t) &= \bar{O} \\ v(t) &= \bar{v} \\ w(t) &= w_{\min} \\ s(t) &= \frac{P(t)}{Y(t)} \end{aligned}\right\} \text{for all } t \geqslant 0 \qquad \begin{aligned} &(\text{i}) \\ &(\text{ii}) \\ &(\text{iii}) \\ &(\text{iv}) \end{aligned}$$

$$P(t) \equiv Y(t) - w(t)L(t) \qquad (\text{v})$$

$$s(t)Y(t) = \frac{dK(t)}{dt} \text{ for all } t \geqslant 0 \qquad (\text{vi})$$

which are sufficient to determine the six basic variables $O(t)$, $v(t)$, $s(t)$, $P(t)$, $Y(t)$ and $w(t)$ given the initial values. From (vi) and (ii) we have

$$G_Y = \frac{s(t)}{\bar{v}} \text{ or } \bar{v}G_Y = s(t)$$

From (v) it follows

$$\frac{P(t)}{Y(t)} = \left[1 - \frac{w_{\min}}{\bar{O}}\right]$$

and hence the share of profit is independent of t. And so, by (iv), $s(t)$ is also independent of t, and hence

$$G_K = \frac{s}{\bar{v}}$$

$$G_K = G_Y$$

$$\frac{P}{K} = G_K$$

$$\frac{P}{Y} = G_K \bar{v} \qquad (\text{I})$$

IV. FULL EMPLOYMENT GROWTH

The first modification I shall introduce is the removal of assumption (6), that of an unlimited supply of labour. We may suppose that there is a certain maximum rate of population growth, λ, determined by fertility rates ; so that (abstracting from technical progress) this rate determines the long-run 'natural rate of growth'. Hence

$$G_n = \lambda.$$

If we suppose, further, that initially

$$G_K > G_n,$$

i.e. the rate of capital accumulation, as determined by the conditions of our previous model, exceeds the maximum rate of growth of population, the economy can only grow at the rate G_K as long as there are reserves of unemployed labour to draw upon. But just because the economy grows at a higher rate than λ, sooner or later capital accumulation must overtake the labour supply. According to Marx this is precisely the situation which leads to a crisis. When the labour reserves are exhausted, the demand for labour will exceed (or tend to exceed) the supply of labour, since the amount of capital seeking profitable employment will be greater than the number of labourers available to employ them with. Owing to the competition between capitalists, this will cause wages to rise and profits to be wiped out, until, in consequence, capital accumulation is reduced sufficiently to restore the labour reserve and thus restore profits.

However, there is no inherent reason why this situation should involve a crisis ; nor does it follow from the assumptions that the maintenance of accumulation requires the continued existence of a labour reserve. Indeed there is no reason why this situation should not result in a neat balanced-growth equilibrium with a higher rate of wages and a lower share of profits, and with a correspondingly lower rate of capital accumulation that would no longer exceed, but be equal to, the rate of increase in the supply of labour. All that is necessary is to bear in mind that every increase in wages (in terms of commodities) lowers the share of profits in income, and every reduction in the share of profits lowers the rate of accumulation of capital and hence *the rate of increase* in the demand for labour. Hence the situation will lead to a balanced-growth equilibrium in which employment at some arbitrary point of time $t = 0$ is taken as given by the size of the working population at that point of time, and where the rate of growth of population λ is also taken as given.

This gives us an alternative model of seven relationships of which four define the assumptions, one is an identity as before and two are equilibrium conditions. Using, in addition, the notation $L^*(t)$ for the maximum amount of labour available at time t, the relationships are as follows :

$$\left.\begin{aligned} L^*(t) &= L^*_{(0)}e^{\lambda t} && \text{(i)} \\ v(t) &= \bar{v} && \text{(ii)} \\ O(t) &= \bar{O} && \text{(iii)} \\ s(t) &= \frac{P(t)}{Y(t)} && \text{(iv)} \end{aligned}\right\}\text{for all } t \geqslant 0$$

$$P(t) \equiv Y(t) - w(t)L(t) \qquad \text{(v)}$$

$$\left.\begin{aligned} s(t)Y(t) &= \frac{dK}{dt} && \text{(vi)} \\ L(t) &= L^*(t) && \text{(vii)} \end{aligned}\right\}\text{for all } t \geqslant 0$$

subject to the inequality

$$w(t) \geqslant w_{\min}$$

which are sufficient to determine the seven basic variables $O(t)$, $v(t)$, $s(t)$, $P(t)$, $Y(t)$, $w(t)$ and $L(t)$, given the initial conditions.

It follows from (i) and (vii) that

$$G_Y = \lambda.$$

From (vi), $s(t) = \lambda v(t)$ and so, by (i) and (ii), $s(t)$ is independent of t. Hence by (iv)

$$G_K = \frac{s}{\bar{v}}$$

$$G_K = G_y$$

$$\frac{P}{Y} = \lambda \bar{v}$$

$$\frac{P}{K} = \lambda$$

Also, by (v), $\qquad w(t) = (1 - \lambda\bar{v})\bar{O},$

subject to the inequality stated. (II)

The difference between this model and the previous one is that while in both, output-per-man and capital-per-man are constant (over time), in this model the rate of profit on capital and the share of profit in income (given v, which is here as a technical constant) are uniquely determined by λ, the population growth rate, which on our present assumptions will alone determine the uniform expansion rate of the economy. There is an equilibrium wage, w, which will exceed the subsistence wage, $w_{\min}$, by the amount necessary to reduce the share of profits to λv. But despite the similarities, this second model is the inverse of the Ricardian (or Marxian) one; for here it is not profits which form a residual after deducting subsistence-wages, but wages form the residual share after deducting profits, the amount of profits being determined independently by the requirements of the (extraneously given) balanced growth rate.[1]

Ricardo did say, in various places scattered around in the *Principles*, that as capital accumulation runs ahead of population, or the reverse, wages will rise above the 'natural price of labour' or may fall below it. But he never drew the immanent conclusion (though in several places he seemed almost on the point of saying it) that the rise or fall in wages resulting from excessive or insufficient rates of accumulation *will itself change the rate of accumulation of capital through changing the profit share*, and thereby provides a mechanism for keeping the rate of accumulation of capital in step with the rate of increase in the labour supply — i.e. that there is an 'equilibrium' level of wages which maintains the increase in the demand for labour in step with the increase in supply. (Had he said so, with some emphasis, one cannot help feeling that the subsequent development of economics, both Marxist and orthodox, might have taken a rather different turn.)

Marx's view that where excessive accumulation leads to a crisis due to the scarcity of labour there is nothing to stop wages from rising until profits are wiped out altogether, clearly assumes a *constant* supply of labour over time. If population is rising, profits cannot fall below the level which provides for a rate of accumulation that corresponds to the rate of growth in the supply of labour; and once 'full employment' has been reached (i.e. the 'reserve army' is exhausted) there is no reason why wages should not settle down to a new equilibrium level, divorced from the cost of subsistence of labour.

There is one other important assumption implicit in this, and in the other growth models, which may be conveniently introduced at this stage. In a capitalist economy continued investment and

[1] This situation is incompatible also with von Neumann's model, which, as mentioned before, implicitly assumes that the effective supply of labour can be increased at the required growth rate, whatever that rate is. But if one introduced labour explicitly as one of the 'commodities' into the von Neumann model (instead of the goods consumed by labour) and assumed that the supply of labour was growing at some autonomous rate that was lower than the maximum potential expansion rate of commodities other than labour, the same result would be reached. For then the equilibrium price system which equalized the rate of profit earned in all the 'chosen' processes would be the one which made the price of labour in terms of other commodities such as to reduce the rate of profit earned in the production of commodities (other than labour) to the expansion rate of labour.

accumulation presupposes that the rate of profit is high enough (in the words of Ricardo) to afford more than the minimum necessary compensation to the capitalists 'for their trouble, for the risk which they must necessarily encounter in employing their capital productively'.[1] Hence growth-equilibrium is subject to a further condition which can be written in the form

$$\frac{P}{K} > r + \rho, \tag{5}$$

i.e. the rate of profit as determined by the model (under our present assumption by λ alone) cannot be less than the sum of the 'pure' rate of interest on financial assets of prime security, and the additional premium required for the risks involved in productive employments of wealth.

We know, since Keynes, that there is a minimum below which the pure long-term rate of interest cannot fall, and that this is determined by the minimum necessary compensation for the illiquidity-risk entailed in holding long-term bonds as against cash (or other short-term financial assets which are close substitutes for cash). We also know (though this has received far less emphasis in the literature) that the risks (whether illiquidity risks or other risks) associated with the direct investment of capital in business ventures are quantitatively far more important than the risks entailed in holding long-term financial assets of prime security. (The rate of profit on business investments in fixed capital [in plant and equipment] in the U.S., for example, is generally taken to be 20 per cent gross, or say 10 per cent net, of taxation, when the 'pure' long-term rate of interest is around 4 per cent.)

The (expected) marginal return on investments in circulating capital (which, by universal convention, are treated as part of the 'liquid assets' of a business) is much more in line with the money rates of interest, though here also, the expected return is likely to be appreciably higher than the (pure) short-term rate of interest. It is indeed highly unlikely that in an economy *without* technical progress, and where *all* profits are saved and re-invested, the rate of profit (as determined by population growth) could be anywhere near high enough to satisfy the above condition. If it is not, there cannot be a moving equilibrium of growth, though this does not mean that the economy will lapse into perpetual stagnation. Accumulation could still take place in periodic spurts, giving rise to a higher-than-trend rate of growth for a limited period.

[1] *Principles*, Sraffa edition, p. 122.

We must now proceed with the relaxation of the various simplifying assumptions made. As we shall see, until we come to technical progress, none of these introduces a vital difference to our results.

V. NEO-CLASSICAL GROWTH

We can allow for variable proportions, instead of strict complementarity, between capital and labour, by postulating that there is a choice of processes of production involving differing quantities of capital per man (i.e. a differing ratio between 'commodities' and 'labour' as inputs). Thus output per man, O ($O \equiv Y/L$), will be a function of K/L, capital per man, the increase in the former being less than proportionate to the latter, if the production function for labour and capital together is homogeneous and linear. Hence

$$O \equiv Y/L = f_1(K/L), \text{ where } f_1' > 0, f_1'' < 0. \tag{6}$$

Assuming that each entrepreneur at any one time has a limited amount of capital at his disposal, the amount of capital per man employed will be such as to maximize the rate of profit ; and this optimum amount of capital per man will be all the greater the higher are wages in terms of commodities, hence

$$K/L = f_2(w), \text{ where } f_2' > 0, f_2'' < 0. \tag{7}$$

(6) in combination with (7) also implies that the capital-output ratio in the 'chosen' process will be all the greater, the higher the rate of wages, hence

$$v \equiv \frac{K}{Y} = f_3(w), \text{ where } f_3' > 0, f_3'' < 0. \tag{8}$$

Further, it also follows that output per man will be the greater the higher the capital-output ratio

$$O = f_4(v), \text{ where } f_4' > 0, f_4'' < 0. \tag{9}$$

Hence as wages rise (with the approach to full employment and the slowing down of the rate of accumulation) v will rise as well ; this in turn will increase the share of investment in output $\left(\frac{I}{Y}\right)$ at any *given* rate of growth of output, and hence the share of profits. It *may* also slow down the rise in wages in terms of commodities, but since the rise in v will increase output per man, as well as the share of profits, this does not necessarily follow. However, on the assumption of diminishing returns (which, as we shall argue later, comes to much the same as the assumption that there is no technical progress) $f_1'' < 0$, the rise in the investment ratio and in the share of profits will not be sufficient to prevent a continued fall in the rate of growth of capital with the continued increase in v. Hence this process of adopting more labour-saving techniques by increasing capital per head will come to an end when the rate of growth of capital declines sufficiently to approach the rate of increase in the supply of labour, λ. From then onwards the system will regain a balanced-growth equilibrium with unchanging techniques and capital per head and proceeding at the uniform expansion rate λ.

Thus the introduction of a choice of processes permitting the substitution of capital for labour will mean that there will be an intermediate stage between the equilibrium of Model I (where G_Y was determined by G_K) and of Model II (where G_Y was determined by G_n, and G_K by G_Y), characterized by the condition

$$G_K > G_Y > G_n,$$

i.e. where the actual rate of growth is greater than the natural rate, as determined by population growth, and lower than the rate of capital accumulation. In other words, the rate of growth of capital will be higher than that of output, and the latter will be declining. The difference thus introduced is best shown in a diagram (Fig. 12)

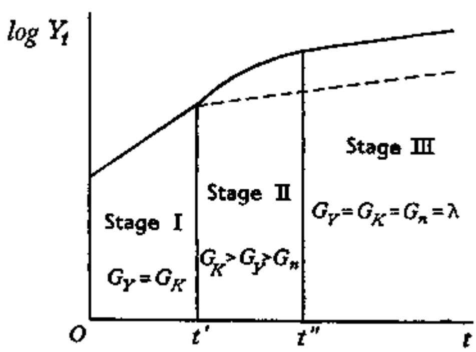

Fig. 12

where output (Y_t) is shown vertically (on a logarithmic scale) and time horizontally. Assuming that from $t = 0$ onwards the economy is in a growth equilibrium with unlimited supplies of labour with $G_Y = G_K$, G_K being determined by the ratio of savings to income when wages are at the minimum subsistence level; and assuming further that the labour reserves are exhausted at the point of time t', then, in the absence of a choice of 'techniques' of a more or less labour-saving character, wages will immediately rise to the point where the share of profits is cut down to the level where the rate of accumulation is brought down to $G_n = \lambda$ and the system attains a new balanced growth equilibrium at this lower rate. If we assume, however, that there are technical possibilities for increasing output per head by using more capital per unit of labour, the transition will be gradual. Wages will rise more gradually, and accumulation will be maintained (temporarily) at a higher rate, serving both the requirements of the growing working population and the increasing amount of capital per unit of labour. But since during this stage the rate of growth of production will be declining, and will be constantly smaller than the rate of capital accumulation, balanced-growth equilibrium will be regained at a certain point (shown by t'' in the diagram). This will occur when wages have risen to the point at which accumulation is brought down to the rate corresponding to the rate of growth of population, and from then onwards the economy will attain the same constant growth rate, determined by λ.[1]

Given the range of alternative processes represented by our f functions, it follows that there is a unique relationship between output per worker and the capital-output ratio (as stated in equation (9) above) and also between the *desired* capital-output ratio and the rate of profit on capital. Hence for balanced growth equilibria (where the actual capital-output ratio corresponds to the desired ratio) we have the further relationship

$$v = \phi\left(\frac{P}{K}\right), \text{ where } \phi' < 0,\ \phi'' > 0. \tag{8a}$$

Writing these relationships in this form, this model will be characterized by seven relationships, of which three are equilibrium conditions.

$$\left.\begin{array}{lr} L^*(t) = L^*(o)e^{\lambda t} & \\ O(t) = f(v(t)),\ f' > 0,\ f'' < 0 & \\ s(t) = \dfrac{P(t)}{Y(t)} & \end{array}\right\} \text{for all } t \geqslant 0 \qquad \begin{array}{r}\text{(i)}\\ \text{(ii)}\\ \text{(iii)}\end{array}$$

$$P(t) \equiv Y(t) - w(t)L(t) \tag{iv}$$

[1] The first of our three stages may be termed the 'classical' stage, the second the 'neo-classical' stage (since it will be characterized by rising capital per man, a rising capital-output ratio, and a declining rate of growth and profit) and the third stage, for reasons set out below, the 'Keynesian' stage.

$$\left.\begin{aligned} s(t)Y(t) &= \frac{dK(t)}{dt} \qquad &\text{(v)}\\ L(t) &= L^*(t) &\text{(vi)}\\ v(t) &= \phi\left(\frac{P(t)}{K(t)}\right) &\text{(vii)}\end{aligned}\right\}\text{for all } t \geqslant 0$$

where $\qquad \phi' < 0,\ \phi'' > 0$

subject to the inequalities

$$w(t) \geqslant w_{\min}$$

$$\frac{P(t)}{K(t)} \geqslant r + \rho$$

By the same argument as employed in Model II above it follows that

$$G_Y = \lambda$$

Hence by (v), $\frac{s(t)}{v(t)}$ is independent of t. By (iii) we have $\frac{P(t)}{Y(t)} = \lambda v(t)$ and so

$$\frac{P}{K} = \lambda$$

and using (vii) we obtain $\qquad \frac{P}{Y} = \lambda\phi(\lambda) \qquad$ (III)

As a comparison with the corresponding equations for Model II shows, the introduction of a 'production function' which makes the capital-output ratio dependent on the rate of profit will not affect the equilibrium growth-rate, or the rate of profit on capital. But it will have an influence on the share of profits, and hence on the savings coefficient, s, for any given rate of growth, since λ and $\phi(\lambda)$ are inversely related to one another: the higher the value of λ, the lower the equilibrium value $\phi(\lambda)$. In the special case where the function $\phi(\lambda)$ is one of constant unit elasticity (i.e. when doubling the rate of growth and the rate of profit involves halving the capital-output ratio, etc.) the investment coefficient, $\lambda\phi(\lambda)$, will be invariant with respect to any change in the rate of growth and the rate of profit on capital, and, in that sense, the share of profits and wages can be said to be uniquely determined by the coefficients of the production function. But the assumption of constant unit elasticity for the ϕ function is by no means implicit in the assumption of homogeneous and linear production functions, and indeed it cannot hold in all cases where there are limits to the extent to which any one factor can be dispensed with. If, in the relevant range, the elasticity of this function is appreciably smaller than one, the share of profit will predominantly depend on the rate of economic growth (and on the propensities to save out of profits and wages discussed below) and only to a minor extent on the technical factors, the marginal rates of substitution between capital and labour (which determine the elasticity of the ϕ function).[1]

VI. THE PROPENSITIES TO SAVE

We can now relax our fourth assumption, the one implicit in all 'classical' models, that there is no consumption out of profits and no saving out of wages. We can allow both for the fact that profits are a source of consumption expenditure and that wages may be a source of savings — provided that we assume that the proportion of profits saved is considerably greater than the proportion of wages (and other contractual incomes) saved.[2] This assumption can be well justified both by empirical evidence and by theoretical considerations. Thus, on U.S. data, gross savings out of gross (company) profits can be put at 70 per cent, whereas savings out of personal incomes (excluding unincorporated businesses) are only around 5 per cent. Statistical evidence from other countries yields very similar results. On theoretical grounds one can expect the propensity to save out of business profits to be greater than that of wage and salary incomes (i) because residual incomes are much more uncertain, and subject to considerable fluctuations, year by year; (ii) because the accumulation of capital by the owners of the individual firms is closely linked to the *growth* of the firms: since a firm's borrowing power is limited to some proportion of its equity capital, the growth of the latter is a necessary pre-condition of the growth in its scale of operations. Apart from this, it could be argued on Keynesian considerations that it is precisely this difference in savings-ratios which lends stability to a capitalist system, under full employment or near-full employment

[1] Empirical evidence, such as it is, lends little support to the supposition that the capital-output ratio is smaller in fast-growing economies than in slow-growing economies, or in economies where the amount of capital per head is relatively small as against those where it is large. But the reason for this, as we shall argue later, is not the lack of substitutability between capital and labour, but the unreality of the postulate of a ϕ function which abstracts from all technical progress.

[2] I am assuming here, purely for simplicity, that the savings functions for both profits and wages are linear (with a zero constant) so that the average and marginal propensities are identical. If this were not so, it would be the difference in marginal propensities which was critical to the theory.

conditions. For if these differences did *not* exist, any chance increase in demand which raised prices would bring about a cumulative tendency: a rise in prices is only capable of eliminating the disequilibrium in so far as the transfer of purchasing power from 'contractual' to 'residual' incomes which it represents reduces effective demand in real terms.

If we denote by α the proportion of profits saved and β the proportion of wages saved,

$$I = \alpha P + \beta W, \text{ where } 1 > \alpha > \beta > 0 \tag{10}$$

$$s = \frac{I}{Y} = (\alpha - \beta)\frac{P}{Y} + \beta \tag{10a}$$

and

$$\frac{P}{Y} = \frac{1}{\alpha - \beta}\,\frac{I}{Y} - \frac{\beta}{\alpha - \beta} \tag{10b}$$

If, in the first approximation, we assumed that βW is zero the equilibrium relationships will remain the same as in Model III, with the exception of (iii) which becomes

$$s(t) = \alpha\frac{P(t)}{Y(t)}.$$

This modification implies that in equilibrium

$$\frac{P}{K} = \frac{\lambda}{\alpha}$$

$$\frac{P}{Y} = \frac{\lambda}{\alpha}\cdot\phi\left(\frac{\lambda}{\alpha}\right). \tag{IV}$$

In other words, the rate of profit on capital will now exceed the rate of growth by the reciprocal of the proportion of profits saved. Similarly, the share of profit in income will also be raised, except in so far as the rise in $\frac{P}{K}$ will reduce v, and hence the investment-output ratio at any given rate of growth.

VII. COMPETITION AND FULL EMPLOYMENT

Before examining the implications of assumption (3), the general rule of competition, I should like to translate our results into terms that are in accord with the Keynesian techniques of analysis. So far we have assumed that the level of production at any one time is limited not by effective demand but by the scarcity of resources available; which meant in the case of Model I that it was limited by the amount of capital (i.e. physical capacity) and in the case of Model II by the available supply of labour. In the 'Keynesian' sense, therefore, the equilibrium in both cases is one of 'full employment'. This is ensured, in the case of Model I, through the assumption, implicit in the model, that it is the 'surplus' remaining after the payment of subsistence wages which determines the rate of accumulation. In the case of Model II, where investment demand per unit of time is independently determined by the accrual of new investment opportunities resulting from the given rate of increase in the labour supply, it is ensured through the fact that the level of wages in real terms, and thus the share of profits, is assumed to settle at the point where savings out of profits are just equal to the required rate of investment. This latter presumes in effect a 'Keynesian' model where investment is the independent variable, and savings are the dependent variable: but the process of adjustment is assumed to take place not in a Keynesian but in a classical manner through forces operating in the labour market. An excess of savings over investment manifests itself in an excess of the demand for labour over the supply of labour; this leads to a rise in wages which reduces profits, and thus savings, and hence diminishes the rate of increase in the demand for labour. There is therefore some particular real wage at which the rate of increase in the demand for labour, resulting from capital accumulation, keeps in step with the rate of increase in the supply of labour, and which therefore is alone capable of maintaining the labour market in equilibrium.

But we are not *obliged* to look upon the equilibrating mechanism in this way; we could equally describe the equilibrating process in the 'Keynesian' manner, through the forces of adjustment operating not in the labour market, but in the commodity markets. In the Keynesian system an excess in the demand for labour in the labour market can only cause a rise in *money* wages, not of *real* wages, since a rise in *money* wages, *ceteris paribus*, will raise monetary demand, and thus prices, in the same proportion. To explain movements in *real* wages (output per man being assumed as given) we need to turn to the commodity markets and examine the conditions of equilibrium for the demand and supply of commodities. It is the most significant feature of Keynes' theory to have shown that equilibrium between savings (*ex ante*) and investment (*ex ante*) is secured through forces operating in the commodity markets. When investment exceeds savings, the demand for commodities will exceed the supply. This will lead *either* to an expansion of supply (assuming the prevalence of 'Keynesian' unemployment and hence a state of affairs where production is less than the short-period maximum) *or* to a rise in prices relatively to costs (assuming 'full employment' in the Keynesian sense, i.e. that supply is limited by physical bottlenecks). In both cases an increase in the demand for commodities will lead to an increase in savings; in the first case, because savings are an increasing function of real income, at any *given* relationship of prices to costs (or of profits to wages); in the second case, because the rise in prices relative to costs implies a rise in profits and a fall in wages (in *real* terms) which increases savings. Keynes, in the *General Theory*, writing in the middle of the big slump of the 1930s, concentrated on the under-employment case, and conceived of the mechanism which equates savings with investment as one which operates through variations in the general level of employment. But in his previous book, *A Treatise on Money* (written in the late 1920s), he described essentially the same mechanism as determining the relationship of prices to costs, with output and employment as given.[1]

To illustrate the nature of this process and to analyse the conditions under which the forces equalizing savings and investment determine the price-cost relationship at full employment, rather than the level of employment at some *given* relationship of prices to costs, I should like to make use of the time-honoured device of the 'representative firm' which is assumed to behave like a small-scale replica of the economy as a whole. I shall assume, in other words, that variations in the output of the 'representative firm' reflect equivalent variations in total production, and that the firm employs a constant fraction of the total employed labour force.

I shall ignore falling average prime costs in the short period and shall assume that average and marginal prime costs are constant up to the point where the optimum utilization of capacity is reached and begin to rise afterwards, as shown by the curves *APC* and *MC* in Fig. 13. I shall assume that our representative firm is fully integrated vertically, so that its average and marginal prime costs consist only of labour cost. (The rate of money-wages is assumed to be given.) And I shall further assume, as is appropriate for a 'developed' economy under conditions of imperfect competition, that the effective bottleneck setting an upper limit to production is labour rather than physical capacity: there is more than enough capacity to employ the available labour force. Hence, since our firm accounts for a constant fraction of total employment, it cannot produce at a rate

[1] *A Treatise on Money* (London, 1930), vol. i, p. 139.

higher than that indicated by the full-employment position (as shown by the dashed line in Fig. 13.) [1]

Finally, I shall assume that whatever the state of demand, our firm will not be forced to reduce prices to the bare level of prime costs ; there is a certain *minimum margin of profit* which competition cannot succeed in eliminating. We can call this minimum profit margin the 'degree of monopoly', or the 'degree of market imperfection', remembering, however, that it does not necessarily *set* the price (in relation to costs), it merely sets a rock-bottom to prices. (In Fig. 13 the dot-and-dash line indicates the minimum price at the given level of prime cost per unit of output.) The greater the intensity of competition the lower will be this minimum margin of profit.

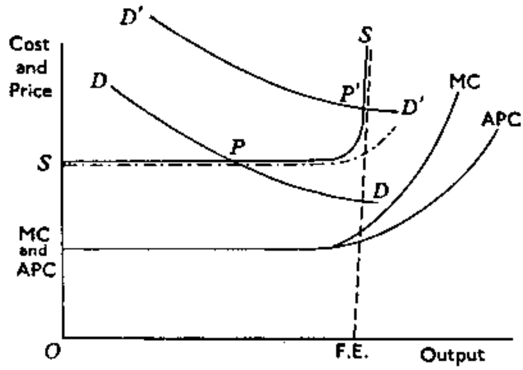

FIG. 13

The assumption that prices cannot fall below some minimum determined by the degree of market imperfection, and that production cannot exceed a certain maximum determined by full employment, yields a short-period supply curve (the curve *S*–*S* in Fig. 13) which exhibits the familiar reverse L-shaped feature : the curve is horizontal up to a certain point (when the supply price is set by the minimum profit margin) and well-nigh vertical afterwards (when production is limited by full employment).

We can now introduce the Keynesian demand function which shows the demand price for each level of output — i.e. it shows for

[1] The assumption that physical capacity is more than sufficient for the employment of the available labour force in 'developed' capitalist economies is empirically supported by the fact that even in times of very low unemployment, double or treble shift utilization of capacity is fairly rare. And it is the existence of considerable spare capacity under conditions of imperfect competition which alone explains the absence of diminishing productivity to labour with increasing employment in the short period, despite the co-existence of physical equipment of varying degrees of efficiency.

any particular output (and employment) that excess of price over prime cost which makes the effective demand in real terms equal to that output. (The excess of price over prime cost is of course the same thing, on our assumptions, as the share of profits in output.) Assuming that investment, I, is an independent variable invariant with respect to changes in output, this demand curve will be falling from left to right, much like the Marshallian demand curve, and its equation, according to the well-known multiplier formula, will be

$$D = \frac{1}{(\alpha - \beta)\frac{p-c}{p} + \beta} I, \tag{11}$$

where D represents aggregate demand in real terms, $\frac{p-c}{p}$ the margin of profit over selling price (which, for the representative firm, is the same as $\frac{P}{Y}$, the share of profits in income) and I the amount of investment (also in real terms), and α and β the coefficients of savings for profits and wages respectively. The higher is I, and the lower are the coefficients α and β, the higher the position of the curve; the greater the difference, $\alpha - \beta$, the greater elasticity of the curve. If $\beta = 0$, the curve approaches the *APC* curve asymptotically; if $\alpha = \beta$ the curve becomes a vertical straight line.

Depending on the relative position of the two curves, this intersection can yield either an under-employment equilibrium (when the demand curve cuts the supply curve in the horizontal segment of the latter, as shown by $D - D$, with the point of intersection P) or a full-employment equilibrium (as shown by $D' - D'$, with the point of intersection P'). In the former case the price-cost relationship (the distribution of income) will be independently given by the degree of market imperfection (marginal productivity plays no rôle in this case since the average productivity of labour is assumed to be constant) whilst the level of output is determined by the parameters of the demand function (the savings-investment relationship). In the latter case, output is independently given, and it is the price-cost relationship which will be determined by the demand function, i.e. by the savings-investment relationship.[1]

However, our demand curve has so far been based on the postulate that the rate of investment is invariant with respect to changes in

[1] It follows also that in so far as β (savings out of wages) is zero or negligible, under-employment equilibrium necessarily presupposes some degree of market imperfection; for if competition were perfect and the minimum profit margin were zero, the intersection of the demand curve with the supply curve would necessarily fall on the vertical section of the latter.

output. In fact, it is the rate of growth of output which governs investment demand ; and, in addition to the growth of output due to the natural rate of growth of the economy, investment in the short period will also vary with the change in output reflecting a change in the level of unemployment. Such 'induced' investment will only come into operation, however, when the degree of utilization of capacity permits a normal rate of profit to be earned ; in other words when receipts cover, or more than cover, *total* costs, including 'normal' profits on the capital invested.

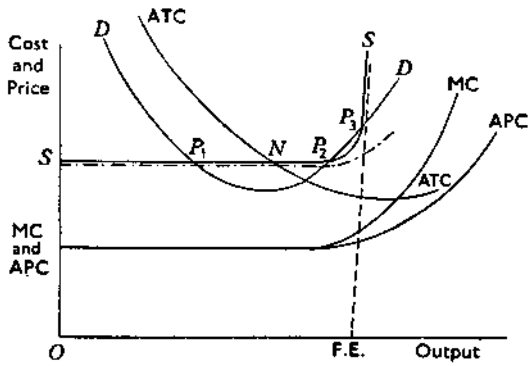

FIG. 14

In Fig. 14 the curve *ATC* indicates average total costs (including 'normal' profits) and the point *N* (where the curve *ATC* intersects the *S–S* curve) the level of production which yields a 'normal' profit on the existing capital equipment. Beyond *N*, any further increase in production will 'induce' investment in the shape of additions to productive capacity, and it is reasonable to suppose that the increase in investment associated with an increase in output will exceed the increase in savings for any *given* distribution of income. Hence the savings-investment relationship will yield a U-shaped demand curve ; the curve will be falling up to *N* (when induced investment is zero)[1] and will slope upwards to the right of *N* (when induced investment is positive). As shown in Fig. 14 this will yield multiple positions of equilibrium, P_1, P_2 and P_3, of which only P_1 and P_3 are stable positions whereas P_2 is unstable (since at P_2, where

[1] Up to point *N*, the position of the demand curve may be regarded as being determined by the existence of some 'autonomous' investment which is independent of the current level of activity, or else by a negative constant in the savings functions, which makes savings zero at some positive level of income and employment.

the demand curve cuts the supply curve from below, a small displacement in either direction will set up cumulative forces away from P_2 until either P_1 or P_3 is reached).

It follows that an under-employment equilibrium is only *stable* under slump conditions when induced investment is zero.

It also follows that it is impossible to conceive of a *moving* equilibrium of growth being an under-employment equilibrium. Such an equilibrium is necessarily one where productive capacity is growing, and where therefore induced investment is positive, and hence the $D-D$ curve slopes upwards and not downwards. It therefore postulates the equilibrium of the P_3 type and not of the P_1 type. In that situation the profit margin must be above the minimum level, and the distribution of income will tend to be such as to generate the same proportion of income saved as the proportion of investment in output.

In a balanced-growth equilibrium, the level of investment must of course also correspond to the rate of accumulation appropriate to the rate of growth of the economy, in other words (in terms of Model II) to $(\lambda v)Y$. This is not necessarily the rate of investment reflected by our (short-period) demand curve at the point P_3; if it is not, the adjustment takes the form of a change in capacity in relation to output (a shift in point N in the diagram) and a consequent change in the investment 'induced' by the excess of actual output over N sufficient to make the volume of induced investment equal to $(\lambda v)Y$.

It further follows that a moving equilibrium of growth is only possible when, given the savings propensities, the profit margin resulting from the equilibrium rate of investment is higher than the minimum profit margin indicated by the height of the horizontal section of the S–S line; and there must be sufficient competition to ensure this. If this were not so, the point P_3 would lie below the S–S line, and the only equilibrium conceivable in that case would be that of the P_1 type at which, as we have seen, induced investment is zero, and the level of output remains stationary over time, irrespective of the growth in population. *It is only under conditions of 'Keynesian' full employment that the growth-potential of an economy (indicated by its 'natural' rate of growth) is exploited in terms of actual growth.*

We must therefore add a further restriction to our models which can be written (putting m for the minimum profit margin, reflecting the degree of market imperfection):

$$\frac{P}{Y} \geqslant m \qquad (12)$$

which, under the assumption of Model II where $\frac{P}{Y} = \lambda v$, can be written in the form

$$m \leqslant \lambda v. \tag{12a}$$

If this condition is not satisfied, the economy will lapse into stagnation.

So far we have not mentioned marginal productivity. Clearly, the equilibrium real wage cannot *exceed* the short-period marginal product of labour: for if it did, the position of full employment could not be reached. Under our present assumptions, where the full-employment position falls within the range of the horizontal section of the average prime cost curve (or very near it), this does not impose any further restriction. For when productivity is constant, the marginal product of labour is the same as the average product, and the condition therefore is necessarily satisfied, so long as the equilibrium wage is lower than output per head (i.e. so long as the equilibrium share of profits is positive). In order to generalize our results, however, to cover the case of diminishing (short-period) returns (i.e. when the full employment line in Figs. 13 and 14 cuts the average prime cost curve in the *rising* section of the latter and marginal costs exceed average prime costs), we need to introduce a further restriction to the effect that the share of wages cannot exceed the marginal product of labour. Writing for a *given* value of K,

$$Y = \Psi(L), \text{ where } \Psi' > 0, \ \Psi'' \leqslant 0$$

for the *short-period* relationship between output and employment (L denoting the amount of employment) the condition is

$$\frac{W}{Y} < \frac{L\Psi'(L)}{\Psi(L)} \tag{13}$$

Under conditions of our Model II, where $\frac{P}{Y} = \lambda v$, this could also be written in the form

$$\frac{\Psi(L) - L\Psi'(L)}{\Psi(L)} \leqslant \lambda v \tag{13a}$$

i.e. the equilibrium share of profits, as determined by the 'dynamic' conditions, cannot be less than the excess of the average product of labour over the marginal product. We can assume, however, that the system will tend to generate sufficient excess capacity (in relation to the labour supply) for this condition to be satisfied.

These two restrictions, (12) and (13), together with that given in

(5), are not additive but alternative, and only the higher of them will apply. For our minimum margin of profit in (12) is not the same thing as the 'optimum' monopoly profit of the text-books, which is the outcome of short-period profit-maximization with reference to some given marginal-revenue schedule to the individual firm. It is more akin to Marshall's notion of a minimum margin of profit on turnover below which producers refuse to go 'for fear of spoiling the market',[1] but which tends to be the lower, the more intense the competition among producers. As such it is related to the average cost of production and not to marginal cost; and as an obstacle to a fall in the profit-share, it overlaps with the technical barrier set by the excess of short-period marginal cost over average prime cost.

VIII. TECHNICAL PROGRESS

We must now proceed to remove the most important of our 'simplifying' assumptions, the absence of technical progress. A moving equilibrium of growth involves continued increase in the productivity of labour, and not only in the working population, *pari passu* with a continued increase in the amount of capital per worker; though in the absence of any reliable measure of the quantity of capital (in a world where the technical specification of capital goods is constantly changing, new kinds of goods constantly appear and others disappear) the very notion of the 'amount of capital' loses precision. The terms 'income' or 'capital' no longer have any precise meaning; they are essentially accounting magnitudes, which merely serve as the basis of calculations in business planning; the assumption that money has a stable value in terms of some price index enables us to think of 'income' and 'capital' as *real* magnitudes only in a limited, and not precisely definable, sense.[2]

Orthodox theory attempts to deal with these problems in terms of the traditional tools — the assumption of a linear and homogeneous production function, coupled with the assumption that with the changing state of knowledge this function is continually shifting upwards and outwards. As depicted in Fig. 15 at any one *point* of time, t, there is assumed to be a unique relationship between capital and output, which conforms to the general hypothesis of diminishing

[1] *Principles* (8th ed.), Book V, ch. 5, section 5, pp. 374-6.

[2] These problems do not appear in a von Neumann type of model of balanced-growth equilibrium with constant technical functions, precisely because the technical specification of goods, their relative composition and their relative values remain unchanged through time; everything remains the same, except for the actual quantities of goods, and there is no problem involved in aggregation.

productivity, but this relationship is constantly shifting with the passage of time. The assumption of 'neutral' technical progress means that the production curve shifts in such a manner that the slope of the tangents of the functions f_t, f_{t+1}, f_{t+2}, etc., remain unchanged along any radius from the origin. This hypothesis is necessary in order to make it possible for a constant rate of profit over time to be consistent with a constant rate of growth and a constant relationship between capital and output (since the rate of profit on capital is uniquely related to the slope of the production function).

There are, however, several basic faults in this procedure — quite apart from the inherent improbability that technical progress should obey any such rigid rules.

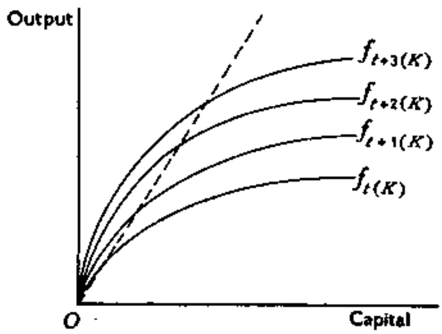

FIG. 15

(1) In the first place the production function assumes that the capital stock in existence at any one time is perfectly adapted to any given capital-labour ratio — that there is a particular assortment of equipment goods corresponding to each successive point of the production curve which is different from the assortment associated with any neighbouring point. (This will be true even in the absence of 'technical progress' so long as the substitution of capital for labour implies the use of different *kinds* of equipment, and not merely the use of relatively greater quantities of the *same* equipment.) Hence the successive points on this curve represent alternative states of long-period stationary equilibrium any one of which could be actually attained only when any given state of capital endowment (i.e., any given capital-output ratio) has obtained unchanged for a long enough period for the actual assortment of capital goods to have become optimally adapted to it. The production curve thus represents a kind of boundary indicating the maximum output corresponding to each particular 'quantity' of capital, a maximum which assumes that

the whole productive system is fully adapted to each particular state of accumulation. In an economy where capital accumulation is a continuous process this boundary is never attained — since the actual assortment of capital goods at any one time (even with a *constant* state of knowledge, whatever that assumption may be taken to mean) will consist of items appropriate to differing states of accumulation, and the output corresponding to any particular 'quantity' of capital will be less than the equilibrium (or maximum) output associated with that quantity. This is only another way of saying that in a society which is *not* in continuous long-run stationary equilibrium, output cannot be regarded as a unique function of capital and labour ; and the slope of the production curve cannot be relevant to the pricing process, since the system does not move *along* the curve, but *inside* it.

(2) In the second place (and quite independently of the first point) the assumption that there is a curve which continually shifts upwards means that technical progress is treated as a variable of the function in a manner perfectly analogous to a second factor of production, like labour (or land). This is evident from the consideration that if, instead of postulating rising technical knowledge and a constant labour force, we postulated a constant state of technical knowledge and a rising labour force, the nature of the shift of the curve (under the hypothesis of a homogeneous and linear function) would be exactly the same. A given rate of shift of the curve, along any radius from the origin, could equally well result from a given percentage increase in the labour supply as from the same percentage increase in the state of 'knowledge'. But unlike labour, the state of knowledge is not a quantifiable factor. A *given* or a *constant* state of knowledge is only capable of being defined implicitly : there is no possible way in which, comparing two different positions, at two different points of time, the change due to the movement along the curve could be isolated from the change due to the shift of the curve. The whole procedure by which this separation is attempted is purely circular : since the *slope* of the curve (under the additional hypothesis that the function is not only homogeneous and linear but a constant-elasticity function *à la* Cobb-Douglas !) is supposed to determine the share of profits in income, the share of profits is taken to be an indication of its slope, and the residual is then attributed to the shift of the curve! There could be no better example of *post hoc ergo propter hoc.*

(3) The hypothesis that the *slope* of the curve determines the share of profits, in accordance with the marginal productivity principle, despite the continued shift in the curve, presumes of course that the factor responsible for the shift is itself rewarded on the same principle, since it is the marginal product of *all* factors taken *together*

which exhausts the total product. This condition can be satisfied when the shift of the curve is due to, say, a certain rate of increase in the quantity of labour, since that part of the increase in the product which is due to the shift is definitely imputed to labour in the form of wages. But knowledge, just because it is not a quantifiable factor which can be measured, or brought under exclusive ownership, or bought and sold, cannot receive its *own* marginal product. It is like other scarce but unappropriated agents of production (like the sea in the case of the fishing industry) whose existence causes divergences between the private and the social product of the *other* factors. This is only another way of saying that we are not free to elevate to the rôle of a 'factor of production' anything we like; the variables of the production function must be true inputs, and not vague 'background elements', like the sun or the sea or the state of knowledge, any of which may be thought to cause the results to diverge from the hypothesis of the homogeneous-and-linear production function. In terms of the *true* variables, Capital and Labour, the production function will not be linear-homogeneous but will be a function of a higher order, when technical knowledge is increasing over time.[1] It is therefore illegitimate to assume that factor rewards are allocated in accordance with their marginal productivities, since the sum of the marginal products of the factors will exceed the total product. When, the quantity of labour being given, an increase in capital by a given proportion yields an increase in output in the same proportion, the 'true' marginal product of capital will *alone* exhaust the total product.[2] For this reason any postulate derived from the hypothesis of diminishing productivity (such as our $v = \phi\left(\frac{P}{K}\right)$ function, given in equation (8a) above) is illegitimate when productivity, for whatever reason, is *not* diminishing. Given the fact of constant or increasing productivity to capital accumulation, the share of profit must necessarily be *less* than the marginal product of capital, and there is no

[1] It is a well-known dodge that any function whatsoever in n variables can be converted into a homogeneous-and-linear function of $n+1$ variables by adding a further variable which is *implicitly* defined. But as Samuelson has pointed out (*Foundations of Economic Analysis*, p. 84), any such procedure is illegitimate, since factor rewards will not conform to the partial differentials of this wider function.

[2] Supporters of the neo-classical approach would argue that the increase in product in this case is not *due* to the change in the quantity of capital alone — it is the joint result of the change in the quantity of the 'factor' capital, and the shift in the 'state of knowledge' which is presumed to have occurred in the interval of time during which the increase in capital occurred. But this is precisely the point: since the accumulation of capital is necessarily a process in time, and cannot be conceived of in a timeless fashion, a movement *along* the curve cannot be isolated from the shift of the curve; indeed it is illegitimate to assume the existence of a 'curve' independently of its shift, since there is no conceivable operation by which the slope of this 'curve' could be identified.

reason why a given capital-output ratio should be associated with a particular rate of profit, or indeed, why the two should be functionally related to each other on account of any technical factor.

(4) Added to this is the further complication that the rate of shift of the production function due to the changing state of 'knowledge' cannot be treated as an independent function of (chronological) time, but depends on the rate of accumulation of capital itself. Since improved knowledge is, largely if not entirely, infused into the economy through the introduction of new equipment, the rate of shift of the curve will itself depend on the *speed of movement* along the curve, which makes any attempt to isolate the one from the other the more nonsensical.[1]

The most that one can say is that whereas the rate of technical improvement will depend on the rate of capital accumulation, any society has only a limited capacity to *absorb* technical change in a given period. Hence, whether the increase in output will be more or less than proportionate to the increase in capital will depend, not on the state of knowledge or the rate of progress in knowledge, but on the *speed* with which capital is accumulated, relatively to the capacity to innovate and to infuse innovations into the economic system. The more 'dynamic' are the people in control of production, the keener they are in search of improvements, and the readier they are to adopt new ideas and to introduce new ways of doing things, the faster production (per man) will rise, and the higher is the rate of accumulation of capital that can be profitably maintained.

These hypotheses can, in my view, be projected in terms of a 'technical progress function' which postulates a relationship between the rate of increase of capital and the rate of increase in output and which embodies the effect of constantly improving knowledge and

[1] None of the above strictures against the postulate of a 'production function' which continually shifts with technical progress invalidates the assumption of a *short-period* relationship between employment and output, which takes the character and composition of fixed equipment of all kinds as given. This short-period production function (as employed in equations (13) and (13a) above) implies that for any given volume of employment a definite 'marginal product' can be imputed to labour, which, as we have seen, sets an *upper limit* to the share of wages in output (the 'rents' to be imputed to capital being the residual, i.e. the difference between the average and the marginal product of labour). This limit, however, only becomes significant when diminishing returns prevail, so that an increase in production is associated with a more-than proportionate increase in employment — with constant or increasing returns, the marginal product of labour will equal to, or exceed, the average product, and the former *cannot* therefore be the governing factor determining distributive shares. Whether diminishing returns prevail or not will predominately depend on the output capacity represented by the existing capital stock and its degree of utilization when labour is fully employed. Under conditions of imperfect competition it is perfectly compatible with 'profit-maximizing behaviour' to suppose that the representative firm will maintain a considerable amount of spare capacity even in relation to the output attainable under full-employment conditions.

know-how, as well as the effect of increasing capital per man, without any attempt to isolate the one from the other.

It is the shape and position of this 'technical progress function' which will exhibit features of diminishing returns. If we plot percentage growth rate of output per head, $\mathring{Y}/Y$, along Oy and percentage growth rate of capital per head, $\mathring{K}/K$, along Ox (Fig. 16), the curve will cut the y-axis positively (since a certain rate of improvement would take place even if capital per head remained unchanged) but it will be convex upwards, and reach a maximum at a certain point — there is always a maximum beyond which a further increase in the rate of accumulation will not enhance further the *rate* of growth of output (Fig. 16). This means that the increase in capital (per head) will yield increasing or diminishing returns in terms of output according as the rate of accumulation is relatively small or large. If the rate of accumulation is less than Op, output will increase faster than capital, and vice versa.

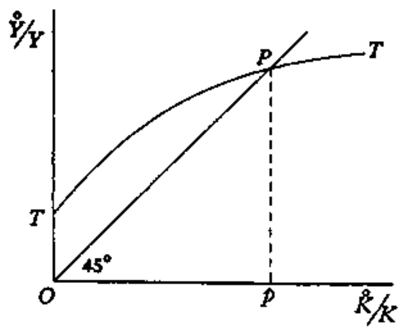

Fig. 16

The *height* of the curve expresses society's 'dynamism', meaning by this both inventiveness and readiness to change or to experiment. But the convexity of the curve expresses the fact that it is possible to utilize as yet unexploited ideas (whether old ideas or new ideas) more or less fully; and it is always the most profitable ideas (i.e. those that raise output most in relation to the investment which they require) which are exploited first. Some are old ideas; some are new ideas; most of the technical improvement that takes place embodies both. We cannot isolate the element of pure novelty in a world where knowledge is constantly improving, and where the actual techniques are constantly lagging behind the very latest techniques that would be selected if everything were started afresh. When capital is accumulated at a faster rate (and technical improvement goes on at a faster rate), productivity will also increase at a faster rate, but the growth in the latter will lag behind the growth in the former, and beyond a certain point a further increase in the rate of accumulation ceases to be 'productive' — it is incapable of stepping up the rate of growth of productivity any further.

There is therefore no *unique* rate of technical progress — no *unique* rate at which alone a constant rate of growth can be maintained. There is a whole series of such rates, depending on the rate of accumulation of capital being relatively small or large.

On this analysis, it is the 'technical dynamism' of the economy, as shown by the height or position of our technical progress curve, which is responsible, in a capitalist economy, both for making the rate of accumulation of capital and the rate of growth of production relatively small or relatively large. It explains why there is no long-run tendency to a *falling* rate of profit, or for a continued increase in capital in relation to output, either in slow-growing or in fast-growing economies. In economies whose technical dynamism is low, both the rate of accumulation and the growth of production will be relatively low, but in either case, growth can go on at a steady rate, without any necessary tendency to diminishing returns and thus to a gradual approach to a stationary state.

On the assumption that this function cuts the y-axis positively (i.e. that there would be some positive rate of growth in output per man, even if capital-per-man remained unchanged — an assumption which is justified by the fact that even a zero rate of net investment implies a certain rate of infusion of new techniques or new designs, through the replacement of worn-out capital; and that there are always *some* improvements which may require no investment at all) and that the curve is convex upwards, there is necessarily a certain point on the curve at which it is intersected by a radius of 45 degrees from the origin — i.e. where the rate of growth of output is equal to the rate of growth of capital (P in Fig. 16). At that point *all* the conditions of 'neutral' technical progress are satisfied: the capital-output ratio will remain constant at a constant rate of growth, constant distributive shares, and a constant rate of profit on capital.

In order to 'close' our model — that is, to produce a model that would account for the empirical features of the growth process as summarized by our 'stylized facts' at the beginning — it is necessary to show, not merely that such a point exists, but that in a capitalist system there is a tendency to move towards this point, which thus represents a long-run equilibrium rate of growth, and which is also stable in the sense that displacements due to shifts in the curve, etc., set up forces to re-establish it.

The hypothesis that *given* the technical progress function, the system tends towards that particular rate of accumulation where the conditions of 'neutral progress' are satisfied, cannot of course be justified on *a priori* grounds; it must be based on empirical evidence — at least in the sense that it can be shown to be consistent with facts which are more difficult to explain on any alternative hypothesis. Supposing that the statisticians were to agree that the capital-output ratio tends to be constant in periods in which the rate of growth of production is constant (in which therefore the rate of technical progress is neither increasing nor decreasing) whilst the capital-output ratio tends to decrease in periods of accelerating growth and vice versa. This would support the hypothesis that the system *tends* towards P: and variations in the rate of growth, and in the movements in the capital-output ratio, are then to be explained in terms of the unequal incidence of technical progress — i.e. in terms of shifts of our technical progress function. If, on the other hand, the statisticians were to agree that there is no correlation between these magnitudes, that periods of steady growth are just as likely to be associated with a steadily decreasing or a steadily increasing ratio of capital to output, this would support the hypothesis that the system tends towards some point on the curve — to some equilibrium rate of growth of output and of capital — which is not necessarily the one at which the two growth rates are *equal*.

IX. ASSUMPTIONS ABOUT INVESTMENT BEHAVIOUR

In either event, to obtain an equilibrium solution — to assert, in other words, that there is some particular equilibrium rate of growth of output and of capital towards which the system is tending — we need to introduce an 'investment function' based on entrepreneurial behaviour. Since we *cannot* say that the rate of capital accumulation depends on the community's propensity to save (since the latter is a dependent variable, depending on the share of profits, and thus on the share of investment) nor on the requirements of the 'natural rate of growth' (because one of the two constituents at least of the natural rate of growth, the rate of growth of productivity, is a dependent variable, depending on the rate of accumulation of capital and thus on the share of investment), we need to introduce, in order to close our model, an independent function governing the investment decisions of entrepreneurs. There are various alternative assumptions that can be made about investment behaviour which lead to divergent results; and at the present stage we cannot say that our knowledge of entrepreneurial behaviour is sufficient to rule out any particular assumption in preference to some other. Hence our final choice of assumption must be based on the admittedly weaker procedure of its yielding results that are more in conformity with the facts of experience than its alternatives.

(1) One hypothesis, originally advanced by Kalecki,[1] is that the subjective risks assumed by entrepreneurs are an increasing function of the rate of capital accumulation (or, as Kalecki put it, the rate of investment decisions is an increasing function of the gap between the prospective rate of profit and the rate of interest). This assumption, at any rate for a given market rate of interest, makes the rate of capital accumulation a single-valued function of the rate of profit

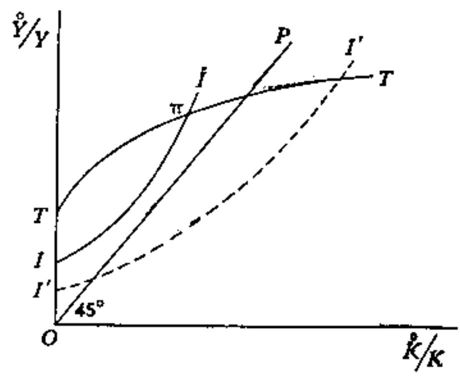

FIG. 17

on capital, and since the latter, in a state of balanced-growth equilibrium, is a single-valued function of the rate of growth, it makes the desired rate of accumulation a single-valued function of the rate of economic growth. Such an 'inducement to invest' function is shown by the curve *I–I* in Fig. 17. The height of this curve (i.e. the point at which it cuts the *y*-axis) reflects the market rate of interest, while the slope of the curve reflects increasing marginal risk. This postulate yields an equilibrium position at point π where the rate of economic growth resulting from the given rate of capital accumulation coincides with the rate of economic growth that is required in order to induce entrepreneurs to accumulate capital at that particular rate. On this hypothesis the equilibrium rate of growth can be anywhere on the *T–T* curve, depending only on the position of the risk preference function (governing the inducement to invest) relatively to the technical progress function (governing the rate of growth resulting

[1] 'The Principle of Increasing Risk', *Economica*, 1937, p. 440.

from varying rates of accumulation). Thus if π is to the left of P, the equilibrium rate of growth will involve a constantly falling capital-output ratio, and if it is to the right of P (as with the dotted line $I'–I'$ in Fig. 17) it involves a constantly rising capital-output ratio. In both cases the rate of growth will be constant over time, but in the first case the equilibrium will involve a steadily falling share of profit in income and in the second case a steadily rising share of profit. On this hypothesis therefore the 'neutral' position at P will only be reached as a result of a coincidence — of the $I–I$ curve cutting the $T–T$ curve at that point.

(2) An alternative hypothesis, which is a variant of the one put forward in my paper 'A Model of Economic Growth',[1] makes the principle of increasing risk applicable, not to the volume of investment decisions as such, but only to that part of investment which is in excess of that required to maintain a constant relationship between output capacity and prospective output. Whenever sales are rising, entrepreneurs will in any case increase the capital invested in the business by the amount necessary to enable them to increase their productive capacity in line with the growth of their sales — there are no greater risks involved in a larger business than a smaller one; and no greater risks are entailed in a higher rate of growth of employed capital, if this proceeds *pari passu* with a higher rate of growth of turnover. Hence if their actual sales are rising at the rate of g (where g may be any particular point on the $T–T$ curve in Fig. 16) we may suppose, in accordance with the 'acceleration principle', that the growth in output in itself will 'induce' sufficient investment to enable that rate of growth of production to be maintained, without requiring a higher prospective rate of profit. As far as this 'induced investment' is concerned, any particular point on the curve could be an equilibrium point. But if a particular rate of growth of output and capital involves the expectation of a *rising* rate of profit in the minds of investors, it will induce an acceleration in the rate of accumulation and hence will cause the system to move to the right (on the curve); if it involves the expectation of a falling rate of profit, it will cause it to move to the left.

The *prospective* rate of profit in the minds of entrepreneurs is based on two things: on the amount of capital required per unit of output, and on the expected profit margin per unit of output. If we assume that all savings come out of profits (i.e. $\beta = 0$) then, given constant rates of accumulation and growth, the realized rate of profit

[1] *Economic Journal*, 1957, p. 604. The form of the 'investment function' given in that paper was justly criticized; the present version, I hope, meets the objections raised against the earlier version by Professor Meade, Mr. Hudson and others.

on capital will also be constant over time, irrespective of whether capital per unit of output is constant, rising or falling (since any reduction in the capital-output ratio will be matched by a corresponding reduction in the share of profits in output, and vice versa). But we cannot assume that the *prospective* rate of profit on current investment will be the same as the *realized* rate of profit on existing capital — the prospective rate of profit will be higher, precisely because the capital required for producing a unit-stream of future output is less than the amount of capital that was (historically) invested in producing a unit-stream of current output. Nor can it be assumed that the prospective rate of profit on new investment will be the same as the actually realized rate of profit in future periods, since the latter magnitude will itself depend on the investment decisions currently made by entrepreneurs. Thus if at some particular rate of accumulation the trend of progress causes a continued fall in the amount of capital required per unit of output,

$$\frac{P}{K}=\frac{P}{Y}\cdot\frac{Y}{K}$$

will remain constant if the rise in Y/K is offset by a corresponding fall in P/Y. This would occur if the fall in K/Y involved a corresponding reduction in I/Y; if, in other words, it left the rate of expansion of capacity unchanged. But if this consequential fall in profit margins is *not* foreseen, or not sufficiently foreseen, the rise in Y/K will involve the expectation of a higher prospective rate of profit, which by increasing the rate of investment may prevent the fall in P/Y from occurring at all. This is a case, therefore, where the movement of the economy, and the nature of the final equilibrium, cannot be predicted independently of the nature of the expectations of entrepreneurs. The assumption of 'static foresight' (i.e. the projection of existing prices, costs and output levels to the future) leads to a different result from the assumption of 'perfect foresight'; the latter assumption moreover leaves the situation indeterminate since the expectations that are capable of being actually realized are by no means unique. It is only in the 'neutral' equilibrium case (at point P) that the two kinds of assumptions (static foresight and perfect foresight) lead to consistent results.

Expectations are invariably based on past experience, and in that sense, are of the 'static' rather than of the 'perfect' kind. In addition, they can be defined as being more or less 'elastic' according as the projections into the future are based on the events of the very recent past, or on the average experience of a longer interval of elapsed time. Expectations are likely to be the more elastic the less past experience justifies the assumption of some norm around which short-term movements fluctuate; the more, in other words, past movements have been subject to a trend. For that reason, business expectations are far more likely to be elastic with respect to volume of sales than with respect to the margin of profit on turnover; the future expectation concerning the margin of profit per unit of sales, which is taken as the basis of business calculations, is far more likely to reflect some standard, or norm, than the experience of the most recent period alone. This provides a further reason for supposing that in situations in which production rises faster than the stock of capital, the prospective rate of profit will be rising relatively to the realized rate of profit; and if, in response to this, the rate of accumulation is accelerated, the rate of growth of production, and the realized rate of profit, will rise as well.

Hence the tendency of the system to move towards a position where output and capital both grow at the same rate, and where therefore the rate of profit on capital will remain constant at a constant margin of profit on turnover, can be justified by the suppositions (i) that the prospective rate of profit on investments will be higher than the currently realized rate of profit on existing capital whenever production is rising faster than the capital stock; (ii) that a rise in the prospective rate of profit causes an increase in the rate of investment, relative to the requirements of a state of steady growth, and vice versa.[1]

X. THE FINAL MODEL

The equilibrium relationships of this final model can thus be set out as follows. It is based on three functions: first, on a savings function on the lines of equation (10) above, which can be written in the form

$$\frac{S}{Y}=(\alpha-\beta)\frac{P}{Y}+\beta, \tag{10a}$$

where $$1>\alpha>\beta\geqslant 0.$$

[1] In the first version of the present growth model (published in the *Economic Journal*, December 1957) I postulated an investment function which made current investment depend (*inter alia*) on the change in the *realized* rate of profit as compared with the previous period. This was unsatisfactory in that it failed to take into account the fact that the inducement to invest depends on the prospective rate of profit, and not on the actual profit earned on existing capital; and that quite apart from the question of expectations, the prospective rate of profit will differ from the currently realized rate whenever (owing to technical progress, etc.) the 'productivity' of capital on new investment (i.e. the amount of investment required per unit of future output capacity) differs from the existing capital-output ratio.

Second, on a technical progress function showing the relationship between the rate of growth of output per worker (G_0) and the rate of growth of capital per head ($G_K - \lambda$), and which (using a linear equation for the sake of convenience) [1] can be written in the form

$$G_0 = \alpha' + \beta'(G_K - \lambda), \text{ where } \alpha' > 0,\ 1 > \beta' > 0. \tag{14}$$

Third, on an investment function based on the assumptions already described, and which makes investment a combination of two terms. The first term of the equation relates to the amount of investment *induced* by the change in output the previous period, and assumes that this investment will be such as to make the growth in *output capacity* in period $(t+\theta)$ equal to the growth in output in period t. Since in view of (14), the rate of capital accumulation per worker $(G_K - \lambda)$, which is required to increase output capacity by G_0 will not (necessarily) be equal to G_0 but to

$$\frac{G_0 - \alpha'}{\beta'}$$

[1] It has been pointed out to me by Professor Meade, Mr. Hahn and others that whilst, in general, the technical progress function cannot be integrated in terms of a production function with a particular rate of time shift, a *linear* technical progress function as given in (14) can be integrated to obtain

$$Y_t = Be^{\alpha' t} K_t^{\beta'} \tag{14a}$$

which appears to be the same as the Cobb-Douglas function (remembering that Y_t and K_t refer to the output and the capital *per unit* of labour). However, as was pointed out to me by H. Uzawa of Stanford University, in integrating the technical progress function, the constant of the integral $B = B(Y_0, K_0)$ is a function dependent on the initial amount of capital K_0 and of output Y_0, whereas a production function of the type

$$Y_t = f(K_t, t) \tag{14b}$$

requires that the function should be independent of the initial conditions.

Apart from this, the aggregative production function of the type (14b), a special case of which is the Cobb-Douglas function, implies the assumption that at any given time t, the output Y_t is uniquely determined by the aggregates, K_t and L_t, irrespective of the age-and-industry composition of the capital stock. However, when the technical progress of an economy depends on its rate of capital accumulation (when, in other words, the improvements in techniques require to be embodied in new equipment before they can be taken advantage of), no such functional relationship exists. To describe the relationship between capital, labour and output we require a function in the form

$$Y_t = \phi(A_t) \tag{14c}$$

where A_t specifies the distribution of capital according to age as well as (in a multi-commodity world) the distribution of both capital and labour between industries and firms. In that case the postulate of a linear technical progress function is perfectly consistent with the ϕ function being neither homogeneous in the first degree nor of constant elasticity. In the short run the age-and-industry distribution is of course given as a matter of past history. But even in a long-run growth equilibrium with technical progress, A_t could not be treated as a unique function of K_t and L_t, since it will also depend on λ and (in view of the varying incidence of obsolescence at differing rates of progress) on γ', the equilibrium value of G_0.

and since
$$G_K \equiv \frac{I}{K},$$

the rate of *induced* investment in period $(t+\theta)$ and which is the first term of our investment equation, will be equal to

$$(G_0(t) - \alpha')\,\frac{K(t)}{\beta'} + \lambda K(t).$$

The second term of our investment equation depends on the change in the prospective rate of profit which, on our assumptions concerning the expected margin of profit turnover (i.e., that the *expected* value of P/Y is based on an average past values), will be a rising function of the *change* in Y/K over time. Assuming this latter relationship to be linear for the sake of convenience the whole function can be expressed in the following form :

$$I(t+\theta) = (G_0(t) - \alpha')\frac{K(t)}{\beta'} + \lambda K(t) + \mu\frac{d}{dt}\left(\frac{Y(t)}{K(t)}\right), \tag{15}$$

where $\mu > 0$.

The first term of this equation gives rise to an amount of investment at any given rate of growth of output that is sufficient to maintain that rate of growth of output — i.e. sufficient to keep the system on any particular point on the T–T curve. It can also be seen immediately that when

$$G_Y > G_K,$$

the second term of the expression is positive, hence G_K will be rising over time. A rise in G_K, in accordance with (14), will raise G_Y but less than proportionately, and hence lead to a further rise in investment in accordance with the first term at the same time as it diminishes the second term. Hence, whatever initial position we start from (defined by given values of K, L, and O at some initial point $t=0$), this process will gradually lead to a situation in which the second term of equation (15) dependent on $\frac{d}{dt}\left(\frac{Y(t)}{K(t)}\right)$ vanishes to zero and where therefore

$$\frac{dv(t)}{dt} = 0. \tag{16}$$

This implies that
$$G_0 = \frac{\alpha'}{1-\beta'} = \gamma', \tag{17}$$

and
$$G_Y = G_K = \lambda + \gamma'. \tag{18}$$

Hence this model, like the earlier ones, also yields a state of moving equilibrium, where the rate of growth, the capital-output ratio and the distributive shares are constant over time — the main difference being that the output-per-worker, capital-per-worker and wages-per-worker are now no longer constant but rising at the equilibrium rate of growth productivity, γ'. However, these assumptions are not yet sufficient to set out a full equilibrium model. The reason is that since we no longer have a technical equation for v on the lines of equation (8a) which was incorporated in Models III and IV, the actual value of v is here left undetermined. From this model it only follows that at the position of equilibrium v will be constant (since this is implicit in equation (15), as shown by (16)); but this is consistent with any particular value for v — or rather v could only be determined in this model historically, if we assumed that it had a certain initial value at some particular point of time, and followed its resulting movement through the successive steps to final equilibrium.

Hence, in order to close the model, we shall introduce two more variables and three additional relationships. These are strictly 'Keynesian' — since they are, on the one hand, necessary to ensure that the reaction-mechanism of the model follows the Keynesian system in which the inducement to invest is independent of the propensities to save; and on the other hand because they incorporate Keynesian notions of the rate of interest and the supply price for risk capital based on liquidity preference or the aversion to risk taking.

We have already argued in connection with (5) above [1] that the inequality

$$\frac{P}{K} > r + \rho$$

is a necessary boundary condition of the model in the sense that the continued accumulation of capital cannot go on unless the ruling rate of profit is *at least* as high as the necessary compensation for risk and illiquidity involved in the productive employment of wealth.[2] Further consideration shows that in order that the investment equation in (15) should hold, it is not enough to make equation (5) into a boundary condition; for so long as P/K is higher than the supply price of risk capital, there is no reason to suppose that

[1] P. 189 above.

[2] A more precise statement of this condition would break down $r+\rho$ further into its component elements, distinguishing between the expected average of short rates of interest and the premium of the long rate over the expected average short rate on the one hand, and the additional leaders', borrowers' and speculative risks, etc., involved in direct investment, on the other hand, but this is not necessary for our present purposes.

investment outlay will be confined to that necessary for the increase in output capacity (i.e. to that given by 'the acceleration principle') or to that resulting from a given increase in the prospective rate of profit in a particular period. Indeed, unless the rate of profit actually corresponds to the supply price of risk capital, one cannot assume that the investment of each period will be confined to the *new* investment opportunities accruing in that period — an assumption necessary for an equilibrium of steady growth. Hence equation (5) should be converted into an equilibrium condition

$$\frac{P}{K}=r+\rho. \tag{19}$$

The second relationship concerns the behaviour of the rate of interest, r, and here we shall follow orthodox Keynesian lines in assuming that the rate of interest is determined by the liquidity preference function and/or monetary policy (summarized in the function $\pi\left(\frac{M}{Y}\right)$, where $\pi' \leqslant 0$ and M is the real quantity of money), subject to the condition that there is a minimum ($\bar{r}$) determined by the risk premium associated with the holding of long-term financial assets, below which the rate of interest cannot fall. This relationship can therefore be expressed in two alternative forms

$$r \geqslant \bar{r}$$

when $r > \bar{r}$,
$$r = \pi\left(\frac{M}{Y}\right) \tag{20}$$

The third relationship concerns the behaviour of ρ, and though this equation can be fully supported on *a priori* grounds, it is put forward here more tentatively, as at present there is insufficient empirical evidence available to support it. It is based on the following considerations.

(1) First, as explained earlier in this paper,[1] it may be assumed that at any given rate of interest the minimum rate of profit necessary to provide inducement for any particular kind of investment will be higher the riskier (or the more 'illiquid') that investment is considered to be;

(2) Second, as was also argued,[2] investment in 'fixed assets' (plant and equipment, etc.) is considered to be far more risky or illiquid than either investment in financial assets or in working capital;

(3) Third, it may be assumed that the turnover-period of circulating capital is invariant (or practically invariant) with respect to

[1] P. 189 above. [2] *Ibid.*

changes in the techniques of production, so that circulating capital stands always in a linear relationship to output ; hence any increase in the ratio of fixed to circulating capital involves an increase in the capital-output ratio.

It follows as a joint result of (2) and (3) that a higher capital-output ratio (including both fixed and circulating capital in the capital employed) requires for any given rate of interest a *higher* minimum rate of profit. Hence when the stage of accumulation is reached in which the actual rate of profit becomes *equal* to this minimum, the capital-output ratio will be uniquely related to the rate of profit ; and, as we have seen, it is only under these conditions that the actual investment in each period is limited by the 'new' investment opportunities becoming available in that period (through λ and γ').

Writing F for fixed capital and C for circulating capital, k for the turnover-period of circulating capital, ρ_F and ρ_C for the marginal risk premium on the two types of investments respectively, and ρ for the marginal risk premium on investment in general, we thus have the following additional assumptions and relationships :

$$K \equiv F + C$$

$$C = kY$$

$$v \equiv \frac{K}{Y} \equiv \frac{F + kY}{Y}$$

$$\rho_F > \rho_C$$

$$\therefore \ \rho \equiv \frac{\rho_F F + \rho_C kY}{F + kY} = \xi_1\left(\frac{F}{Y}\right)$$

$$\rho = \xi_2(v), \text{ where } \xi_2' > 0. \tag{21}$$

It will be noted that the relationship expressed in (21) operates in a reverse manner to equation (8a) which determines v in the 'neo-classical' model ; since in the case of (8a), ϕ' is negative, not positive.

We have argued at some length that equation (8a) can no longer be assumed to hold when technical progress is a continuing process and there is *no* unique function relating output to the capital stock, in which case, depending on the factors determining the rate of growth, varying shares of profit in income and varying rates of profit on capital can be associated with any *given* capital-output ratio. It is now seen that when equation (21) holds, equation (8a) *cannot* hold — at least not within the framework of a model which assumes that the money rate of interest is determined by 'monetary' factors and that there is a minimum below which the rate of interest cannot fall.[1]

We can now set out our final Model V in a formal manner. It contains ten equations and ten variables — $Y(t)$, $O(t)$, $L(t)$, $P(t)$, $v(t)$, $s(t)$, $w(t)$, $G_o(t)$, $\rho(t)$ and $r(t)$. We shall continue to assume for simplicity that β is zero (there are no savings out of wages) and we shall take the simpler form of (20), treating the money rate of interest as a constant. We shall also bring together the various boundary conditions that emerged in the course of the analysis (cf. equations (4), (12) and (13) above), including a further one that is implicit in the relationship expressed in (21).

Assumptions

$$\left.\begin{aligned} L^*(t) &= L^*(o)e^{\lambda t} && \text{(i)}\\ G_o(t) &= \alpha' + \beta'(G_K(t) - \lambda) && \text{(ii)}\\ s(t) &= \alpha\frac{P(t)}{Y(t)} && \text{(iii)}\\ \frac{dv(t)}{dt} &= 0 && \text{(iv)}\\ r(t) &= \bar{r} && \text{(v)}\\ \rho(t) &= \xi(v(t)) && \text{(vi)}\\ \xi' &> 0 \end{aligned}\right\} \text{for all } t \geqslant 0$$

Identity

$$P(t) \equiv Y(t) - w(t)L(t) \qquad \text{(vii)}$$

Equilibrium Conditions

$$\left.\begin{aligned} s(t)Y(t) &= \frac{dK(t)}{dt} && \text{(viii)}\\ L(t) &= L^*(t) && \text{(ix)}\\ \frac{P(t)}{K(t)} &= r(t) + \rho(t) && \text{(x)} \end{aligned}\right\} \text{for all } t \geqslant 0$$

subject to the inequalities

(a) $$w(t) \geqslant w_{\min}$$

(b) $$\frac{P(t)}{Y(t)} \geqslant m$$

[1] It might be argued that the two equations could be made compatible with one another by an appropriate movement of the money price level which brought the 'real' rate of interest (*à la* Fisher) into an appropriate relationship with the other factors. But the movement of the price level depends on the behaviour of money wages (relatively to the change in productivity, γ') and this factor cannot, in turn, be treated as a function of the other variables.

(c)
$$\frac{W(t)}{Y(t)} \leqslant \frac{\frac{dY(t)}{dL(t)}L(t)}{Y(t)}$$

(d)
$$\rho_F + \bar{r} > \frac{\lambda + \gamma'}{\alpha} > \rho_C + \bar{r}. \qquad \text{(V)}$$

It is readily seen that the above yields a determinate system provided that the solutions fall within the limits indicated by the boundary conditions (a) – (d). By (ii) and (iv) we have

$$G_o = \frac{\alpha'}{1-\beta'} \equiv \gamma' \text{ (say)}$$

Hence by (i) and (ix) $\quad G_Y = \lambda + \gamma'$

But by (vii) $\qquad G_Y(t) = \frac{s(t)}{v(t)} = \lambda + \gamma' \equiv N \text{ (say)}$

By (iii), (v), (vi) and (x)

$$\frac{P(t)}{K(t)} = \frac{s(t)}{\alpha v(t)} = \frac{N}{\alpha} = \bar{r} + \xi(v(t)).$$

Hence by solving the last equality for $v(o)$, we can obtain all the remaining unknowns of the system.

If inequality (a) does not hold, $\frac{P}{Y}$ will be compressed below its equilibrium level, and hence the rate of accumulation and the rate of growth will be less than that indicated. As long, however, as we abstract from diminishing returns due to limited natural resources, and assume continuous technical progress, so that $G_o(t)$ rises over time, sooner or later the point must be reached where this inequality becomes satisfied.[1]

If, on the other hand, any one of the inequalities (b), (c) or (d) are not satisfied, $\frac{P}{Y}$ will be larger than its equilibrium value, and full-employment growth equilibrium becomes impossible. As regards (c) we may assume that there is always some degree of excess capacity (i.e., some relationship between output capacity and the full-employment labour supply) which satisfies this condition, and the system will tend to generate the required amount of excess capacity, if it did

[1] Allowing for diminishing returns, however, it is possible that (depending on the relative values of λ, α' and β') balanced growth equilibrium will necessarily settle at the point where the fall in $G_o(t)$ due to λ is precisely offset by the rise in $G_o(t)$ due to γ'; where, in other words, constancy of $G_o(t)$, and $w(t)$ over time, becomes a necessary condition of equilibrium. (This case seems to have application for many of the under-developed countries.)

not obtain initially.[1] It is possible, however, that the conditions (b) or (d) represent genuine obstacles to the attainment of balanced growth equilibrium.[2] In that case the system cannot grow at a steady rate. This does not mean, however, that the economy will lapse into permanent stagnation. As investment opportunities accumulate during periods of stagnation (owing to continued technical progress and population growth), it becomes possible for the system to grow, for a limited period, at a rate appropriately higher than $(\lambda+\gamma')$, thus generating the required value of $\frac{P(t)}{K(t)}$.

Finally, if condition (d) is not satisfied, a steady rate of growth is incompatible with the assumed rate of interest $\bar{r}$. Two cases are possible. If $\frac{\lambda+\gamma'}{\alpha} > \rho_F + \bar{r}$, equilibrium requires a higher money rate of interest. If $\frac{\lambda+\gamma'}{\alpha} < \rho_C + \bar{r}$, and the money rate of interest is already at its minimum level, it requires a rate of increase in money wages that would permit a rate of increase in the price level which reduced the real rate of interest to the appropriate figure.

Of all the relationships assumed in this model, that represented by (vi) and the inequality (d) are perhaps most open to doubt. Yet it can be shown that the assumption that ρ is a variable of v is the only one which makes the condition expressed in (x) — that the rate of profit is *equal* to the supply price of risk capital, consistent with the rate of profit being also determined by the growth factors, λ and γ' and by α. Equation (x) taken alone is incompatible with the rest of the model if the money rate of interest is assumed to be determined independently. But as indicated earlier, until there is more empirical evidence available to show that ρ_F is *appreciably* higher than ρ_C (or alternatively, that ρ_F itself is a rising function of the fixed-capital-output ratio, $\frac{F}{Y}$) and in consequence, the rate of profit is higher in industries and/or economies where the capital-output ratio is higher. I hesitate to put forward the relationship expressed in (vi) as more than a tentative suggestion, which I would be prepared to discard in favour of a better alternative, if such could be found.[3]

[1] Page 202 above. One may assume that the reaction mechanism here operates via the in- and out-flow of new firms as well as the investment behaviour of the representative firm.

[2] It is evident that these two restrictions are alternatives, of which only the higher one will apply.

[3] For the reasons given I regard Kalecki's assumption

$$\rho = \theta(G_K), \text{ with } \theta' > 0$$

as a worse alternative, apart from the fact that in the context of the present model it serves as a substitute for equation (15), not for equation (21), and hence is not sufficient for closing the model.

Chapter 11

A DYNAMIC GROWTH MODEL INVOLVING A PRODUCTION FUNCTION

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I. THE REDUCED FORM OF THE PRODUCTION FUNCTION

IT is a convenient simplification to suppose that in a given state of technical knowledge and with a given supply of land, the quantity of output produced per unit of time is a function of the amount of labour and the quantity of capital in use. It is even more convenient if, under conditions of perfect competition, the wage rate may be equated to the marginal product of labour, and the rate of profit on capital may be equated to its marginal efficiency.

So long as we are comparing a set of alternative possible 'stationary states' there is no great difficulty in framing a plausible set of axioms [1] which will justify the representation of output of consumption goods by an equation

$$O=\phi(M, N), \tag{1.1}$$

where M is an 'index-number' of the quantity of machinery and N is the number of persons employed and where if P and p denote money prices of machinery and consumption goods, W denotes money wages and r denotes the rate of profit on capital, then

$$p\frac{\partial\phi}{\partial M}=rP \text{ and } p\frac{\partial\phi}{\partial N}=W. \tag{1.2}$$

But so soon as we depart from such a comparison of stationary states, the innocent-looking assumption $O=\phi(M, N)$ begins to land us in difficulties. The production function, like Alice's flamingo, will

[1] D. G. Champernowne, 'The Production Function and the Theory of Capital', *Review of Economic Studies*, 1955, p. 112.

not keep still but wriggles about and regards us with an injured expression so soon as we attempt to make use of it.[1, 2]

Intuition suggests that provided accumulation is slow and machinery is not very durable, the errors introduced by using the reduced form of the production function should be of the second order of small quantities and of no great practical importance. But in order to throw some light on the question of when the production function can safely be employed and when not, it is of some interest to examine various particular models of capital accumulation, and to see whether reduced-form production functions can be obtained for them.

Many of the causes which make the production function alter when we change such parameters as the growth rate of population or the saving propensitites of wage-earners or capitalists, arise from the fact that machinery is durable but not indestructible. This property of machinery faces us with a dynamic model with a hotchpotch of machinery of various ages, best suited to the conditions of various past dates, and the relative values of different items of this hotchpotch keep altering ; it is largely this which makes the production function change when we alter the parameters of growth or thrift. Similar difficulties can be caused by changes in the distribution of income reacting on the relative prices of different kinds of consumption goods.

We shall study a model in which these particular sources of trouble have been rooted out by suitable assumptions. On the other hand, instead of supposing that there is only one type of consumption good, or equivalently, that consumption goods, although of many kinds, are always bought in baskets of the same composition, we shall allow substitution between consumption goods in response to price changes. We shall also allow investment in machinery for producing some consumption goods to run ahead of investment for producing others : indeed that will be a main feature of the process of accumulation which we shall study.

The model to be described owes much to the ideas of Mr. D. M. Bensusan-Butt although it differs in many important respects from his own model.[3] As in his model, the form which the fall in the rate of profit on capital and the rise in real wages will take, is that the prices of the various consumption goods begin each to fall as their

[1] N. Kaldor, 'A Model of Economic Growth', *Economic Journal*, 1957, p. 591.

[2] D. G. Champernowne, 'Capital Accumulation and the Maintenance of Full Employment', *Economic Journal*, 1958, p. 211.

[3] D. M. Bensusan-Butt, 'Some Elementary Theory about Accumulation', *Oxford Economic Papers*, 1954, p. 306 ; and 'A Model of Trade and Accumulation', *American Economic Review*, 1954, p. 511.

production is mechanized. Capital accumulation takes the form not of the gradual modification of production techniques to embody more and more machinery, but by the once-for-all mechanization in turn of each of a great many different processes. These processes are represented in the model as processes producing finished consumption goods — that is only one stage of production of consumption goods is recognized (apart from the manufacture of machinery to use in this one stage).

It turns out that if the savings parameters A and B, the population growth parameter v, and technical knowledge parameter λ are held constant, and if land is freely available, then real output per head is a homogeneous function $\phi(M, N)$ of degree one in N and M, namely men employed and the amount of robots. Moreover, the wage rate and the rate of profit on capital satisfy the equations (1.2) above. As we should expect, the function $\phi(M, N)$ is altered if we change the technical knowledge parameter λ, but it is distressing to find that a change in the savings parameters A and B can also alter $\phi(M, N)$.

This difficulty can be overcome if we make our production function $\phi(M, N)$ refer only to the output of consumption goods and to the quantities of factors M and N_1 directly employed in making them. The functions $\phi(M, N_1)$ then turns out to be unaffected by alterations to A and B and v and the marginal equations (1.2) remain valid.

If we are given the form of the function $f(x)$ (which shows how the cost of machines varies with the kind of goods in whose manufacture they are used) and know also the parameters λ, for technical knowledge, A and B for savings propensities and v for population growth, then the function $\phi(M, N_1)$ can be derived and, under suitable circumstances, the whole development of the economy can be predicted.

II. DESCRIPTION OF THE MODEL

1. *Definitions and Assumptions*

The Unit of Time is the *Week*.

Labour is homogeneous and its unit is the *Man*.

Consumption goods are of various kinds (x) where x is a continuous variable which may take any value between 0 and 1.

Robots are of various kinds (R_x) specialized for making consumption goods of the various kinds (x); their efficiency remains unimpaired for ever.

Production of consumption goods of any specified kind (x) in a flow of K units per week is possible by means of either

(i) K men working unaided.
(ii) λK men working with the aid of K robots of kind R_x. (λ is a constant depending on the state of technical knowledge but independent of x; K is any positive constant.)

Money is measured in units of 100 dollars.

Money wage rates are fixed for ever at one per time-unit ($100 per week).

Money expenditure on consumption goods is assumed to have a fixed pattern in the sense that, for any band of values of x, the proportion of total money expenditure on consumption which goes on goods of kinds (x) within that band is constant over time.

The scale of measurement of x is so chosen that the proportion of consumption expenditure devoted to goods of kinds $x(x<X)$ is equal to X.

Production of robots of type R_x in a flow of K per week is possible only by $Kf(x)$ men working unaided. Robots will not make robots.

The ordering of x is so arranged that $f(x)$ is a monotonic increasing function of x (i.e. the cheaper is the robot for making a kind of good, the smaller is the value of x denoting that kind).

Perfect competition is assumed in the sense that

(i) the money price of all kinds of consumption goods made by men unaided is unity, thus exactly covering their production cost;
(ii) the money prices of all kinds of goods made by men aided by robots are such as to allow the same short-term rate of profit on the cost-value of each kind of robot in use;
(iii) without increasing prices it would not be possible, by introducing robots into the production of any kind of good, to earn a higher short-term rate of profit than that actually earned by any robot.

Constant savings-proportions of A and B are assumed to apply to profits and wages respectively ($1 \geqslant A \geqslant B \geqslant 0$).

Full employment of robots is assumed. This is taken to imply that the distribution of new robots among kinds is nicely adjusted so that the price-pattern of consumption goods just allows the short-term profit rates on all kinds of robots in existence to remain equal.

The number of men is assumed either to remain constant or to expand with a constant positive growth rate.

2. *Some Remarks on the Working of the Model*

By assuming that robots are made by men alone and that their efficiency is unimpaired for ever, we sidetrack all the difficulties of measurement of capital. We can, without a blush, use J.R. units of capital,[1] namely man-weeks and, since we have fixed money wages at 100 dollars a week, we can take a unit of capital to be 100 dollars worth of robots. It is easiest to think of this as robots *costing* 100 dollars, but our assumptions about perfect competition and full employment of robots also ensure that the robots will indeed be worth their cost to anybody caring to use them, so there is no difference here between their cost-value and their demand-value.

When our model is working normally the short-term rate of profit on robots will be falling as accumulation proceeds. In this normal case, the level of this profit rate $r(t)$ at any moment t will divide robots into three classes :

(i) those that need a price greater than unity for their products, if they are to earn the profit rate $r(t)$. These robots cannot yet have been produced ;
(ii) robots that need a price less than unity for their products in order to earn the profit rate $r(t)$. Robots of these kinds must already exist and must already have driven out all labour (except that working with the robots) ;
(iii) robots that need a price equal to unity for their products in order to earn the profit rate $r(t)$. These robots will now be built for the first time.

Corresponding to the three classes of robots there will be three kinds of consumption goods ; those not yet made with the aid of robots ; those already made only with the aid of robots ; and those now for the first time made with the aid of robots. This last class will be a single kind of good only X, say, and owing to our choice of scale of x, the three sets of goods are (i) kinds $x>X$, (ii) kinds $x<X$, and (iii) the kind X.

We may refer to these sets as goods outside, inside and on the margin of mechanization.

3. *Stagnation*

When the model is working normally the profit rate continually falls and the margin of mechanization continually advances so that

[1] Joan Robinson, 'The Production Function and the Theory of Capital', *Review of Economic Studies*, 1955, p. 81 ; and *The Accumulation of Capital*.

X increases from 0 towards 1. However, even with a fixed working force X will not necessarily ever reach 1, and with an expanding working force X may actually stop short of 1. A great deal depends on the form of the function $f(x)$ which shows how the cost of robots increases as the margin of mechanization is advanced.

However, under many circumstances the margin of mechanization will reach the limiting position $X=1$. By then every worker in the consumption sector will be aided by $1/\lambda$ robots. This, however, is not quite the end of the story, as there are still two outlets for investment. First, the expansion of population will involve the need for more robots of every kind : this, however, will not be enough to absorb all the savings. In addition, if the profit rate falls, so will the proportion of savings to income and there will be a surge of workers from the robot-making sector to the consumption sector ; the need to provide robots to aid these workers will act as a stimulus to investment. Indeed, now that the margin of mechanization has reached the limiting position, prices and the profit rate will fall just fast enough to ensure that the demand for robots is exactly sufficient to absorb the available savings.

If the population growth rate is sufficient,[1] a dynamic equilibrium can be reached with profit just low enough to keep savings down to the level required to balance the need for new robots. But if the population growth rate is insufficient then profits will disappear altogether and stagnation will ensue.

In so far as this breakdown had been foreseen it would of course have lowered the demand for new robots at an earlier date. In order to make sense of our assumptions about investment in robots we must therefore suppose that nobody ever foresees the eventual stagnation of the economy ; or that, if they do, they place it beyond their planning horizon ; or alternatively that the government undertakes to buy up their robots at cost if such stagnation occurs.

4. *The Raising of Living Standards*

Even when population is stationary, the output of consumption goods will be expanding so as to raise living standards both for workers and robot-owners. This expansion will not be the same for all kinds of goods. Indeed the output of goods still outside the margin of mechanization will actually fall if the distribution of income is moving in favour of wages. For since the total money wages bill is fixed if population is constant, a redistribution of income away from profits implies a reduction in total money profits and hence a re-

[1] The level of sufficiency is the ratio of B to the capital-output ratio when $X=1$.

duction in money expenditure on each kind of good. Since money prices of goods outside the margin of mechanization are fixed at unity, any reduction in money expenditure upon them must cause a reduction in their output. This will be accompanied, of course, by a shift of labour across the margin of mechanization to man the new robots and expand the output of these robot-made goods, whose prices will fall just sufficiently to secure an adequate market for them despite the fall in money expenditure on them. It is this reduction in prices which constitutes the increase in real wages. The rise of living standards takes the form of a greatly increased consumption of the cheapened robot-made goods, partly offset by a reduction in the capitalist consumption of the relatively dearer hand-made goods. Workers' consumption of hand-made goods remains unaltered. In the contrary case where income shifts towards profits, the output of all kinds of goods expands.

In order to measure the increase of living standards it is convenient to define an index number of the prices of consumption goods. In the normal case, all prices which change will be falling and there is no difficulty in defining such an index. We simply take a chain index p with very short links ; the index taken over any such short period, being a Laspeyre index based on the beginning of the period. The reciprocal of this index p may be considered an index of real wages.

If we deflate money expenditure on consumption by the price-index, we obtain the quantity of output of consumption goods. It can be shown that this quantity O satisfies a relation of the form $O=\phi(M, N_1)$ where M is the money-value of robots, N_1 is the number of men employed in the consumption sector and ϕ depends only on λ and $f(x)$ (the robot-cost function), but is unaffected by changes in the savings parameters A and B or by changes in the rate of growth of population. The same result cannot, it appears, be obtained for a production function including robots as well as consumption goods.

The function ϕ also satisfies the marginal conditions.

$$p\frac{\partial\phi}{\partial M}=r \qquad p\frac{\partial\phi}{\partial N_1}=1$$

$$\phi=M\frac{\partial\phi}{\partial M}+N_1\frac{\partial\phi}{\partial N_1}$$

(r =short-term rate of profit on robots ; 1 =rate of money wages).

These assertions are proved in the Mathematical Appendix.

III. THE PRODUCTION FUNCTION

If a particular form $f(x)$ is assumed for the robot-cost function, and a particular value is assumed for λ, then we can express many important economic variables in terms of r, the short-term rate of profit on robots.

For example, for any kind of good inside or on the margin of mechanization X, the price must be

$$p(x) = \lambda + rf(x), \tag{3.1}$$

where λ is the wage-cost and $rf(x)$ the profit or interest-cost. Again, at the margin of mechanization, X, price must be unity; thus

$$p(X) = 1, \tag{3.2}$$

whence, by (3.1), the margin of mechanization is given by

$$f(X) = \frac{1 - \lambda}{r}. \tag{3.3}$$

It is further shown in the Mathematical Appendix that the money-value of output per man in the whole consumption sector is given by

$$y(r) = 1/[1 - h(r)], \tag{3.4}$$

where $h(r)$ is the ratio of profits to the value of output in the consumption sector and

$$h(r) = \int_0^X \left\{1 - \frac{\lambda}{p(x)}\right\} dx, \tag{3.5}$$

X being the margin of mechanization, obtainable from (3.3).

The average value of robots per man employed in the *whole* consumption sector can also be obtained as a function of r: it is

$$u(r) = h(r)/r\{1 - h(r)\}. \tag{3.6}$$

This expression can be obtained directly from (3.4) by using the condition that the money-value of output per man $y(r)$ is the sum of wages per man (unity) and profits per man, $ru(r)$.

The price-index p of consumption goods can also be obtained as p_r, a function of r: it is shown in the appendix that

$$\frac{d}{dr} \log_e p_r = h(r)/r, \tag{3.7}$$

so that

$$\log_e p_r = \int_{r_0}^{r} \frac{h(r)}{r} dr, \tag{3.8}$$

where r_0 is the initial level of the short-term profit rate.

The above relations are all independent of the savings propensities and the rate of growth of population, and enable us to use the relations

$$p_r\phi(M, N_{\mathrm{I}}) = N_{\mathrm{I}}y(r), \tag{3.9}$$
$$M = N_{\mathrm{I}}u(r), \tag{3.10}$$

to find all we may wish to know of the consumption sector production function $\phi(M, N_{\mathrm{I}})$.

Examples

EXAMPLE I

Suppose $\lambda = 0{\cdot}2$, then one man can operate 5 robots to produce a flow of 5 units a week of some particular kind of consumption good.

Let $f(x) = 100/(1-x)$ (3.11), then the cheapest kind of robot (R_0) costs 100 units, i.e. 10,000 dollars. A robot of kind $R_{0{\cdot}5}$ would cost 20,000 dollars and so forth.

Initially, when the first robot is built, we have $X = 0$ and the short-term rate of profit will be given by (3.3) as

$$r_0 = 0{\cdot}008, \tag{3.12}$$

which means 0·8 per cent per week.

After that, as r falls, the margin of mechanization will advance and will be given by (3.3) as

$$X = 1 - 125r. \tag{3.13}$$

The prices of consumption goods of kind x will by (3.1) be given by

$$p(x) = 0{\cdot}2 + \frac{100r}{1-x} \quad (x < X)$$
$$p(x) = 1 \qquad\qquad (x \geqslant X). \tag{3.14}$$

The function $h(r)$ defined in (3.5) may be found as

$$h(r) = 500r \log_e \{1 + 500r/625r\}. \tag{3.15}$$

Hence by (3.4), the value of output per man in the consumption sector is

$$y(r) = 1 \Big/ \left\{1 - 500r \log_e \frac{1+500r}{625r}\right\}. \tag{3.16}$$

and by (3.6) the average value of robots per man employed in the whole consumption sector is

$$M/N_{\mathrm{I}} = u(r) = 500 \log_e \frac{1+500r}{625r} \Big/ \left\{1 - 500r \log_e \frac{1+500r}{625r}\right\}. \tag{3.17}$$

TABLE 23

RESULTS FOR EXAMPLE I

(Assuming 50 million men employed in Consumption Sector)

Rate of Profit ‰ per Week r	8	7	6	5	4	3	2	1	0·5	0·2	0·1
Margin of mechanization X	0·000	0·125	0·250	0·375	0·500	0·625	0·750	0·875	0·9375	0·975	0·9875
Value of robots (million dollars)	0	781	2,001	2,953	7,174	12,651	22,170	38,926	53,039	69,479	82,119
Money value of consumption goods output (million dollars per week)	50·0	55·5	62·0	69·8	78·7	88·0	94·3	88·9	76·5	63·9	58·2
Price-index of consumption goods	100·0	99·3	97·1	92·9	86·4	77·0	64·0	46·5	35.4	27·3	24·2
Volume of consumption goods output (million dollars per week at base-year prices)	50·0	55·8	63·9	75·1	91·1	114·3	147·4	191·3	216·4	233·7	240·7
Capital-output ratio (weeks)	4	14	32	57	91	144	235	438	693	1087	1411

Comparing (3.16) and (3.17) we can see that the capital-output ratio in the consumption goods sector is

$$a(r) = \frac{u}{y} = 500 \log_e \{(1 + 500r)/625r\}. \tag{3.18}$$

The price-index p_r of consumption goods, by equation (3.7) must satisfy

$$\log_e p_r = \int_r^{0\cdot008} 500\{\log_e (1 + 500u) - \log_e 625u\}du,$$

whence
$$p_r = 0\cdot2(1 + 500r)^{1+500r}(625r)^{-500r}. \tag{3.19}$$

Combining (3.16) and (3.17) and (3.19), we find that when N_1 men are employed in the consumption goods sector, and the profit rate is r, the value of robots is

$$M = uN_1 = 500N_1 \log_e \frac{1 + 500r}{625r} \Big/ \left\{1 - 500r \log_e \frac{1 + 500r}{625r}\right\} \tag{3.20}$$

and the quantity of consumption goods produced is

$$\phi(M, N_1) = N_1 y(r)/p_r$$

$$= 5N_1(625r)^{500r} \Big/ \left\{1 - 500r \log_e \frac{1 + 500r}{625r}\right\}\{1 + 500r\}^{1+500r}. \tag{3.21}$$

Taking $N_1 = 50{,}000{,}000$ Table 23 gives the position of the margin of mechanization, the value of machinery, the value of consumption goods output and the volume of consumption goods output for various values of r.

EXAMPLE II

As a second example we may consider the case where $\lambda = 0\cdot2$ and $f(x) = 500x$. The corresponding equations for the various economic indices in terms of r are then

$$r_0 = \infty \tag{3.22}$$

$$X = 0\cdot0016/r \tag{3.23}$$

$$p(x) = 0\cdot2 + 500rx \tag{3.24}$$

$$h(r) = \beta/r \qquad (\beta = 0\cdot00095622) \tag{3.25}$$

$$p_r = e^{-\beta/r} \tag{3.26}$$

$$M = \beta N_1/r(r - \beta) \tag{3.27}$$

$$\phi(M, N_1) = re^{\beta/r}N_1/(r - \beta). \tag{3.28}$$

TABLE 24

RESULTS FOR EXAMPLE II ON THE BASIS OF 50 MILLION MEN EMPLOYED IN CONSUMPTION SECTOR

Margin of mechanization	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	0·95	0·98	1·00
Rate of profit per week		16·00	8·00	5·33	4·00	3.20	2·67	2·29	2·00	1·78	1·68	1·63	1.60
Value of robots (million dollars)	0	199	848	2048	3927	6659	10482	15733	22903	32735	33995	43293	46417
Money value of consumption goods output (million dollars per week)	50·00	53·18	56·79	60·92	65·71	71·31	77·95	85·96	95·81	108·20	115·68	120·68	124·26
Price-index of consumption goods	100	94·2	88·7	83·6	78·7	74·2	69·9	65·8	62·0	58·4	56·7	55·7	55·0
Volume of consumption goods output (million dollars per week)	50·00	56·45	64·00	72·89	83·45	96·14	111·57	130·61	154·53	185·27	204·08	216·78	225·90
Capital output ratio (weeks)	0	3·73	14·94	33·62	59·76	93·38	134·47	183·0	239·0	302·6	337·1	358·7	373·5
Proportionate share of profits per cent	0	5·98	11·95	17·93	23·91	29·88	35·86	41·84	47·81	53·79	56·78	58·57	59·76

Table 24 gives, for $N=50$ million, the position of the margin of mechanization, the value of machinery, the value of consumption goods output and the volume of consumption goods output for various values of r.

In this example we can even express $\phi(M, N_1)$ as an algebraic function of M and N_I, since by solving (3.27) as a quadratic in r we find

$$r = \beta/2 + \sqrt{\beta^2/4 + \beta N_\text{I}/M}, \tag{3.29}$$

whence substituting in (3.28)

$$\phi(M, N_\text{I}) = \frac{M\beta}{2}\left\{\sqrt{1 + \frac{4N_\text{I}}{M\beta}} + 1\right\} e^{\frac{M\beta}{2N_\text{I}}\left\{\sqrt{1+\frac{4N_\text{I}}{M\beta}}-1\right\}}. \tag{3.30}$$

IV. THE GROWTH RATE OF CAPITAL

1. *The Warranted Growth Rate*

If A and B are the proportions saved out of profits (Mr) and wages (N), then total savings are

$$\frac{dM}{dt} = AMr + BN. \tag{4.1}$$

But since wages are unity and all costs in the investment sector are wage-costs, the value of investment is $N - N_\text{I}$, and hence

$$N - N_\text{I} = AMr + BN. \tag{4.2}$$

This may be rewritten in the form

$$N/M = Ar/(1 - B) + N_\text{I}/M(1 - B). \tag{4.3}$$

Hence by (3.6)

$$\frac{N}{M} = \left\{\frac{r}{1 - B}\right\}\left\{A + \frac{[1 - h(r)]}{h(r)}\right\}. \tag{4.4}$$

Dividing (4.1) by M we find the growth rate of capital as

$$\frac{dM}{dt}\Big/M = \left\{\frac{r}{1 - B}\right\}\left\{A + \frac{B[1 - h(r)]}{h(r)}\right\}. \tag{4.5}$$

This growth rate, which is what is necessary to absorb the available savings and so preserve full employment, is known as the warranted growth rate of capital.[1]

2. *The Natural Growth Rate*

With this rate we may compare the natural growth rate of capital, namely that which would have to be provided for out of savings if

[1] Trevor Swan, 'Economic Growth and Capital Accumulation', *Economic Record*, November 1956.

the rate of profit r was to be held steady. In the absence of technical progress, i.e. if λ and $f(x)$ remained fixed, this natural growth rate of capital must simply equal the growth rate of population. Before considering how technical progress, in the shape of a falling value of λ, will increase the natural growth rate of capital, we may first consider the calculation of the warranted growth rate in our two examples. Suppose $A = 0{\cdot}5$ $B = 0{\cdot}1$ are the proportions saved out of profits and wages :

Example 4.1 $\quad \lambda = 0{\cdot}2 \quad f(x) = 100/(1-x)$
$\quad A = 0{\cdot}5 \quad B = 0{\cdot}1.$

As shown in example 3.1 above

$$h(r) = 500r \log_e \{(1+500r)/625r\}.$$

Hence by equation (4.5) the warranted growth rate of capital is

$$W(r) = r\left\{1 + 2000r \log_e \frac{1+500r}{625r}\right\}\Big/ 4500r \log_e \frac{1+500r}{625r}. \tag{4.6}$$

Example 4.2

$$\lambda = 0{\cdot}2 \quad f(x) = 500x \quad A = 0{\cdot}5 \quad B = 0{\cdot}1.$$

As shown in example 3.2 above equation (3.25)

$$h(r) = \beta/r \quad \beta = 0{\cdot}00095622.$$

Hence by equation (4.5) the warranted growth rate of capital is

$$W(r) = r(r+4\beta)/9\beta. \tag{4.7}$$

Table 25 compares for various selected values of r the positions of the margin of mechanization and warranted growth rates of

TABLE 25

WARRANTED GROWTH RATE W AND MARGIN OF MECHANIZATION X FOR VARIOUS LEVELS OF THE PROFIT RATE r

(per thousand per week)

r	8·0	6·0	4·0	3·0	2·0	1·6	1·0
W 1st example	∞	6·11	3·00	2·11	1·36	1·12	0·70
W 2nd example	10·99	6·85	3·64	2·38	1·35	1·01 stop	—
X 1st example	0	0·250	0·500	0·625	0·750	0·800	0·875
X 2nd example	0·200	0·167	0·400	0·533	0·800	1·000	—

capital in these two examples. Fig. 18 compares the warranted growth rates.

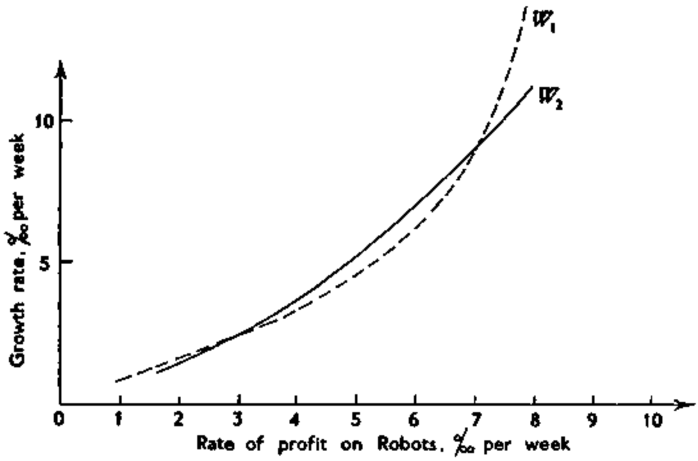

FIG. 18

In the second example, if we are given also the growth rate of the population to be 0·0001 per week (i.e. about 5 ‰ per annum), it is possible to find the time schedule for the fall of r and hence for the other economic variables. This is done by solving the differential equation (4.8) which is explained in the Mathematical Appendix.

$$-\frac{dt}{dr}=(4r-\beta)/r(2r-\beta)\left\{\frac{r^2}{9\beta}+\frac{4r}{9}-0\cdot0001\right\}. \tag{4.8}$$

Table 26 and Fig. 19 show the time-schedules thus obtained for the fall of r.

TABLE 26

LENGTH OF TIME TAKEN FOR RATE OF PROFIT TO FALL TO VARIOUS LEVELS

Rate of profit per thousand per week	10	8	6	5	4	3	2	1·6
Margin of mechanization	0·16	0·22	0·267	0·32	0·4	0·533	0·8	1·0
Time in weeks	70	105	175	241	354	575	1124	1623
Time in years of 50 weeks	1·7	2·1	3·5	4·8	7·1	11·5	22·5	32·5

At the end of the 32½ years the economy would have reached saturation. In interpreting this result it must be remembered that we are allowing no technical progress and that the average value of robots per man cannot rise above 50,000 dollars : a large proportion of income is profits and that half of these are saved : furthermore, no robot wears out. Moreover, accumulation can continue a little longer after X has reached the value 1 through capital widening as profits fall further, savings are correspondingly reduced and labour flows from making robots to making consumption goods.

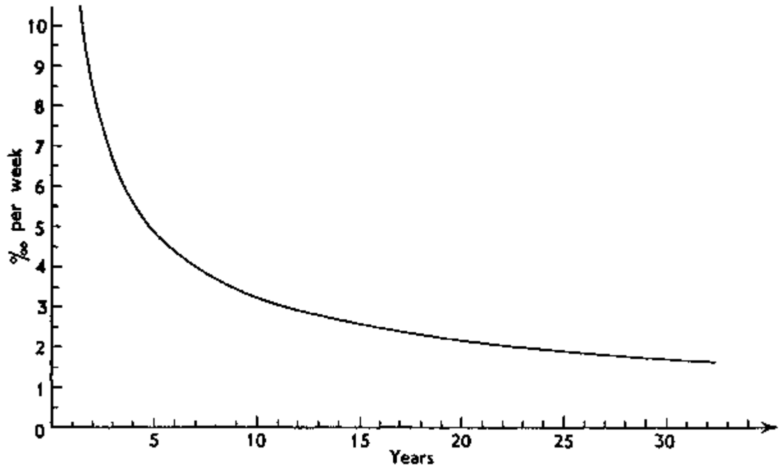

FIG. 19

3. *Technical Progress*

Any advance in technical knowledge is likely to increase the natural growth rate of capital.

In our model such advance can be represented either by a steady lowering of λ or else by a steady reduction of the robot cost-functions $f(x)$. The former corresponds to labour-saving inventions and the latter to capital-saving inventions.

The effect of the former type of invention may be briefly considered.

We suppose that there is no change in the kinds or cost of robots, but methods are discovered of saving labour in their use ; thus there is a steady increase in $1/\lambda$, the number of robots attended by each man in those industries which lie inside the margin of mechanization.

In order to find the corresponding effect on the natural rate of growth of capital, we have to find what growth rate of the total value of robots would be required with fixed population in order to keep the rate of profit on robots constant despite a given rate of fall of λ.

This can be found from the equation (4.4)

$$\frac{M}{N}=\frac{(1-B)h(r)}{r[1-(1-A)h(r)]}. \tag{4.9}$$

A change in λ will alter M only through its effect on $h(r)$.

Regarding h now as a function $h(r, \lambda)$ of both r and λ, we see that the growth rate in capital necessitated by the change in λ if r is to remain constant with fixed population is

$$T=\frac{\partial h}{\partial\lambda}\left\{\frac{1}{h}+\frac{1-\lambda}{1-(1-A)h}\right\}\frac{d\lambda}{dt}=\frac{\frac{\partial h}{\partial\lambda}\;\frac{d\lambda}{dt}}{h[1-(1-A)h]}. \tag{4.10}$$

We may call this value T, the rate of technical progress. If we add T to the growth rate v of population, their sum

$$N=T+v \tag{4.11}$$

is the natural growth rate of capital which must be achieved if r is to be held constant. The rate of profit, r, will rise or fall accordingly as the natural growth rate exceeds or falls short of the actual rate.

Examples

Suppose $f(x)=500x \quad A=0{\cdot}5 \quad B=0{\cdot}1 \quad v=0{\cdot}0001$

$$\lambda=0{\cdot}5e^{-0{\cdot}0005t}.$$

From equations (3.1) (3.3) (3.5) it follows that

$$h(r\lambda)=(1-\lambda+\lambda\log_e\lambda)/500r$$

$$=\left\{\frac{1-0{\cdot}5e^{-0{\cdot}0005t}(1+\log_e 2+0{\cdot}0005t)}{500r}\right\} \tag{4.12}$$

and that $$\frac{\partial h}{\partial\lambda}=\log_e\lambda/500r=(-0{\cdot}0005t-\log_e 2)/500. \tag{4.13}$$

We also have $$\frac{d\lambda}{dt}=-0{\cdot}00025e^{-0{\cdot}0005t}. \tag{4.14}$$

Hence by equation (4.10)

$$T=\frac{(\text{Log}_e\, 2+0{\cdot}0005t)e^{-0{\cdot}0005t}}{4000-2000e^{-0{\cdot}0005t}(1+\log_e 2+0{\cdot}0005t)}, \tag{4.15}$$

and $$N=T+0{\cdot}0001 \tag{4.16}$$

we find that the natural growth rate of capital is independent of the rate of profit but that it is changing through time in the manner shown in Table 27.

TABLE 27

THE FALL OF THE NATURAL GROWTH RATE OF CAPITAL

(Example 3)

Time elapsed: Years of 50 weeks	0	5	10	15	20	25	30	35	40	45	50
Profit rate ‰ per week	1·23	1·01	0·36	0·73	0·64	0·56	0·50	0·45	0·41	0·37	0·34

V. INTRODUCTION OF LAND INTO THE MODEL: SUMMARY OF INTERIM RESULTS

It would be interesting to discover whether if homogeneous land were introduced as a third factor of production into the model, it would be possible to establish that the production function for the consumption sector would now take the form $\phi(L_1, M, N_1,)$, where L_1, M and N_1 were the quantities of land, robots and men used in the consumption sector, and whether ϕ would be independent of the savings parameters and would satisfy the marginal equations

$$p_r\frac{\partial\phi}{\partial L_1}=\rho \quad p_r\frac{\partial\phi}{\partial M}=Pr \quad p_r\frac{\partial\phi}{\partial N_1}=w \tag{5.1}$$

where ρ is rent per unit of land; P the price index for robots; w the money-wage rate; and p_r the price-index of consumption goods.

At the moment of writing, this problem remains unsolved, but there seems some prospect that the following approach would be fruitful. We suppose that land is *not* needed for production involving robots as well as men: thus robots are to some extent a substitute for land. But men unaided by robots need land for production. Instead of producing a unit flow of any good, a man produces a flow of $R(z)$ where z is the number of units of land that he works with. We suppose

$$R(1)=1 \quad \frac{dR}{dz}>0\frac{d^2R}{dz^2}<0. \tag{5.2}$$

We also suppose that $f(x)$ men working with $zf(x)$ units of land can produce a flow of $R(z)$ robots of the type of making goods x.

If we now choose units of money such that the costs of goods *outside* the margin of mechanization still satisfy

$$p(x)=1 \quad (x>X) \tag{5.3}$$

provided only the most economical proportions of land and labour are used; then it will follow that with perfect competition,

$$P=\frac{dR}{dz} \quad w=R(z)-z\frac{dR}{dz}, \tag{5.4}$$

where z is the amount of land available per man employed in the land-using industries.

It can also be shown that the value of production per man employed in the consumption sector is given by the equation

$$\frac{Y}{N_1} = \frac{R(z)}{1 - h(r)}, \tag{5.5}$$

and the price-index for consumption goods still satisfies the equation

$$\log p_r = \int_{r_0}^{r} \frac{h(r)}{r} dr. \tag{5.6}$$

The proportionate shares of the three factors employed in the consumption sector can be found as

$$\frac{wN_1}{Y} = w[1 - h(r)] \tag{5.7}$$

$$\frac{Mr}{Y} = (1 - w)X + wh(r) \tag{5.8}$$

$$\frac{\rho L_1}{Y} = (1 - w)(1 - X). \tag{5.9}$$

A model with land thus introduced could perhaps be used to illustrate the effects of a fixed land supply and an expanding population on the distribution of income between the three factors. Or, at the same time, it would illustrate the effects of the substitution of capitalistic methods in mitigating the effects of the land shortage.

MATHEMATICAL APPENDIX

Notation

N	Number of men = Total wages
N_1	Number of men making consumption goods
$N - N_1$	Number of men making robots = Value of investment
x	Kind of consumption good
X	Margin of mechanization
M	Total value of robots
Mr	Profits
$h(r) = Mr/Y$	Ratio of profits to value of output in consumption sector
r	Rate of profit (short term) on robots
$m(x)$	Density function of robots along scale of x
$f(x)$	Cost of robot to make goods of type x

$1/\lambda$	Number of robots per man in mechanized industry
$Y = N_I + Mr$	Value of output of consumption goods
$p(x)$	Price of consumption goods of kind x
p_r	Price-index of consumption goods
$y = Y/N_I$	Value of consumption good output per man employed
$Y/p_r = \phi(M, N_I)$	Production function for consumption goods
$u = M/N_I$	Value of robots per man in whole consumption sector
$\alpha = M/Y$	Capital-output ratio
S, A, B	Savings ratios for all income, profits and wages

Our assumptions ensure that money-wage rates are all unity and that when the margin of mechanization is at X, the prices of the various kinds of consumption goods satisfy.

$$p(x) = 1 \quad (x \geqslant X) \tag{1}$$

$$p(x) = \lambda + rf(x) \quad (x \leqslant X), \tag{2}$$

hence

$$r = (1 - \lambda)/f(X), \tag{3}$$

where r is the short-term rate of profit on robots and $f(x)$ is the cost of a robot.

The total expenditure Y on consumption goods satisfies the equation

$$Y = N_1 + Mr, \tag{4}$$

where N_1 is the number of men employed in the consumption sector and M is the total value of robots, since all robots are employed in this sector.

Since this expenditure is spread evenly along the x-scale the density of spending is Y along this scale from 0 to 1. Hence since each robot is associated with a unit flow of goods

$$m(x)p(x) = Y \quad (0 < x \leqslant X). \tag{5}$$

Now the value of profits in the consumption sector is by (5)

$$Mr = \int_0^X m(x)\{p(x) - \lambda\}dx = Y\int_0^X \left\{1 - \frac{\lambda}{p(x)}\right\}dx. \tag{6}$$

Hence the ratio of profits to value of output in this sector is

$$h(r) = Mr/Y = \int_0^X \left\{1 - \frac{\lambda}{p(x)}\right\}dx. \tag{7}$$

The total wages N_1 in this sector must therefore be

$$N_I = Y - Mr = \{1 - h(r)\}Y. \tag{8}$$

Thus the value of output per man employed is in this sector

$$y(r) = Y/N_I = 1/\{1 - h(r)\}, \tag{9}$$

whereas the value of robots per man employed in this sector is

$$u(r) = M/N_{\mathrm{I}} = h(r)/r\{1 - h(r)\}, \tag{10}$$

and the capital-output ratio is

$$\alpha(r) = M/Y = h(r)/r. \tag{11}$$

We define the price-index of consumption goods to be a chain-index with very short links always weighted according to current expenditure. Hence since expenditure is always spread uniformly along the x-scale

$$\frac{d}{dt}\log p_r = \int_0^1 \frac{d}{dt}\log p(x)dx = \int_0^X \frac{d}{dt}\log p(x)dx. \tag{12}$$

Let p_r denote the value of p when the rate of profit is r then

$$\frac{d}{dr}\log p_r = \int_0^X \frac{d}{dr}\log\{\lambda + rf(x)\}dx = \int_0^X \frac{f(x)}{p(x)}dx$$

$$= \frac{1}{Y}\int_0^X m(x)f(x)dx = M/Y = h(r)/r.$$

Thus
$$\frac{d}{dr}\log p_r = M/Y = h(r)/r. \tag{13}$$

Hence
$$\log p_r = -\int_r^{r_0}\frac{h(r)}{r}dr, \tag{14}$$

where r_0 is the value of r initially when $X = 0$.

Since by definition of $\phi(M, N_{\mathrm{I}})$

$$p_r\phi(M, N_{\mathrm{I}}) = Y = Mr + N_{\mathrm{I}} \tag{15}$$

we obtain, by differentiation with respect to M, holding N_{I} constant and regarding M as a function of r and of N_{I},

$$p_r\frac{\partial\phi}{\partial M} + \phi(M, N_{\mathrm{I}})\frac{dp_r}{dr}\bigg/\frac{\partial M}{\partial r} = r + M\bigg/\frac{\partial M}{\partial r}. \tag{16}$$

But by (15) and (13)

$$\phi(M, N_{\mathrm{I}})\frac{dp_r}{dr} = Y\frac{d}{dr}\log p_r = M, \tag{17}$$

so that by (16)
$$p_r\frac{\partial\phi}{\delta M} = r. \tag{18a}$$

Similarly, differentiating (15) with respect to N_{I}, holding M constant and regarding N_{I} as a function of M and r,

$$p_r\frac{\partial\phi}{\partial N_{\mathrm{I}}} + \phi(M, N_{\mathrm{I}})\frac{dp_r}{dr}\bigg/\frac{\partial N_{\mathrm{I}}}{\partial r} = 1 + M\bigg/\frac{\partial N_{\mathrm{I}}}{\partial r},$$

and we may again use (17) to obtain

$$p_r \frac{\partial \phi}{\partial N_{\mathrm{I}}} = 1. \tag{18b}$$

Equations (18a) and (18b) establish that the partial derivatives of the production function when multiplied by the price-index equal the rate of interest and the wage rate of unity.

The differential equation given in the text as (4.8) was obtained as follows:

By equation (4.1), with $A = 0{\cdot}5$ and $B = 0{\cdot}1$

$$\frac{dM}{dt} = 0{\cdot}5Mr + 0{\cdot}1N. \tag{19}$$

Also by (4.4), since $h(r) = \beta/r$

$$\frac{N}{M} = \frac{5r(2r-\beta)}{9\beta},$$

$$\frac{d}{dr}\left(\frac{N}{M}\right) = \frac{5(4r-\beta)}{9\beta}. \tag{20}$$

We are assuming that

$$\frac{dN}{dt} = 0{\cdot}0001N \tag{21}$$

$$\therefore\ 0{\cdot}0001N = \frac{N}{M}\{0{\cdot}5Mr + 0{\cdot}1N\} + M\frac{5(4r-\beta)}{9\beta}\,\frac{dr}{dt},$$

$$\therefore\ \frac{0{\cdot}0005r(2r-\beta)}{9\beta} = \frac{5r(2r-\beta)}{9\beta}\left\{0{\cdot}5r + \frac{0{\cdot}5r(2r-\beta)}{9\beta}\right\} + \frac{5(4r-\beta)}{9\beta}\,\frac{dr}{dt},$$

$$\therefore\ 1{\cdot}000\frac{dr}{dt} = \frac{r(2r-\beta)}{10(4r-\beta)}\,\frac{9\beta - 45{\cdot}000r\beta - 5{\cdot}000r(2r-\beta)}{9\beta},$$

$$\therefore\ -\frac{dt}{dr} = (4r-\beta)\Big/ r(2r-\beta)\left\{\frac{r^2}{9\beta} + \frac{4r}{9} - 0{\cdot}0001\right\}, \tag{22}$$

which is the equation (4.8).

This equation is easily integrable and thus yields the figures given in Table 26 in the text.

Chapter 12

NOTES TOWARD A WICKSELLIAN MODEL OF DISTRIBUTIVE SHARES

BY

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It is always a comfort to read or re-read Wicksell. No other great economist is so candid, so cheerful about putting all his cards on the table and so unassuming about the strength of his hand. Even so technical a work as his 'Mathematical Analysis of Akerman's Problem' [1] is attractive for this reason as well as for the power and modernity of the capital theory it contains. In this brief essay (part of a book review written at the age of 72!) Wicksell works out a sort of streamlined, Austrian capital theory and does it in such a way that it would fall neatly into place in a full Walrasian general equilibrium scheme. At the same time the analysis is cast formally in terms of a simple macro-economic model, and it is from this point of view that I want to consider it.

Wicksell himself was not particularly concerned with the question of distributive shares and makes only a few remarks in passing. In this paper I propose to see what propositions about distribution can be teased out of the Akerman-Wicksell model. For this purpose it is necessary to complicate and extend the model a little so that it can accommodate, however crudely, capital accumulation and technical progress. There are still other extensions that fairly cry out to be made, but it is remarkable how soon the analysis becomes tedious and opaque and the pithy conclusions so dear to economists are no longer to be had.

I. THE AKERMAN-WICKSELL MODEL

I begin with the model much as Wicksell left it, in order to establish a notation, provide a standard of comparison, and make it as easy as possible to see what is going on. The scene is an isolated

[1] Reprinted in *Lectures on Political Economy* (London, 1935), vol. i, pp. 274-99.

economy engaged in producing a single consumption commodity (or a 'take it or leave it' combination of goods) by means of labour (all workers are alike) and a single type of durable machine. Wicksell makes it an axe; alternatively we might borrow Trevor Swan's fiction of the miraculous Meccano [1] set. In any case, the assumption that only one kind of durable asset exists clearly needs lifting. Labour and machines can be combined in continuously variable proportions and since the economy is a competitive one we avoid trouble by supposing that all production is carried on under constant returns to scale. Land and natural resources are to be thought of as abundant and free.

The economy also produces its own machines, again by use of labour and machines. Wicksell has axes produced by labour alone; this is a slight simplification, but for later use it is better to recognize that it takes machines to produce machines. The economy is in a state of long-run stationary equilibrium. Techniques do not alter, and there is no capital accumulation. The age distribution of the stock of machines is balanced and production of machines is confined to the replacement of worn-out ones. Unlike old soldiers, machines do not fade away; they die. They are built for a certain durability — the optimal length of life is one of the unknowns of the problem — and keep their productive characteristics intact until the end of that period, when they become valueless. On the other hand all production is instantaneous; there are no delay periods between input and output.[2] As Wicksell notes, there is a side of mechanization that is not captured in so simple a model. But since we already have three ways of reflecting changing capital intensity (the durability of machines and the machine-labour ratios in the production of consumption goods and machines) we may let it go at that.

Wicksell conducts his analysis on the assumption that all of the production functions involved are of constant-elasticity or Cobb-Douglas type. I shall do the same for definiteness. Up to a point this makes no difference: in any formula, one can think of the Cobb-Douglas constant as being simply the elasticity of an output with respect to an input, evaluated at the equilibrium point. But when we begin to consider changes in data and the corresponding

[1] Economic Growth and Capital Accumulation', *Economic Record*, 1957, p. 344.

[2] There would be no fundamental difficulty in introducing delay periods. If the delay periods were technically fixed then, as Lange showed long ago, they add nothing essential to the theory. ('The Place of Interest in the Theory of Production', *Review of Economic Studies*, 1935–1936, pp. 185 *et seq.*) If the delay periods are open to choice then we get some more equilibrium conditions and some new possibilities but, as long as there is only one kind of machine, nothing very different. (It is a bit startling to realize that Lange's paper is now twice as old as Wicksell's essay was when Lange wrote.)

displacements of equilibrium, then the assumption that these elasticities are constant does count for something. How much it counts for depends of course on how close it comes to being true.

To be precise, let Q_c represent the rate of output of consumption goods, and L_c and R_c the inputs of labour and machinery devoted to the manufacture of consumption goods. Then with a proper choice of units the production function for commodities is

$$Q_c = L_c^{\gamma} R_c^{1-\gamma}. \tag{1}$$

Because of competition we can also write

$$\gamma \frac{Q_c}{L_c} = w, \tag{2}$$

$$(1-\gamma)\frac{Q_c}{R_c} = r, \tag{3}$$

where w and r are the wage rate and the rental of machine time, in terms of consumption goods. Similarly let Q_R be the rate of output of machines, N_R their durability, and L_R and R_R the inputs of labour and machinery in the machinery industry. Then we have another production function

$$Q_R = L_R^{\alpha} R_R^{1-\alpha} N_R^{-\beta}. \tag{4}$$

Now if ρ is the instantaneous rate of interest, the value of a new machine in terms of consumption goods is easily calculated to be

$$\frac{r}{\rho}(1 - e^{-\rho N_R}),$$

and since in competition this must equal the unit cost of construction we have

$$\frac{r}{\rho}(1 - e^{-\rho N_R}) = \frac{wL_R}{Q_R} + \frac{rR_R}{Q_R}, \tag{5}$$

and the further equilibrium conditions

$$\frac{r}{\rho}(1 - e^{-\rho N_R})\frac{\alpha Q_R}{L_R} = w. \tag{6}$$

$$\frac{r}{\rho}(1 - e^{-\rho N_R})(1-\alpha)\frac{Q_R}{R_R} = r. \tag{7}$$

Obviously (6) and (7) imply (5), so that there are only two independent equations here. In addition, the optimal choice of durability, and of input-combination in the production of machines must maximize the difference between the right- and left-hand sides of (5) subject to the production function (4). This yields two more conditions:

$$e^{-\rho N_R} = \frac{\beta}{1-\alpha}\frac{R_R}{Q_R}\frac{1}{N_R}. \tag{8}$$

$$\frac{\alpha}{1-\alpha}\frac{R_R}{L_R} = \frac{w}{r}. \tag{9}$$

Of these, (9) is not new; it can be obtained by dividing (6) and (7). But (8) simply balances the addition to the present value of a machine as a result of an increment of durability against the marginal cost of adding to durability. Another relation is worth writing down: (5), (8) and (9) together imply

$$e^{\rho N_R} = 1 + \frac{\rho N_R}{\beta}. \tag{10}$$

This is interesting because it is a single equation in the one unknown ρN_R; thus in this model ρN_R is a constant, depending only on β. Any change in data which brings about a change in ρ is counter-balanced by an equiproportionate opposite change in N_R.[1]

Finally we have some balance equations. If the total labour force available is L and the existing stock of machines (without distinction as to age) is R, then

$$L = L_c + L_R. \tag{11}$$

$$R = R_c + R_R. \tag{12}$$

But since this is stationary equilibrium, the age distribution of machines is uniform and $1/N_R$th of the stock must be replaced per unit time; thus

$$Q_R = \frac{R}{N_R}. \tag{13}$$

Adding up, we have 10 independent equations in 12 unknowns. Wicksell takes the total supply of labour to be given; this makes L a constant and reduces the number of unknowns to 11. I shall do the same, although it would not be too difficult to let the supply of labour depend on the real wage. Whether this makes any real difference or not will be clearer in a moment. However the supply of labour is handled, the system is still one equation short. Indeed everything that has so far been said is compatible both with a capital-

[1] Equation (10) holds even if β is not a constant. But then a change in data will in general change β. If the change in β is negligibly small, so will be the change in ρN_R.

rich, high real-wage, low profit equilibrium and with the opposite. What is lacking is some principle to determine whether the economy comes into stationary equilibrium with a lot of machinery or with a little.

In Wicksell, as Mrs. Robinson remarks,[1] the value of the stock of capital in terms of commodities is simply given. In effect, the capitalists of the society come to some independent decision as to the asset-value (in terms of consumption goods) that they wish to hold in the form of machines, and that's that. Then Wicksell proceeds to make comparisons among equilibria in which the decision was to hold a little more or a little less by way of asset-value. Note that this is something different from a decision to hold more or fewer machines. It is easy to calculate that the commodity-value of the balanced stock of machinery is

$$V_R = \frac{rR}{\rho}\left(\frac{\rho N_R + e^{-\rho N_R} - 1}{\rho N_R}\right). \tag{14}$$

Wicksell's procedure amounts to taking V_R as a parameter and observing how the whole system's equilibrium shifts as V_R is varied.[2]

This seems to leave the stock of capital hanging in mid-air, which is not necessarily a bad thing in an historical world, but seems a little unnatural. It would be more plausible to suppose that capitalists decide on the asset-value they wish to hold in relation to their annual real income. It would be even more plausible, to my mind, to admit that in making decisions of this kind capitalists, even entrepreneurs, take more than a passing interest in the going rate of profit. Such considerations become more pressing and more interesting when we study situations in which net saving and capital formation are occurring in some regular way. At this stage of the game it is not so important to take a stand on the precise way in which the system is completed.

We can now return to the supply-of-labour equation. In Wicksell's version of the theory, the decision as to the asset-value of the stock of capital may be thought of as fixing the scale of the economy. (If (14) is put equal to some constant, this at once determines Rr/ρ.) Then (14) together with all the rest of the equations determines the rate of profit, real wage, amount of employment, etc. The exact shape of the labour-supply function may

[1] *The Accumulation of Capital*, p. 391.

[2] It is only fair to remark that the *Lectures*, written some twenty years earlier, does contain a brief section 'On the Accumulation of Capital' (vol. i, part *iii*, pp. 207-18). But even this contains only a few inconclusive remarks.

matter a great deal. If, on the other hand, one chooses the alternative device of having capitalists decide on the number of machines (or their value) relative to annual profits or some other extensive quantity, and with or without reference to the rate of interest, or if one introduces some kind of savings function, the situation is different. Then the labour-supply equation fixes the scale of the economy and has essentially no other function. So it does not much matter and we may as well imagine the supply of labour to be inelastic. To see this, note that all the equations of the system except (14) have this property: given any solution, one gets another solution by multiplying all extensive quantities (L's, Q's, R's) by the same constant, and leaving all prices (and N_R) unchanged. Then if (14) is replaced by one of the suggested alternatives, all of which share the property described, one can imagine solving the system as follows. Ignore the labour-supply function, choose an arbitrary number for the labour force and solve the rest of the equations. This will yield among other things a real wage. Now turn to the labour-supply curve, find what offer of labour corresponds to the equilibrium real wage and blow up all the extensive quantities to match.

II. DISTRIBUTIVE SHARES UNDER STATIONARY CONDITIONS

As long as we stick to stationary conditions, the distributive implications of the Akerman-Wicksell model do not depend on what we eventually decide about the motives for capital accumulation. In an economy such as we have just characterized it is easy enough to calculate the proportions in which the national income will be divided between wages and profits or interest.

Since no net capital formation is going on, the national income or net national product consists entirely of the output of consumption goods, Q_c. The share of the return to capital is thus $\rho V_R/Q_c$, and can be computed from (14) together with (3), (7), (12) and (13). Equivalently, we can calculate the share of wages as wL/Q_c. From (2) and (6) aggregate real wages is a fraction γ of the output of consumption goods plus a fraction α of the value of the gross output of machines. Carrying out the computation and subtracting from unity we again get the share of property income. Either way the relative share of profits turns out to be

$$\frac{\rho V_R}{Q_c} = (1-\gamma)\frac{\rho N_R + e^{-\rho N_R} - 1}{\rho N_R - (1-\alpha)(1-e^{-\rho N_R})}. \tag{15}$$

If $\rho N_R < 2$, we can expand the exponential in powers of ρN_R, neglect powers higher than the second and approximate (15) by the somewhat simpler form

$$\frac{\rho V_R}{Q_c} \simeq \frac{(1-\gamma)\rho N_R}{\rho N_R + \alpha(2-\rho N_R)}. \tag{16}$$

The interesting thing about the expressions is that they show the relative share breakdown to depend *only* on the technical parameters describing production possibilities. According to (10), the product ρN_R is constant in this model and depends only on the value of β and on nothing else. Thus the right-hand sides of (15) and (16) are fixed once α, β, and γ are fixed. It does not matter whether the Wicksellian economy has accumulated a lot of capital or a little, or whether the real wage is high or low. Once stationary conditions prevail, distributive shares will settle at the value given by (15) and approximated by (16).

The meaning of constants like α and γ is familiar and requires no comment. The higher α and γ, i.e. the higher the shares imputed to labour in the two industries, the higher the over-all share of labour. β perhaps requires some comment. According to (4), β measures the technical difficulty of extending the lifetime of machines. The higher β, the more labour and machinery it takes to construct more durable machinery. We may assume $\beta < 1$, since otherwise it would never pay to construct durable machines. If the two sides of (10) are represented on the same axes it is apparent that the larger the value of β, the smaller will be the equilibrium value of ρN_R. And from (16) it is easily seen that the smaller the value of ρN_R, the smaller will be the share of profits in the national income. Thus the harder it is, technologically speaking, to extend the durability of machines, the more the relative distribution of income shifts in favour of wages.

Purely by way of numerical example, if we take $\beta = 0{\cdot}8$, equation (10) yields that $\rho N_R = {\cdot}44$, so that if the interest rate is about 4 per cent, machines would have a durability of eleven years. If we put $\alpha = \gamma = 0{\cdot}7$, then the exact share of profits according to (15) works out at about 7·3 per cent of national income, and the approximate (16) gives 8·6 per cent.[1] There is of course no paradox in the fact that the profit share is only one-fourteenth while both Cobb-Douglas elasticities for machines are 0·3. The latter govern the share of gross

[1] (16) will in general overestimate the property share. One can of course take cubic terms in approximating (15); this leads to the approximation

$$\frac{(1-\gamma)(3-\rho N_R)\rho N_R}{6\alpha + (1-\alpha)(3-\rho N_R)\rho N_R}$$

which will be better than (15). In the numerical example it works out to 7·5 per cent.

machine rentals, from which depreciation allowances still need to be deducted.

The result that relative shares are the same in all stationary states regardless of the number of machines accumulated can be looked at in another way. Equations (3) and (7) also entail that the division of the stock of machines (and also of the labour force) between producing consumables and replacing machines is also fixed, regardless of the amount of accumulation that has occurred. Any difference from one stationary state to another in the stock of machines is exactly compensated by differences in durability.

III. THE MODEL IN A GOLDEN AGE

The first and most important generalization to be made is to extend the model to situations in which net investment is occurring. The trouble is that the central mechanism of the model — the rate of accumulation of machines and their changing durability — depends fundamentally on the way entrepreneurial expectations about the future are formed, and about this we have no commanding hypothesis. The only assumption that has been explored with any degree of thoroughness is the assumption of perfect foresight — that entrepreneurs expect with confidence precisely those prices and interest rates which in fact occur. For some problems this may do as a start, but as a method for dealing with the basic problem of uncertainty it is like closing your eyes because you're afraid of the dark. One can make sense of the assumption of perfect foresight by restricting consideration to states of balanced growth, golden ages as Mrs. Robinson calls them. A situation in which all extensive magnitudes (labour force, stock of machines, both outputs) are growing exponentially at the same rate is compatible with constant prices, a constant interest rate, and a fixed optimal durability of machines. Moreover, once a golden age is established, simple extrapolations hold good and it seems reasonable that stable anticipations should be formed — in fact any other assumption would become unreasonable.

If we imagine a golden age as a kind of moving equilibrium, the natural comparison is among golden ages with different rates of growth. The stationary state of the last two sections becomes a special case of a golden age with zero rate of growth. If we ask how relative shares change when the rate of growth changes, we are asking how one golden age compares with a different golden age. Nothing entails that a system starting off in a state of unbalance will ever achieve a golden age, nor that a state of balanced growth will, if disturbed, restore itself or work toward a different state of balanced growth. If that should be the case, however, then our comparisons could be thought of as comparison over long periods of time.

Imagine then a Wicksellian economy which is no longer stationary. The labour force is increasing like e^{gt}, at relative rate g. So is the output of consumer goods. The gross output of machines is precisely enough to make up the momentary depreciation and to increase the stock of machines at the same rate as everything else. The age distribution of machines will be moved toward the young end. The real wage, the gross rental of a machine, the interest rate and the length of life to which machines are built are all constant.

Fortunately, nearly all of the earlier equations can be used again, with a slight difference in interpretation. Take (1), for example. If we replace Q_c by $Q_c e^{gt}$, L_c by $L_c e^{gt}$, R_c by $R_c e^{gt}$, all the exponentials will cancel out and we are left with an equation that looks exactly like (1) except that Q_c, L_c and R_c are to be interpreted as ratios to an exponential trend e^{gt}. In this version of the model the prices remain prices, but the physical quantities represent the proportions in which labour, machines and output are mixed, all growing at rate g. Everything remains unchanged until we come to (13), which fails on two counts. Less than $\frac{1}{N_R}$th of the stock of machines needs to be replaced per unit of time, because of the skewed age distribution; in addition the output of machines must now provide for net capital formation. Remembering that the symbols represent ratios to a trend, it is easily calculated that

$$Re^{gt} = \int_{t-N_R}^{t} Q_R e^{gt} dt,$$

so that

$$Q_R = \frac{gR}{1 - e^{-gN_R}}. \tag{13'}$$

Since

$$\frac{g}{1 - e^{-gN_R}} > \frac{1}{N_R},$$

it follows that the ratio of the annual output of machines to the stock of machines is greater in a growing system than in a stationary one. For vanishingly small g, (13′) and (13) coincide. As for the value of the stock of machines in terms of consumption goods, a similar calculation yields

$$V_R e^{gt} = Q_R e^{gt} \frac{r}{\rho}\left[\frac{1 - e^{-gN_R}}{g} - \frac{e^{-\rho N_R} - e^{-gN_R}}{g - \rho}\right], \tag{14'}$$

and once again if we let g tend to zero we get (14).

For later use it will be handy to know the value of gross and net investment in terms of consumables. The total output of machines is

$$Q_R e^{gt} = \frac{gRe^{gt}}{1 - e^{-gN_R}},$$

with a total value of $\quad \dfrac{gRe^{gt}}{1 - e^{-gN_R}} \cdot \dfrac{r}{\rho}(1 - e^{-\rho N_R}).$

A direct computation of the volume of depreciation gives

$$\frac{gRe^{gt}}{1 - e^{-gN_R}} \cdot r \frac{(e^{-\rho N_R} - e^{-gN_R})}{g - \rho}.$$

The difference,

$$\frac{gRe^{gt} r}{1 - e^{-gN_R}} \left[\frac{1 - e^{-\rho N_R}}{\rho} - \frac{e^{-\rho N_R} - e^{-gN_R}}{g - \rho} \right], \tag{17}$$

should be the value of net capital formation. A check is provided by recalling that in a golden age, with all relative prices constant, net investment $= \dfrac{dV_R}{dt} = gV_R$. Using (13′) and (14′), net investment should amount to

$$\frac{gRe^{gt} r}{\rho} \left[1 - \frac{g}{g - \rho} \frac{e^{-\rho N_R} - e^{-gN_R}}{1 - e^{-gN_R}} \right], \tag{18}$$

and a little manipulation shows that (17) and (18) are indeed identical.

IV. BALANCED GROWTH AND RELATIVE SHARES

National income now contains a net investment component, and this makes it a little more laborious to compute the share of national income which will be competitively imputed to machines and their owners. National income (valued in terms of consumer goods) consists of the output of consumables, Q_c, plus the value of net investment, from (17) or (18). Aggregate interest payments can be calculated as $\rho V_R e^{gt}$ from (14′). Alternatively, aggregate real wages consists of γ times the output of consumables plus α times the value of gross investment. Subtracting the wage share from unity we again get the share of profits in national income.

Either way, the calculation is tedious but finally yields

$$D = \frac{\rho V_R}{Q_c + \text{Net Investment}}$$

$$= \frac{\rho(1-\gamma)\left[\dfrac{1-e^{-gN_R}}{g} - \dfrac{1-e^{-\rho N_R}}{\rho}\right]}{(\rho-\gamma g)\left[\dfrac{1-e^{-gN_R}}{g} - \dfrac{1-e^{-\rho N_R}}{\rho}\right] + \alpha(\rho-g)\dfrac{1-e^{-\rho N_R}}{\rho}}. \tag{19}$$

As a check, if we put $g=0$, (19) reduces to (15).[1] Before looking more closely at the determinants of distributive shares, we can approximate (19) as we did (15) by expanding the exponentials in series up to the square terms. As before, this approximation is useful only if ρN_R and gN_R are both less than 2, and the smaller they are the better.[2] In a model like this, unlike closed von Neumann-like systems with no final demand, one can say nothing *a priori* about the relative magnitudes of the rate of interest ρ and the rate of growth g. But there seems to be some presumption that, at least for small rates of growth, ρ will exceed g. In general this will be so, in a golden age, if the consumption of capitalists exceeds the savings of workers. Moreover, the model of a stationary state described earlier has $g=0$ and a positive rate of interest. By continuity, then, as g inches up into positive values there must be a range in which it still falls short of ρ.

For small values of g we can write the approximation

$$\frac{\rho V_R}{Q_c + \text{Net Investment}} \simeq \frac{(1-\gamma)\rho N_R}{(\rho-\gamma g)N_R + \alpha(2-\rho N_R)}. \tag{20}$$

Once again, if we put $g=0$, we return to (16).

A couple of the implications of (20) are worth stating explicitly. In the first place, relative shares are no longer a matter of technological constants alone. Not only does the rate of growth itself make a difference, but in addition the durability of machines, N_R, appears in the denominator divorced from its partner ρ. Thus any change in data which has the effect of shifting the optimal durability of equipment (or of changing the rate of interest, which comes to the same thing) will also cause a change in the distribution of income. Other things equal, an increase in the durability of machines goes

[1] Since $gN_R = \frac{g}{\rho}.\rho N_R$, inspection shows that (19) depends only on α, β (via ρN_R), γ, and the ratio g/ρ. The latter is shown in the footnote following equation (23) to depend in turn on D, σ_ρ and σ_w. If σ_ρ is sensitive to ρ, then g/ρ will change with g.

[2] Of course one can get a better approximation, and one valid over a wider range of values, by taking third-degree terms in the series. See the next footnote.

along with an increase in the fraction of national income distributed in profits.

Secondly, another conclusion can be drawn. If we compare a golden age where the system is growing at a small but positive rate with a stationary state, the former will have a larger share of its income going to profits. This is because whatever happens to N_R when g rises above zero, gN_R must be positive and therefore the denominator of (20) smaller. Note that this conclusion follows without any assumption as to the nature of the savings function (as long as enough savings are provided to allow for expansion at rate g). Whether an increase in the rate of expansion always means an increase in the share of profits requires a closer analysis, because of the possibility that N_R may decrease sufficiently with an increase in g to keep the product gN_R from rising.

Before looking a little further into this question I shall carry the earlier numerical example into this more general case. Keep $\alpha = \gamma = 0{\cdot}7$ and $\beta = 0{\cdot}8$. But suppose $g = 0{\cdot}02$, and let us assume that in equilibrium the optimum length of life of machines is 11 years (which implies, since $\rho N_R = 0{\cdot}44$, that the interest rate is 4 per cent). Then according to (19) the share of profits rises from 7·3 per cent in a stationary state to almost 8 per cent with an instantaneous rate of growth of 2 per cent. The simple approximation (20) overestimates again, at 9·6 per cent.[1]

In principle, the question whether an increase in the rate of expansion will cause the profit share to improve or deteriorate can always be settled by an appeal to (19). But the derivative of (19) leads to nothing very translucent, and we are reduced instead to the two approximations that have been given above. Over the range of growth rates and values of ρN_R for which they are sufficiently accurate, something can be said.

In terms of (20), it is clear that when g increases, the profit share rises or falls according as gN_R rises or falls, hence according as $N_R + g\dfrac{dN_R}{dg}$ is positive or negative. I have already remarked that since N_R is positive when $g = 0$, this expression is positive for a sufficiently small g.

For somewhat larger values of gN_R and ρN_R, the formula with the higher degree of approximation in the preceding footnote can be

[1] The next best approximation to the exact formula (19) is

$$\frac{(1-\gamma)\rho N_R(3-\rho N_R-gN_R)}{(\rho-\gamma g)N_R(3-\rho N_R-gN_R)+\alpha(6-3\rho N_R+\rho^2 N^2_R)} \qquad (20')$$

and in the present numerical example this comes out at 7·6 per cent, a slight underestimate of the exact 7·9 per cent.

used. It yields the following proposition: When g increases slightly, the profit share rises or falls according as

$$\left(N_R + g\frac{dN_R}{dg}\right)[\gamma(3 - \rho N_R - gN_R)^2 - \alpha(6 - 3\rho N_R + \rho^2 N^2{}_R)]$$

is positive or negative. Now, as one would expect, the relative magnitudes of γ and α play a rôle. When the rate of growth increases, there is a shift of demand from consumer goods to machines, and what happens to relative shares depends on the proportions in which the two industries pay out their revenues in wages and quasi-rents. Thus if $N_R + g\frac{dN_R}{dg}$ is positive, the profit share is favoured by a high value of γ relative to α, for this means that the machine-building industry pays a relatively high fraction of its revenues to owners of machines.

Finally, if we consider near-zero rates of growth, but do not require ρN_R to be small, the profit share rises or falls in response to an increase in the growth rate according as the expression

$$\left[\gamma - \frac{\alpha}{2}(1 - e^{-\rho N_R})\right]\rho N^2{}_R - (2\gamma - \alpha)(1 - e^{-\rho N_R})N_R + (\gamma - \alpha)(1 - e^{-\rho N_R})^2/\rho$$

is positive or negative. This comes from differentiating (19) and letting $g \rightarrow 0$.

This most complicated result is the most important as well. The assumption underlying the earlier approximations — that ρN_R is small — is by no means innocuous. Equation (10) shows that ρN_R is small only when β is large, close to one; that is, only if extending the lifetime of machines is technically very difficult. This is what makes the earlier approximation yield such definite results apparently independent of γ and α. It is easily seen from the last given criterion that in the opposite case where ρN_R is large (β small), a sufficient condition for the profit share to increase with the growth rate is that $\gamma > \alpha/2$.

V. THE RÔLE OF THE SAVINGS FUNCTION

Rather than ring changes on this formula, we may look a bit more closely at the expression

$$N_R + g\frac{dN_R}{dg}$$

and the conditions under which it will be positive. The inequality

$$N_R + g\frac{dN_R}{dg} > 0$$

is equivalent to
$$\frac{g}{N_R}\frac{dN_R}{dg} > -1,$$

i.e. that the elasticity of N_R with respect to g shall be greater than -1. Now since ρN_R is constant in this model, it follows that

$$\frac{dN_R}{dg} = \frac{-N_R}{\rho}\frac{d\rho}{dg}.$$

Making this substitution we arrive at the equivalent condition

$$\frac{g}{\rho}\frac{d\rho}{dg} < 1, \tag{21}$$

i.e. that the proportionate change in the interest rate shall be less than the proportionate change in the rate of growth.

A full discussion of the relation between the rate of profit and the rate of growth would be too ambitious an undertaking. So far nothing has been said about the origin of the growth rate in a golden age — we have simply discussed how one golden age differs from another. What *makes* one golden age differ from another is a much deeper question. It is often simply assumed that population grows autonomously at some fixed rate and the rest of the system, notably the rate at which capital accumulation proceeds, adapts to this independent demographic fact. A deeper theory would relate the rate of population increase to economic as well as non-economic circumstances. For present purposes, however, I shall take the easy way out and treat g as given. (It might be given from some other direction than the rate of population increase, and then the assumption would be that the supply of labour simply adapts to whatever demands are made upon it.)

If we make the simplifying assumption that all profits are saved and all wages consumed, the model takes on a von Neumann-like character, and the rate of growth and the rate of interest or profit coincide. If we go to the other extreme and allow both wage and profit incomes to be divided — perhaps in different proportions — between consumption and saving, then the relation between interest and the growth rate depends on how consumption out of profits compares with saving out of wages. This can be seen as follows. Let σ_w and σ_p be the fractions of wage and profit incomes saved. In a golden age all prices are constant and so the value of the stock of machines grows at rate g. Thus $gV_R = \sigma_w wL + \sigma_p \rho V_R$. It then follows that $\rho - g \gtreqless 0$ according as $(1-\sigma_\rho)\rho V_R \gtreqless \sigma_w wL$, i.e. as capitalists' consumption exceeds, equals or falls short of workers' saving. Thus the answer turns not only on the saving habits of the two classes but also on the relative shares themselves.

On the intermediate assumption that capitalists consume but workers do not save, we put $\sigma_w = 0$ and it follows that the interest rate always exceeds the rate of growth.[1] Now suppose that the fraction of their incomes that profit-receivers will save is an increasing function of the rate of return ρ. Then we have the equation

$$g = \rho\sigma_\rho. \tag{22}$$

Thus given the rate of growth, the interest rate is determined. If we tread cautiously we are not thus committed to a view of the causal substructure underlying (22). Nor is it implied that a mechanism exists for gilding an ungolden age. All (22) states is that if golden ages characterized by different rates of growth are compared, their interest rates will be found to differ in accord with (22).

Since $\sigma_\rho < 1$, we again verify that $\rho > g$, and in addition we can calculate that

$$\frac{g}{\rho}\frac{d\rho}{dg} = \frac{1}{1 + \dfrac{\rho}{\sigma_\rho}\dfrac{d\sigma_\rho}{d\rho}} < 1, \tag{23}$$

provided only that the savings ratio increases with the yield on capital. Comparing with (21), it follows that

$$N_R + g\frac{dN_R}{dg} > 0$$

and therefore that over the (possibly very narrow) range for which (20) is accurate, the profit share increases with the growth rate. The implications from (20′) are much more complicated, but there is perhaps some presumption that for values of g and ρN_R not too large the same statement will hold.[2]

[1] For a similar treatment of the von Neumann model, see a recent paper by E. Malinvaud, 'Programmes d'expansion et taux d'intérêt', *Econometrica*, 1959, p. 215.

[2] Essentially the same conclusion holds good if we go back to the more general situation where wage-earners also save. With σ_w, the savings ratio from wage income, presumably also a function of ρ, and letting D stand for the share of capital in national income, it is easily calculable that (22) is replaced by

$$g = \rho\left(\sigma_\rho + \frac{1-D}{D}\sigma_w\right).$$

From this together with (20) it can be shown that the profit share increases with the growth rate provided only that

$$d\sigma_\rho/d\rho + \frac{1-D}{D}d\sigma_w/d\rho > 0$$

and therefore certainly if both savings ratios respond positively to increased yields. If σ_w depends on the real wage, which is certainly plausible, the calculation is more complicated.

VI. NEUTRAL TECHNICAL PROGRESS

The whole argument has been conducted so far on the assumption that productive techniques are fixed. In fact there is reason to believe that much, perhaps most, of the increases in real income per head over the last century has stemmed from technical progress rather than from capital accumulation as such. (Not that anyone for a moment believes the two to be independent.) I propose to lift that assumption now, but only very slightly. The only kind of technical progress I can afford without wrecking the whole balanced-growth framework within which the model operates is 'neutral' technological change.

In order to keep things manageable I have to suppose that technical progress does not in any way change the nature of the machinery, but only the way in which it is organized and used. (Professor Swan's Meccano set would come in handy here.) Then neutral technical progress in Mrs. Robinson's sense [1] boils down to what might be called 'uniform' technical progress in the consumption goods industry alone. That is, the output of consumables obtainable from any given inputs of man-hours and machine time grows at a steady exponential rate (the same rate no matter what the ratio of inputs is). In the machine-building industry the production function does not change.

Suppose neutral progress in this sense is proceeding at a constant rate λ.[2] In terms of the model, the production function for consumer goods must be re-written

$$Q_c e^{(\lambda+g)t} = e^{\lambda t}(L_c e^{gt})^{\gamma}(R_c e^{gt})^{1-\gamma}. \tag{1'}$$

Thus we return again to (1), with yet another interpretation. L_c and R_c remain ratios to the trend e^{gt}. But now Q_c is a ratio to the trend $e^{(\lambda+g)t}$. The stock of machines and the labour force both grow at the old rate g, but the output of consumables increases at the rate $\lambda+g$. As a consequence, the real wage and the real rental of a machine (both in terms of consumer goods) rise at the compound rate λ, and the symbols w and r are to be taken as ratios to $e^{\lambda t}$.

This of course has an effect on the value of a machine, and on the best length of life. Under steady growth conditions and with a steady rate of technical progress, one must assume that entrepreneurs

[1] See her discussion in *The Accumulation of Capital*, p. 133.

[2] Inventions do not fall like manna (neither does manna, for that matter) and it would be better to make λ a function of g at least and perhaps other things. But I cannot think of any way to handle this relationship which is not excessively mechanical.

expect the real rental to increase over the lifetime of a machine and take this into account in their calculations. A steady increase in rentals at rate λ is the same thing from this point of view as a reduction in the force of interest by λ. The value of a new machine in terms of consumer goods becomes

$$\frac{re^{\lambda t}}{\rho-\lambda}(1-e^{-(\rho-\lambda)N_R}).$$

Thus, remembering the new interpretations, the old equations (5)-(8) and (10) become (5′)-(8′) and (10′) in which ρ is replaced by $\rho-\lambda$. In particular it follows from (10′) that $(\rho-\lambda)N_R$ is constant, depending only on the value of β. Hence when there is neutral technical progress the product ρN_R is greater than when there is not.

So far as distributive shares are concerned, not much has changed. All of the earlier formulas (19), (20) and (20′) hold good, except that ρ is replaced by $\rho-\lambda$. Thus, for instance, the crudest approximation becomes

$$\frac{(1-\gamma)(\rho-\lambda)N_R}{(\rho-\lambda-\gamma g)N_R+\alpha(2-(\rho-\lambda)N_R)}. \tag{20″}$$

It would be a mistake to conclude from this that since $(\rho-\lambda)N_R$ now has the same numerical value as ρN_R had before, therefore neutral technical progress has no effect on the distribution of income. It might be so, but (20″) does not prove it. For the expression gN_R appears in (20″). Suppose we forget that g and λ are bound up together, and imagine fixing g and varying λ. Then in general ρ and N_R will shift. Unless ρ changes by exactly the same amount as λ, N_R will change and so will the share of profits in national income.

Once again the outcome depends on the behaviour of savings. Although the value of a machine in terms of consumption goods is expected to increase, and does increase at rate λ, the yield on capital to the saver is still ρ. In anticipation of capital gains, machines are priced to yield $\rho-\lambda$ from quasi-rents alone. But now in a state of steady growth the value of the stock of machines increases at rate $g+\lambda$. If the only saving is done out of profits, the equation corresponding to (22) is now

$$g+\lambda=\rho\sigma_\rho. \tag{22′}$$

Since the saving ratio increases with ρ it follows that an increase in the rate of technical progress is accompanied by an increase in the interest rate. To be precise, ρ increases by more or less than λ according as the slope of $\rho\sigma_\rho$ is less or greater than one, i.e. according as $\sigma_\rho+\rho\dfrac{d\sigma_\rho}{d\rho}$ is less or greater than one. And since $(\rho-\lambda)N_R$ is constant throughout, we find that N_R increases or decreases according as $\sigma_\rho + \rho\frac{d\sigma_\rho}{d\rho}$ is greater or less than unity. Finally, if g is taken as fixed while λ varies, the share of profits increases or decreases as N_R increases or decreases. Thus a high savings ratio on the part of profit receivers, and a savings ratio highly responsive to the rate of return, both favour an increase in the profit share when the rate of technical progress improves. This conclusion is valid only for small values of gN_R and values of β close to one. Otherwise we must turn to the modified forms of (19) or (20′) and the output elasticities play a rôle.

In the more realistic case, where we recognize that the rate of growth and the rate of progress are interdependent, the array of possible golden ages does not include all possible values of g and λ. Instead we can scan only an array of situations in which higher values of g and λ go together. Since the variation of the profit share depends (if (20″) is accurate) on what happens to gN_R, the likelihood increases that situations with higher values of g and λ will also have higher profit shares. For purposes of comparison it does not matter whether we imagine g to depend on λ or vice versa, although of course these imply different views of the causal dynamics. If we write g as a function of λ, the condition for (20″) to yield a positive correlation between λ and the share of profits is

$$\frac{1}{g}\frac{dg}{d\lambda} + \frac{1}{\rho - \lambda}\left(1 - \frac{1 + dg/d\lambda}{\sigma_\rho + \rho\, d\sigma_\rho/d\rho}\right) > 0.$$

If $dg/d\lambda = 0$, this reduces to the earlier criterion.

VII. CONCLUSION: FURTHER PROBLEMS

Instead of recapitulating the results of this enquiry, I might perhaps suggest some of the ways in which the simple Wicksellian model needs to be improved if it is to look more like a live economic system.

1. *Many Consumption Goods.* If the bundle of goods consumed is always of fixed composition, then no problem arises. But if consumables differ in income elasticity of demand then the whole game of comparing golden-age situations gets harder to play. Without technical progress, with income per head constant, it's still not so hard. As the rate of growth varies factor prices also change. This will have an effect on the relative prices of consumer goods ; commodities paying out a large fraction of their costs on the more

expensive inputs will rise in price relatively to others. The composition of the consumption bundle will in general be different from golden age to golden age. And this will have some influence on relative shares. The exact nature of the effect is not too difficult to calculate. It will depend, of course, on the fractions of income devoted to the different commodities, on the elasticity of substitution between factors in each production function, on the composition of their costs and on the elasticity of substitution between commodities in consumption.

But when technical progress enters the door, much simplicity goes out of the window. For then income per head rises through time, even within a state of balanced growth, and the make-up of the consumption budget changes too. The whole concept of the golden age begins to blur at the edges.

2. *Many Capital Goods.* Surprisingly, the introduction of more than one kind of machine (say two, for instance) does not offer many difficulties in principle. It is easy enough to write down the additional equations that characterize a golden age. What I have not succeeded in doing is to work out a simple formula for distributive shares. The reason is that the proportions in which different capital assets are combined will vary with the rate of expansion. To disentangle the effects on distribution involves the finer details of the production functions for consumer goods as well as for machines. It doesn't seem to come out neatly.

3. *Unbalanced Growth and Biased Technical Progress.* The reason why balanced growth and neutral technical change are so handy is that they enable us to keep prices and interest rates constant over time or changing in some simple and foreseeable way. This helps to make sense of the assumption that entrepreneurs anticipate future events confidently and correctly. Once away from these simple assumptions, one must recognize that the valuation of durable goods and the making of investment decisions take place in a fog of uncertainty and so does the theory of capital.

APPENDIX: AN ALTERNATIVE ASSUMPTION ABOUT DEPRECIATION

In the body of this paper I have retained Wicksell's assumption that an axe or a machine retains its productive efficiency unimpaired to the end of its finite lifetime and then all at once disappears. An alternative, and in most respects simpler, assumption is that machines decay exponentially, like cakes of dry ice. This is what Samuelson, in his paper in this volume, calls 'radioactive' depreciation. Mathematically, if X new machines are constructed at time zero, then at time t there will remain $Xe^{-\delta t}$ equivalent machines. The constant $\delta > 0$ is the 'force of mortality', and the average lifetime of a machine is $1/\delta$. The reason why this is a simpler assumption is that we need no longer distinguish machines by age. The $Xe^{-\delta t}$ remaining machines at time t are in every way equivalent to the same number of new machines.

With this new assumption, the production function for machines becomes

$$Q_R = L_R{}^{\alpha} R_R{}^{1-\alpha} \delta^{\beta}, \tag{4a}$$

where the average lifetime replaces the fixed lifetime used before. The value of a new machine in terms of consumption goods is simply $\frac{r}{\rho+\delta}$ and so (5) becomes

$$\frac{r}{\rho+\delta} = \frac{wL_R}{Q_R} + \frac{rR_R}{Q_R} \tag{5a}$$

and the marginal productivity conditions can be written

$$\frac{r}{\rho+\delta} \alpha \frac{Q_R}{L_R} = w \tag{6a}$$

$$\frac{r}{\rho+\delta}(1-\alpha)\frac{Q_R}{R_R} = r. \tag{7a}$$

Corresponding to (8) for the optimal choice of lifetime is an equation for the choice of δ

$$\frac{(\delta+\rho)^2}{\delta} = \frac{1-\alpha}{\beta} \frac{Q_R}{R_R}. \tag{8a}$$

Now a little calculation gives the equation corresponding to (10).

$$\frac{1}{\beta} = 1 + \frac{\rho}{\delta}. \tag{10a}$$

Just as (10) showed ρN_R to depend only on the value of the technological constant β, (10a) shows that β also uniquely determines the parallel quantity ρ/δ, the interest rate multiplied by the average life of a machine.

Finally (13′) is replaced by

$$Q_R = (g+\delta)R, \tag{13a}$$

which says that when the stock of machines is growing at rate g, the output of machines consists of δR for replacement and gR for net investment. And (14′) becomes something much simpler.

$$V_R = \frac{r}{(\rho+\delta)(g+\delta)}. \tag{14a}$$

There is no need to repeat the details of the argument. In a golden age with growth rate g, the exact share of capital in net income comes out to be

$$D = \frac{1-\gamma}{1+\frac{\alpha\beta}{1-\beta}+(\alpha-\gamma)\frac{g}{\rho}}. \tag{19a}$$

This is simple enough so that no special approximations seem called for.

Since this formula and some related ones were the object of discussion at the Conference, I make only three brief remarks about it.

First, as David Champernowne so clearly pointed out with regard to my original equation (19), the breakdown into distributive shares depends only on the technological parameters α, β and on g/ρ, the ratio of the rate of growth to the rate of interest.

Second, if one uses the fact, pointed out in the footnote following equation (23) of the paper, that

$$g/\rho = \sigma_\rho + \frac{1-D}{D}\sigma_w,$$

(19a) can be written

$$D = \frac{(1-\gamma)-(\alpha-\gamma)\sigma_w}{1+\frac{\alpha\beta}{1-\beta}+(\alpha-\gamma)(\sigma_\rho-\sigma_w)}. \tag{19b}$$

Only if σ_ρ and σ_w are independent of ρ is D determined independently of the growth rate g.

Third, it will be noted from equation (19a) that, if we compare two golden ages with different growth rates, the nature of the difference depends not only on what happens to g/ρ, but even if we agree that g/ρ will increase with g, it still depends on the sign of $\alpha-\gamma$, i.e. on whether the consumption goods industry or the machinery industry distributes more of its product in wages.

Chapter 13

CATEGORIES OF CAPITALISTS IN THE THEORY OF THE DISTRIBUTION OF THE NATIONAL INCOME[1]

BY

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THE subject of this Round Table is capital theory and it has been divided into three groups of problems : measurement and definition ; the relation of capital to growth ; and the reward of capital in the theory of distribution. This paper is intended as a contribution towards the solution of the last group of problems.[2]

I. METHODOLOGY

Let me begin by stating my view about the manner of posing the problems and about the type of analysis suitable for their solution.

A. I believe that the processes of the distribution of national income among individuals cannot be properly analysed in terms of factors of production alone. The factors of production are owned by human beings and we need to consider them, too, in the infinite variety of their circumstances and in the setting of their environment.

Within the limits set by the subject of this Round Table, one can say that capital is an interesting concept which certainly has its

[1] Translated from French by Elizabeth Henderson.

[2] The author has been working for some years on problems of the distribution of national income. His publications on this subject include : 'Esquisse d'une théorie moderne des salaires et d'une théorie générale de la répartition', *Revue économique*, 1955, p. 553 ; 'Les Disparités de salaires entre qualifications et le comportement des travailleurs non manuels dans la répartition', *Revue économique*, 1957, p. 746 ; 'The Construction of a New Theory of Profit', *American Economic Review*, 1951, p. 549 ; 'Die Theorie der Verteilung bei den englischen Klassikern', *Zeitschrift für Nationalökonomie*, 1954 ; 'Wage Theory and Social Groups', in *The Theory of Wage Determination*, edited by John T. Dunlop (London, 1957) ; and, chiefly, in collaboration with J. Lécaillon, *La Répartition du revenu national*, two volumes published so far (Paris, 1958), and 'Is the Income of the "Cadres" a Special Class of Wages ?' in *Quarterly Journal of Economics*, 1958, p. 166.

The present paper rests on the author's previous published work, but attempts to take it further on some points.

place in economic theory but which, in the context of income distribution, is eclipsed by the concept of the capitalist. For our purposes it is obviously not enough to define a capitalist simply as one who owns capital; if we want to improve the theory we must look for other criteria as well. This line of approach at once suggests that there may be several types of capitalists.

We can adduce two reasons in support of this assumption.

(a) The first reason is that, even with given structures, that is, among other things, with a given degree of competition in the economy, a given organization of the agricultural produce markets and given relations between employers and workers, the owners of a factor of production do not all employ it in the same fashion in order to earn an income. They do not all behave in the same way so far as income distribution is concerned.

In other words, even on the assumption of unchanging structures we cannot define different categories of income recipients for each category of factors of production.

(1) The same factor used differently — for instance, lent on a contractual basis or risked in a production venture — may yield incomes of different kinds and subject to different laws.

(2) Any one factor of production may, in combination with others, give rise to a special type of income which cannot be decomposed simply into parts attributable to the factors employed, except to the grave detriment of economic analysis. This kind of procedure is admissible from an accounting point of view, but accounting analysis is only one, and not necessarily the determining, aspect of economic analysis. Accounting analysis is certainly unable to furnish a complete explanation of the mechanism of income formation.

(3) There is a third argument which, however, takes us somewhat outside our terms of reference. It happens in modern societies that individuals who do not currently contribute any factors of production nor have done so in the past manage, rightly or wrongly, to establish certain claims on the national budget, say, or on social security funds, and so acquire an income. We may describe these people as primary or secondary income recipients, but we still have to consider them, to analyse their behaviour and to make them fit into our theoretical scheme.

These distinctions would be irrelevant for economic analysis if there were fluid limits between these types of behaviour. All those who supply a factor of production would simply be making it effective, and we would have to consider only the factor itself and its objective characteristics. But no such mobility exists in fact. The suppliers of factors of production change their behaviour only slowly and reluctantly. Similar behaviour patterns sometimes merge or interchange, but dissimilar ones never do. Consequently there are discontinuities. Factors of production are linked to different individuals and individuals themselves to different environments, and in these circumstances even the apparently least specific factors of production, like capital, lose much of their mobility. They agglomerate, as it were, around individuals of distinct types.

If, then, we want to lay bare the processes by which individual incomes are formed, and understand the mechanism of the distribution of national income, we must go beyond the simple categories of income recipients defined by ownership of a factor of production. Even if we artificially immobilize the economic structures which, in reality, always change, we must look for more sophisticated categories defined by the procedures used for obtaining an income. These procedures often involve the use of one or more factors of production, but need not necessarily do so.

(b) The definition of new categories becomes imperative when, as we should, we introduce into the analysis the idea that the structures within which incomes are formed change and indeed that they change largely, though not exclusively, under the pressure of income recipients rightly or wrongly convinced that their absolute or relative share of the national income is too small.

In actual fact the distribution of national income takes place through two sets of actions, which we shall call actions within structures and actions on structures. The two are closely related to each other. Any category of income recipient which does not, within the existing structure, get an income which it regards as adequate for its needs, tries to modify the structure. So far as distribution is concerned, action on structures is more important than action within structures.

The workers who in the middle of the nineteenth century, with its competitive labour market, earned wages which they judged insufficient and which were in fact very low, set about changing the existing structures by creating trade unions, negotiating collective agreements and pressing for government regulation of working conditions. Similarly, agricultural producers, whose profits were being squeezed and subject to damaging instability on the free market, claimed and nearly always obtained a strict limitation of production and stocks and the establishment of minimum and maximum prices.

In these circumstances it seems to me altogether impossible to maintain the traditional distinction between what belongs to economics and what belongs not to economics but to sociology. The two sets of actions, within structures and on structures, are related and the latter are more important than the former. If we were to examine only the former, we should at once give up any attempt at constructing a true theory of distribution and any hope of arriving at more than the most superficial view of reality.

The global approach to distribution problems in all their related aspects may oblige the economist to give up certain cherished habits and may on occasion force him to fashion new analytical tools. A case in point is precisely the need to base definitions of new categories of income recipients on more than the sole criterion of the nature of the factor of production they employ to obtain an income.

B. If this is so, then one can think of two types of analysis by which we may try to dissect the processes which, in modern societies, bring about the distribution of national income among the members of the community ; an analysis in terms of the mechanism of flows, and an analysis in terms of the struggle between groups. The most refined form of the former type of analysis is probably the neo-Keynesian one ; the latter owes much to the theory of games, to operational research and to matrix techniques, or what we may call the mathematics of human behaviour. Several French economists are at present working on the latter type of analysis.

(a) The former analysis [1] is open to two objections. First, since it uses categories of income recipients defined solely in terms of factors of production, it is insufficiently explicit and realistic.

If it is the purpose of a theory or model to provide a basis for practical action, then it is certainly not enough merely to distinguish between those who contribute labour and earn wages, and those who contribute capital and earn profits. The categories need to be made more specific and diversified in terms of behaviour, which means that instead of the factors of production we must consider the people who supply the factors, and we must consider them with all their characteristics and in the setting of their environment. Within the framework of flow analysis we need to move on from capital to categories of capitalists.

The second objection is that too many restrictions are built into the assumptions of these models and obscure the major part of the explanation of distribution. In other words, the data and the variables of flow analysis seem to be ill-chosen for purposes of investigating income distribution. Incidentals are brought to light while essentials remain obscure. The relationships need to be shifted from the data to the variables and this can only be done by

[1] Cf. N. Kaldor, 'Alternative Theories of Distribution', *The Review of Economic Studies*, 1955–1956, p. 83 *et seq.*, and J. Robinson, 'La Théorie de la répartition', *Économie appliquée*, 1957, p. 523.

means of analysis in terms of struggles between groups. This latter type of analysis does not invalidate flow analysis, but does deprive it of autonomy and of application to the general case. Analysis in terms of the mechanism of flows becomes dependent upon and subject to analysis in terms of struggles between groups.

We may say, then, that in so far as we continue to use analysis in terms of the mechanism of flows, we should make it more significant by a more detailed study of income recipients, going beyond the sole consideration of the factors of production they supply. In so far as this analysis is subordinated to one in terms of struggles between groups, it becomes even more clearly necessary to take account of all the characteristics of the individuals, considered in relation to their environment.

(b) To forestall criticism, it will be useful to make it clear that analysis in terms of struggles between groups must display the following characteristics :

(1) It should be conducted in countries with well-defined structures. In this connection we shall deliberately choose advanced capitalist countries, such as the United States, Great Britain or France.[1] This does not imply that an analysis of income distribution in under-developed countries is devoid of interest. On the contrary, we believe that such analysis, conducted by the same methods, is indispensable for any informed development policy in these countries.

(2) The analysis of income distribution should be linked to an analysis of the factors determining the growth or decline of national income. The two analyses have many points in common, but we may concentrate more particularly either on the distribution or on the size of national income.

(3) Finally, if analysis in terms of the struggle between groups attaches primary importance to power relationships, this does not by any means preclude considerations of productivity. The fact that a category of people contributes a factor of production is an element of power for that category, but it is in no case the only one that matters.

Hence, analysis in terms of struggles between groups requires attention to be focussed on two points. First, we must determine what groups, or better, what categories of income recipients are relevant. Secondly, we must describe the nature and the result of the processes by which they enter into contact with each other. The two problems are obviously connected, the categories of income

[1] A precise definition of this type of country may be found in J. Marchal and J. Lécaillon, *La Répartition du revenu national*, *op. cit.* vol. 1, p. 12.

recipients being determined by their actions and the part they play in these processes. In this paper, I shall elaborate only the first point.

II. TYPES OF INCOME

If we wish to construct a realistic model of the distribution of national income in an advanced capitalist country, what categories of income recipient are relevant ? The terms of reference of this Round Table oblige us to consider only one part of the problem and to leave out of account those who obtain an income either by contributing only labour or by contributing nothing at all. The question then simply becomes how, in the present state of economic structures, it is possible to obtain an income wholly or partly as a result of owning capital, in the technical sense of the word. Is only one type of behaviour possible or several, and if so, what are they ?

A first distinction must be made between behaviour patterns according as they are connected with the use of capital only or with the joint use of capital and labour — the term labour being used in the widest sense, meaning activity. In the first case, the income recipients no doubt also display some activity in that they place capital at the disposal of others and discuss the terms of its remuneration ; but this activity is reduced to a minimum and statisticians do not customarily count the people involved among the active population. If occasionally they are included in the active population, it is for other reasons and this fact has no bearing at all on the reward they draw as capitalists. In the second case, the income recipients have the double characteristic of being capitalists and members of the active population, and the two are closely linked. In the first case we may speak of their behaviour as that of lenders, in the second case as that of entrepreneurs. Both types of behaviour are those of capitalists.

Let us look first at the behaviour of the entrepreneur. If we define entrepreneurs as individuals trying to obtain an income by the joint contribution of capital and labour, is this a homogeneous category from the point of view of the processes of income distribution ?

Here we have to make three further distinctions, which we shall try to justify.

A. We must first distinguish between agricultural entrepreneurs and those in industry and trade, the reason being that farmers are in a very special situation. In an analysis in terms of the mechanism of flows, the following observations have to be made.

(1) Part of the capital contributed by farmers necessarily takes the form of land. Now this portion of capital is less flexible than others ; it is less dependent on the will of the owner. The farmer may, of course, try to increase the land capital at his disposal, but in so doing he meets with specific obstacles which he cannot surmount except at the cost of delays which may be very long. If there is no more land available in the neighbourhood, he must, pending a change in the situation, adapt his farming to the amount of land he possesses.

(2) Land also has certain features which the farmer can modify, but only to some extent. For instance, he can do nothing about the position of his fields, nor about whether they are in a dry or wet region.

(3) The technical process of production depends much more than in industry on external circumstances, especially the weather. The farmer who incurs a certain total expenditure cannot know in advance what volume of output he will obtain. He can calculate his average costs only afterwards.

We may conclude that on free markets agricultural entrepreneurs find themselves in a different, and generally weaker, position than industrial or trade entrepreneurs. Apart from the circumstances we have mentioned, the fact that farmers are nearly always very numerous, and spread throughout the country, makes it more difficult for them to watch over their own interests by quantitative control of supplies on the produce markets.

Hence — and this brings us to an analysis in terms of group struggles — farming entrepreneurs tend to rely heavily on action outside the framework of the market. For example, they put pressure on the government to limit cultivated acreage, to put part of the harvest in stock, or to modify the mechanism of price formation.

The number and dispersion of farmers are to their disadvantage from the point of view of the mechanism of flows, but become an advantage from the point of view of the struggle between groups. Being numerous and dispersed, farmers have more opportunities of putting their case to members of Parliament of all parties and so to speed up the desired government intervention.

We cannot elaborate the point any further,[1] but it does seem that the behaviour of agricultural entrepreneurs differs from that of industrial and trade entrepreneurs and that the procedures followed

[1] Cf. J. Marchal and J. Lécaillon, *La Répartition du revenu national*, *op. cit.* vol. 2, p. 134. See also Marc Latil, *L'Évolution du revenu agricole* (Les agriculteurs devant les exigences de la croissance économique et des luttes sociales), A. Colin, Paris, 1956, and Jean Féricelli, *Le Revenu des agriculteurs* (Matériaux pour une théorie de la répartition), doctoral thesis of the Faculty of Law and Economics in the University of Paris, 1958 (mimeographed but about to be printed in the near future, ed. M. Th. Génin; Lib. de Médicis, Paris).

by the former in obtaining an income and a share in the national product are not the same as those followed by the latter. We must distinguish the two categories in our model.

B. If we take the group of entrepreneurs in industry and trade, we must make a further distinction between individual, or 'personal' entrepreneurs and joint-stock companies.

In both cases, entrepreneurial behaviour is characterized by the joint application of capital and labour. But in the first case the entrepreneur is a person, in the second an institution. A whole series of differences follows.

(a) From the point of view of the mechanism of flows, there are differences in the flexibility of the capital and activity which underlie the enterprise.

A personal firm is headed by an individual, who puts into the business his own capital and sometimes whatever more he can raise within a fairly narrow family circle. Similarly, the same individual runs the business, possibly with the help of close relatives. The firm is organized around this strictly limited double contribution. Some capital may be raised on the capital market or borrowed from banks, but it always remains in some definite proportion to the original capital, for two reasons. First, the lenders have to be given security, and secondly the owners want to keep the firm as a family business. Similarly, extra staff may be introduced by the appointment of managers who do not belong to the family and have no share in the business, but again, there will be few of them and every opportunity will be seized to incorporate them into the family group.

In short, the intention is more than anything else to ensure the continuity of the family capital and to enable members of the family to lead a certain way of life. The family with its capital resources and managing personnel remains the basis of the whole structure, the business is only a means to an end. In the absence of this characteristic behaviour and of the limitations it implies the firm, whatever its juridical form, ceases to be a personal enterprise and becomes a joint-stock company.

In these companies there may still be managers who occupy a central position in the business, have a seat on its board and bring in some of the capital — 2 per cent on the average, according to Gordon and Hurff. But outside capital grows as much as circumstances require, equity and bond issues are floated, the representatives of banks join the board. A whole new group of higher managers comes in, who draw a salary but often also have an interest in the outcome of the business.

In abstract terms, we may say that the firm ceases to be linked to a family group and to depend on a family group's resources of capital and business leadership. The firm becomes an end in itself, an institution. Instead of the firm adapting itself to given amounts of capital and managing activity, which are there and must be used, capital and activity now assume the extent required by the firm. The source of capital and managing activity loses its importance.

(b) There are also differences in the nature of the profit which determines entrepreneurial decisions. We are still within the terms of flow analysis here, but are beginning to trespass on the territory of analysis in terms of group struggles.

The individual entrepreneur looks at the global profit from the business, without attributing portions of this profit in any precise manner to the work he has furnished or to the capital he has ventured. This profit can, of course, be divided up, for purposes of accounting analysis, into interest on his own capital, entrepreneurial wages and true profit. But experience shows that frequently there is not only no profit, but that, according to interpretation, either the interest on his own capital or entrepreneurial wages may be negative magnitudes. It may seem astonishing that an income can be negative ; but more than that, it would seem that this circumstance does not necessarily cause the entrepreneur to draw the appropriate conclusion and go out of business.

The reason is that the profit which determines the behaviour of the individual entrepreneur, both on the plane of the movement of flows and on that of group reactions, is the global profit considered as an indivisible whole. So long as the global profit enables the entrepreneur to subsist, even, if necessary, thanks only to very hard and badly paid work and very slow amortisation of his capital ; so long can he continue in business. On the level of the mechanics of flows, he takes no action because he is not master of the variables which might enable him to act.

But on the level of group action, things are different. The entrepreneur can join forces with other entrepreneurs who find themselves in a similar situation and together they can try to obtain from the government tax relief, restrictive regulation of production and stocks, changes in the system of price formation — in short, structural intervention tending to push the over-all profit of the individual entrepreneur back to a level which provides a standard of life which society considers justifiable for someone in his position.

Companies behave differently. They are institutions, their managers personally own only a modest fraction of the capital and have a staff of specialists to advise them. Moreover, their behaviour is generally of a kind which economists would regard as more rational. Studies of investment decisions have shown that managers deliberately forgo some chances of increasing the income of the owners of the company whenever the proposed extension or change would yield less than the normal profit rate.

The soundness of the business is judged by the maintenance of a high profit rate and this aim replaces the pursuit of maximum over-all profit, the latter being of no interest to the managers except in so far as they are themselves shareholders. Furthermore, a longer view is taken of profits, which is natural enough since these companies are long-lived institutions.

Finally, and here we definitely enter the sphere of group struggle analysis, there are differences in the behaviour of the two types of firm with respect to the institutional framework.

Companies have large funds enabling them to employ elaborate and costly techniques for mobilizing public opinion in an effective way. Senior management and the higher civil servants usually have common origins and this facilitates contacts. Claims are carefully documented and, in the words of an investigator, given 'the gloss of general ideas'. Ultimately, the interests of large concerns often coincide to some extent with the general interest and it is easy enough for them to stress this aspect.

All in all, business companies have great bargaining power *vis-à-vis* the government. By virtue of this bargaining power, the companies try to get tax advantages and profitable public orders, and above all to prevent any measures which might limit their own power on the market. What the companies try to achieve on the plane of group struggle, is that they be left free to take advantage of the power they possess to control, without too much difficulty, certain variables dominating flows.

Private entrepreneurs are not so well placed in bargaining with the government. They defend only their own interests and this reduces their ability to exercise pressure. But small business has two trump cards : number and dispersion. Votes count, and therefore small entrepreneurs can exercise effective pressure on members of Parliament and get favourable laws passed. What personal entrepreneurs try to achieve is, generally, market regulation of a kind which would give them full or partial power over variables normally beyond their control.

Thanks to their small size and dispersion, private firms also find it easier to evade fiscal or parafiscal obligations. Surveys conducted by the French Finance Ministry have indeed shown that tax fraud is much more common among personal enterprises than among joint-stock companies.

This analysis [1] suggests the conclusion that the behaviour of personal entrepreneurs in trying to obtain an income and to safeguard its size, is different from the behaviour of companies. The two categories should be kept apart in a model of income distribution.

C. Finally, within the group of personal entrepreneurs it would seem that the members of the liberal professions are a group apart. They, too, obtain their income by a joint contribution of capital and activity, but they display a behaviour pattern of their own.[2]

In a modern economy, labour is hardly ever applied in the raw, as it were. Everybody has some general and technical education, which tends to spread more and more. However, there are still great inequalities. Even in countries where general education has gone farthest, some individuals are much better educated than others, either because they had rich parents or because some private or public institution paid for their education.

Naturally, to take full advantage of these educational opportunities, the people concerned must have certain talents. But from the point of view of the economist what matters is that these talents are supplemented by private or public investment. There is a process of capital formation. The labour of certain individuals is enriched in some way; it becomes better and more scarce. This labour comes to contain not only effort but, like producer goods, also time. Or, more exactly, since in a modern economy all labour contains time, this labour contains more time than the labour of most people, at the period and in the society under consideration.

Persons so educated have a choice between two solutions. Some of them take up the so-called liberal professions, that is to say, they establish themselves on their own and sell their services to clients, at more or less high prices. Others go into business and put their knowledge at the disposal of management for a salary. In common usage, we say that the first group belongs to the entrepreneurs, the second to the wage-earners.

If we consider the first category, there can be no doubt that it has a behaviour all its own and different from that of other entrepreneurs, both in the sphere of the mechanism of flows and of the struggle between groups. It is true that income is obtained by the joint application of capital and labour, but instead of capital and labour both being used to produce goods, capital serves to improve labour and to transform it into a producer good. What one ultimately sells on the market are the services of this producer good. Hence there is

[1] For more detail, see J. Marchal and J. Lécaillon, *La Répartition du revenu national*, vol. 2, p. 7.

[2] See my previously cited articles in *Revue économique*, September 1957, and the *Quarterly Journal of Economics*, 1958.

a certain shift in the variables which these entrepreneurs control or try to get control of by institutional action.

III. THE RETURN FROM CAPITAL

Let us now turn to the case where an income from capital is not linked to any real activity on the part of the owner of the capital. Again different types of behaviour are possible.

If we say that the activity of the income recipient is reduced to a minimum, we are in fact speaking of loans, in the widest sense. But these loans may take various forms and lead to results that are not sufficiently similar to each other for us to be able legitimately to describe the rewards obtained as one single type. In other words, there are discontinuities, and if we want to be realistic we must distinguish different kinds of income.

A. The capital itself may take different forms. It may be money, land or houses.

In these three cases we normally describe the income as interest. But in my view one cannot lump together interest on money, rent on land and rent on houses without neglecting some very important circumstances. With so high a degree of abstraction one arrives at a theory entirely devoid of practical significance.

The neglected circumstances are the following.

(a) Demand on the three markets is quite different and not substitutable. The market for money capital, that is, the money market plus the capital market, is the largest. Industrialists, merchants and farmers alike need money to carry on their business. By virtue of the predominant position of the first two, one can assume that the rate of interest on money capital depends mainly on conditions in industry and trade and that farmers must, as best they may, adapt themselves to a situation over which they have practically no control.

On the market for agricultural land only farmers count. We have seen that they behave differently from other entrepreneurs. The position of farmers has decisive weight in determining the level of rent for agricultural land. It has been shown in the case of France that there is a definite link between the share in national income of agricultural income and of capital income from agricultural land.[1]

On the housing market, finally, demand is exercised mostly by consumers rather than by producers. This demand is determined by such things as the rate of growth of the population, the degree of urban concentration, the average size of family. Moreover, it has

[1] Cf. J. Marchal and J. Lécaillon, *op. cit.* vol. 2, p. 331 *et seq.*

rightly been pointed out that the need for housing is the most compressible of all ; as progress creates new needs, the proportion of income which consumers are prepared to lay out on rent tends to fall. Often, and especially in countries like France, the government is under strong pressure to contain any rise in rents.

(b) This differentiation of demand on the three markets would not lead to any differentiation between the three kinds of income formed on these markets, if capital supply moved freely from one to the other. But this is not so.

Traditional economic theory, assuming a stable value of money and absence of any government intervention tending to modify the fiscal and juridical structure of the three markets, usually holds that the obstacles which obstruct free movement of capital from one market to the other can be overcome in the long run, and that consequently capital yields tend towards a uniform level. In these circumstances it is thought unnecessary to distinguish three types of income in the analysis of distribution.

In our view this is to concentrate on the incidental and ignore the essential. If we take a longer period, the general price level changes, sometimes quite considerably and often as a result of struggles over the distribution of national income. Moreover, it is perfectly normal for governments to take measures designed to raise or lower the rate of interest and to affect land and housing rents. Finally, taxes are always being changed. To neglect these factors, or, what amounts to the same thing, to take them as data in the system, is to leave aside some of the most important elements in the explanation of income distribution.

Now these factors obviously have a bearing on the relative position of those who lend money, land or houses. When the value of money falls, money owners have more difficulty than owners of real capital in maintaining their income or even keeping their capital intact. Direct government interference or an increase in the tax burden do not necessarily affect all capital in the same way. And even if it were so, the owners of real capital would still have less chance of evasion and be more vulnerable than money owners.

(c) Finally, the three categories of lenders are very differently placed with regard to the kind of government action they want. They all want some kind of government action, but if we are anxious to probe at all deeply we cannot rest content with this bare statement. We must define the forms and conditions of people's actions in this field, on the basis of the economic characteristics of the three groups of lenders.

All the investigations made go to show that the three categories of lenders do not necessarily display the same reactions from the point of view of the general strategy they employ in the economy as a whole and in the struggle between groups. We must make distinctions. It is true that so long as the value of money remains fairly stable, government intervention in the economy remains limited and the tax burden remains low, the action taken by the three categories of lenders remains interconnected and of the same type. Economists often underrate this kind of action, but it is important. Strong pressure is brought to bear on the authorities in the sense of seeking a strict maintenance of the value of money, a minimum of direct interference and the avoidance of the introduction of any direct or indirect taxes. Whatever category they belong to, lenders defend property. They set property up as a natural right. Their strategy, which incidentally is most effective, is a defensive strategy. The stress is placed on their common interest, namely property rights, and so the three categories of lenders merge into one.[1]

Nevertheless, even then there remains room for separate and specific action on the part of each category. One such type of action is protectionism, which is not uncommon among land owners. Marc Latil [2] has made the point that during the first half of the nineteenth century the protectionism of British landlords 'resulted in high agricultural prices, and high rents caused the institutional income so created to accrue to the landlords'. There are plenty of other examples.

However, the differences between the action brought to bear by the three categories of lenders are much sharper in modern economies, where, as a result of two World Wars, enormous changes have taken place in the general price level, the tax burden has become much heavier and government interference has multiplied. Schumpeter [3] showed in a general way that there are two types of response by individuals to a change in environment : a creative response and an adaptation response. All lenders, in whatever category, have proved incapable of creative response. They simply adapted themselves to deteriorating circumstances. But their adaptation responses took on specific forms, connected with the type of loan.

Money lenders shortened the duration of loans, subscribed to equities rather than bonds and tried to tie interest to the price level, thereby connecting income from capital in some measure with industrial or trade profits. Land owners could not so easily reduce the length of leases, but they tried to introduce a real element into

[1] J. Marchal and J. Lécaillon, *op. cit.* vol. 2, p. 347 *et seq.* [2] *Op. cit.*
[3] J. A. Schumpeter, 'Creative Responses in Economic History', *Journal of Economic History*, 1947, p. 149.

rents by linking them to agricultural profits — which latter, we repeat, are of a different nature from industrial or trade profits. Both money lenders and land owners tried to obtain tax reliefs, but obviously in different forms.

As for house landlords who make a living from housing rents, they are a vanishing group. They are being replaced by real estate companies whose approach is quite different. For these companies, the question is not long-term capital investment for the purpose of preserving the capital and drawing an income from it, but short-term finance of building ventures and rapid re-investment in another as soon as the first is completed. Houses are sold to co-partnerships of residents and the companies aim at the most rapid possible turnover of their capital. They are no longer lenders of real capital but entrepreneurs.

In conclusion, one may say that if one wanted to give a full explanation of the process of distribution of the national income and, therefore, to take account of all the relevant actions both within and on the existing institutional framework, then the analysis would have to distinguish three categories of lender and hence also three categories of capital reward : interest on money, rent on land, and rent on houses — the latter being a vanishing category.

B. Apart from the nature of the capital lent, we should also consider the form of the loan.

Owners of money capital who want to use it to obtain an income may make a loan in the strict sense of the word ; that is to say lend money at fixed interest. In that case they will buy bonds. Or they may buy equities, carrying variable yields.

In the latter case, it is often said that the capitalists behave not as lenders but as entrepreneurs. This seems misleading, because the characteristic feature of an entrepreneur is the joint supplying of activity and capital. But ordinary shareholders, unless they also sit on the board of the company, generally contribute no activity at all. Quite often they do not even attend the company's meetings of shareholders, and if they do, they simply ratify decisions in which they have no part in any real sense. It is therefore more correct to say, for analytical purposes, that shareholders are money lenders, that is to say that they contribute capital only.

But it is equally correct to say that the position of ordinary shareholders is not exactly the same as that of debenture holders, since the reward of the former varies with the results of the business and is not guaranteed in the same way. The distinction should not be pressed too far, for several reasons. First, while in principle dividends are a function of earnings, manipulation of appropriations and reserves enable the management, at its discretion, to distribute more or less than earnings. Secondly, while the debenture holder's claims have absolute precedence over the ordinary shareholder's when business is bad, even the debenture holder will get a true repayment of his capital only on condition that the value of money has not changed in the meantime. Finally, in between the two types of lenders there are the owners of indexed bonds with yields in direct proportion to turnover, production, etc.

In these circumstances it is not at all certain that we should establish two separate categories for holders of equities and holders of bonds. It would be essential to do so only if discontinuities between the two types of lender were proved to exist. But the limits are rather fluid. Investigations have, it is true, shown that fixed-interest bonds and more particularly short- and long-term government bonds tend to be taken up by all sections of the population, while securities with variable yields are held by a fairly well-defined minority. This minority comprises officials, managers of businesses and private entrepreneurs, etc., that is to say people who act rationally and know their way about business life. But there is a fringe of people ready to switch from one kind of investment to the other and this tends to maintain their respective yields at a steady level.

IV. CONCLUSION

In conclusion, let me try to put the foregoing study in its proper perspective. To this end it seems necessary to stress two points on which we know from experience that confusion may arise.

The first point is that this study is meant to remain strictly on the plane of economic theory. I am not concerned with history nor with sociology. I merely believe that the economist's concepts, and more particularly the concept of the capitalist, are defined by criteria which are too few to provide a full understanding of the processes by which national income is distributed among the members of the community. I believe that we should go beyond the mere ownership of capital, in the objective sense, and consider other elements too, including more especially the way in which, with given structures, the owner uses his capital to obtain an income. These other elements are in essence those which modify the owner's economic calculation, subject to the qualification that, in the long run, this calculation must be seen in relation not only to existing structures but also to action designed to modify existing structures.

The second point is that this study does not represent a final result. It is only the first link of a chain which will ultimately lead to the construction of a general model. In order to remain within the terms of reference of this Round Table, I have discussed only categories of capitalists, and have defined these categories with a view to investigating the processes which different types of income recipients may adopt. But the processes themselves have not been described. I am engaged on a study of these processes, but prefer to say nothing more precise about them while this research is under way, with all its inherent elements of chance.

There is obviously a close connection between the definition of categories of income recipients and the definition of the processes they use. But one must begin somewhere and treat one aspect of the question after another. I have chosen to start with the categories, which presupposes at least some investigation of processes. When the processes in their turn are examined in detail, new light will be obtained for the categories.

What I present here is, as it were, the first of several stages of a rocket. I am fully aware of the fact that this first stage can be meaningful only in relation to the others. But I am also convinced that in such a difficult field any isolated effort can have a chance of success only if it is constantly submitted to the widest possible range of critics. This is the sole justification of this paper.