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Georg von Charasoff's Theory of Value, Capital and Prices of Production

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Georg von Charasoff's Theory of Value, Capital and Prices of Production

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Abstract:

The present paper on the now partly well known Russian mathematician and "amateur economist" v. Charasoff was originally written in 1987 together with H. Duffner three years after Charasoff's remarkable contribution of 1910 "Das System des Marxismus" (The system of Marxism) had been rediscovered by the Italian economists Gilibert and Egidi. It was then the second mathematical formulation of Charasoff's contribution on prominent but partly still unresolved topics in Marxian economics. However, though our paper circulated as mimeo it had not been published in a regular journal of economics. Meanwhile, several contributions on Charasoff appeared by such authors as Egidi, Gilibert, Kurz and Salvadori, Stamatis and Mori. But none of them seems to deal with Charasoff's economics in an exhaustive manner. Therefore and nevertheless, the paper may be still of some interest to the, nowadays regrettably rather narrow, audience of economists specialized in linear models of production, Marxian economic theory and Neoricardianism.

Keywords: Marxian economics, labor theory of value, transformation problem, prices of production

JEL classification: B14, B51, C67, D24

0. Introduction

The present paper deals with the rather unknown economic studies of the Russian mathematician Georg Charasoff.

Charasoff, who was born in 1877 at Tiflis, studied medicine at Moscow, until he was expelled from the university because of his participation in the 1896 student riots. He left Russia and matriculated at the mathematical faculty of the University of Heidelberg, where he obtained his doctorate in 1901. Since 1903 he lived in Switzerland and returned to Russia in 1916.

In 1909 Charasoff published *Karl Marx über die menschliche und kapitalistische Wirtschaft* as the first book of a planned trilogy, which was devoted to a systematic analysis of Marxian and neoclassical economic theory. His second book *Das System des Marxismus*, which will be reviewed here, appeared in 1910. The third one, *Die Probleme der Produktion und der Verteilung*, which - as far as we know - never ap-

Abstract

G. Charasoff, a Russian mathematician, is a forerunner of Leontief, v. Neumann and Sraffa in the theory of linear economic models. As early as in 1910 he anticipated concepts nowadays familiar to economists, such as duality, sub-system and Markov-process for instance. His central concept of "original capital", based on the property of convergence of primitive and productive input-matrices, is an unusual but sophisticated device to calculate prices of production and v. Neumann-output structures. By the same token he obtains a solution to Marx's transformation problem.

peared, was announced in 1910 as a critique of the theories of Walras, Menger and v. Böhm-Bawerk.

In our view Charasoff's importance is based on the fact that his ideas - developed at a time when the use of linear algebra was completely unknown to economists - must be regarded as a remarkable anticipation of modern mathematical reformulations of economic theory in the classical and Marxian tradition. Though his contribution was noticed in the debates of several Marxist economists, the essence of his ideas was not grasped at all.[1] The only modern discussion of Charasoff is to be found in the exposition of his theory of prices of production in Egidi/Gilibert (1984).[2]

Charasoff's system, however, is much more than just a theory of prices. Therefore our aim is to give an exhaustive description and interpretation of Charasoff's economics in modern mathematical terms.[3]

1. Marx's theory of reproduction as a linear model

In the context with his critique of Marx's formula of the profit rate Charasoff formulates the following scheme of simple reproduction:[4]

$$C' + V' + M' = K' + M' = K$$

$$C^{\circ} + V^{\circ} + M^{\circ} = K^{\circ} + M^{\circ} = M \qquad (1)$$

C', V', M' resp. C°, V°, M° denote - as in Marx - the constant an variable capital of the two departments and their respective surplus values. $K = K' + K^{\circ}$ is society's total capital, $M = M' + M^{\circ}$ is the total surplus value. Somewhat different from Marx, Charasoff aggregates the departments I and IIa, which produce the constant and the variable capital, into the "<u>basic production</u>", and calls department IIb "<u>secondary production</u>".

Charasoff decomposes the above aggregates into the components of quantities and values and transforms this system into a three-sectoral linear model of production[5], which serves him as an illustration for all further argument.

To introduce modern mathematical notation we generalize Charasoff's model into a multisectoral scheme of simple reproduction :

$$x_B A_{CB} \lambda_B + x_B l_B \phi b \lambda_B + s_B \lambda_B = x_B \lambda_B$$
$$x_N A_{CN} \lambda_B + x_N l_N \phi b \lambda_B + s_N \lambda_N = x_N \lambda_N$$

The value of gross output of the basic production $x_B \lambda_B$, where

- $x_B \in \mathbb{R}^m$ is the vector of activities in the basic production and
- $\lambda_B \in \mathbb{R}^m$ is the vector of values of basic products.

is composed of the constant capital used up in production

$x_B A_{CB} \lambda_B$, where

- $A_{CB} \in \mathbb{R}^{mxm}$ is the matrix of coefficients of inputs in the basic production,

and the used up value of variable capital $x_B 1_B \phi b \lambda_B$ and the surplus value $s_B \lambda_B$.

According to Charasoff's system we form a matrix of inputs of consumption goods for the workers' subsistence by the vector of labour inputs, $1_B \in \mathbb{R}^m$, of the basic production and the vector of real wages, $b \in \mathbb{R}^m$. $s_B \in \mathbb{R}^m$ is the surplus product of the basic production.

Similarly, for the second department where the surplus product of the basic production, s_B, is transformed into luxury goods, one has $s_N \lambda_N = x_N \lambda_N \in \mathbb{R}^n$. The constant and variable capital of the secondary or luxury production, $x_N A_{CN} \lambda_B$ and $x_N 1_N \phi b \lambda_B$, is fed by the surplus product of the basic production, where $x_N \in \mathbb{R}^n$ is the vector of activities, $A_{CN} \in \mathbb{R}^{nxm}$ the matrix of input coefficients, $1_N \in \mathbb{R}^n$ the vector of labour input coefficients and $\lambda_N \in \mathbb{R}^n$ is the vector of labour values.

Because Charasoff takes the real wage b as given and the length of the working day T as a variable, we introduce $\phi \in \mathbb{R}$ as an index of real wages. ϕ transforms the daily value of labour-power $b\lambda$ into the hourly real wage $\phi b \lambda$ and, thus, depends on T, i.e. $\phi = \frac{1}{T}$. Therefore we have

(2) $T = (1 + \mu)b\lambda$,

where $0 \lesssim \mu = \dfrac{1 - \phi b\lambda}{\phi b\lambda}$ is the rate of surplus value.

2. The series of production

In chapter 10 Charasoff generalizes his argument to multi-sectoral systems and leaves the simple structure of Marxian theory of reproduction of equation (1). In reality, Charasoff remarks, a given capital physically consists of numerous different goods being themselves produced by different industries, each of which, in general, is characterized by a peculiar physical composition of inputs.

Suppose X is an arbitrary product and X' is the vector of inputs used up in the production of X, then $X \sim X'$ indicates, that X' is put into the production process at the beginning of the period of production and X is obtained at the end.

The product X can be an arbitrary commodity - a luxury good as well as means of production or, finally, <u>any composition of commodities</u>. (119, emphasis added)

X', however, represents means of production alone. If the "capitals of first order" X' are produced themselves by the composition of capitals X'', i.e. $X' \sim X''$, and if this process is continued,

then the concept of a *<u>production series</u>* $X \sim X' \sim X'' \sim \ldots \sim X^{*} \sim$ *is obtained, which has the peculiar property, that each term in it is simultaneously the product of the following one and the capital of the preceding one.* (120)

If A is a productive matrix of input coefficients with m+n rows and columns, then for an arbitrary vector of outputs q $\in \mathbb{R}^{m+n}$ the series of production is

$$q \sim q' \sim q'' \sim q''' \sim \ldots \iff q \sim qA \sim qA^2 \sim qA^3 \sim \ldots$$

Charasoff maintains, that in the production series luxury goods, which are exclusively consumed by capitalists, are eliminated from the capital of first order qA. In the capital of second order, qA^2, those means of production are eliminated, which are exclusively used up in the production of these luxury goods.

In the same manner further means of production - means of production of second order for luxury goods - are certainly eliminated from the capitals of third order X'''.
However, since conceptually the series of production can be continued to infinity, obviously, beyond some finite point any further elimination is unnecessary, and all remaining elements of the production series will be composed by the very same means of production, which, ultimately, are indis-

pensable in the production of each and every product, and which, therefore, we may denote as __*basic products*__ *[Grundprodukte]*. (120 - 121)

Charasoff's idea can be represented by a decomposable matrix of input coefficients of the following structure:

$$(3) \qquad A = \begin{bmatrix} A_B & 0 \\ A_{NB} & A_N \end{bmatrix} = \begin{bmatrix} A_{CB} & 0 \\ A_{CN} & A_N \end{bmatrix} + \begin{bmatrix} 1_B \phi b & 0 \\ 1_N \phi b & 0 \end{bmatrix}.$$

where $0 \leq A_B \in \mathbb{R}^{m \times m}$ is the indecomposable matrix of the basic system.

$0 \leq A_{NB} \in \mathbb{R}^{n \times m}$ is the matrix of input coefficients of those basic products, which enter into the production of the non-basics, and

$0 \leq A_N \in \mathbb{R}^{n \times n}$ is the matrix of non basics, which enter exclusively into the production of non-basics.

In order to eliminate all non-basics from the production series by Charasoffs procedure, A_N must be a nilpotent matrix, i.e. $[A_N]^k = 0$ for some positive integer k. This is guaranteed, if - as Charasoff assumes - non-basics do not enter directly or indirectly into their own reproduction. This is tantamount to the possibility of reducing the production of luxuries and their means of production ultimatly to basic products - in an "Austrian" sense. If k steps are required for this reduction process, A_N is nilpotent of order k.

Means of production for luxuries

are themselves gained by other auxiliary materials, hence forming a transition from means of production proper to luxuries. (119)

Because the series of production reduces itself to basic products, Charasoff concludes:

Hence the entire problem of pricing boils down to the determination of the prices of these basic products. (121)

Therefore he confines his analysis to the basic system.

3. The original capital

3.1. The series of production and the original capital

Charasoff formulates the basic system as a system of linear equations and derives from it the idea of the "original capital" (*Urkapital*) by expanding the series of production to capitals of increasingly higher order:

Let us denote by A, B, C,... the yearly produced quantities of the different products, and by A', B', C', ... their capitals of first order, then by A'', B'', C'', ... the capi-

tals of second order etc., then the capitals A', B', C', ... therefore are compositions of the products A, B, C, ... which can be stated by the following equations:

$A' = lA + mB + nC + \ldots\ldots$

$B' = qA + rB + sC + \ldots\ldots$

$C' = uA + vB + wC + \ldots\ldots\ldots\ldots$ *etc.,*

where the letters l, m, n, q, r, s, ... represent certain numerical quantities given by the technical conditions of production which. for a given technique, must be regarded as constants. (121 - 122)

To the capitals of first order A'. B'. C'. ... of the basic system the corresponding vector is $q' = qA$.[6] In the same manner the vector of capitals of second order is $q'' = qA^2$. or. as Charasoff puts it:

$A'' = lA' + mB' + nC' + \ldots\ldots$

$B'' = qA' + rB' + sC' + \ldots\ldots$

$C'' = uA' + vB' + wC' + \ldots\ldots.$

From this one can see, that the capitals of second order are composed by those of first order. ***But, since the physical composition of a sum of capitals obviously is always an average of the physical composition of the summands, it follows that the capitals of second order are always less divergent in their type as is the case for the capitals of first order.*** (123)

Since the capitals of higher order are themselves produced by capitals increasingly similar in their composition, the series of production converges to a structure of uniform physical composition. This feature of the series of production is the basis of Charasoff's theory of original capital. For the series of production, $q \sim qA \sim qA^2 \sim \ldots$ of a given product q, the capital of order t+1 is qA^t. To verify Charasoff's proposition, it has to be shown that the series of production converges to a stable structure - the original capital of the technique A:

As is well known for systems which yield a surplus product the series of production converges to the null vector, because for $t \rightarrow \infty$ one has $\lim A^t = 0$. But for a peculiar normalization of A the structure in question is obtained.[7]
By a theorem of Nikaido (1968) a nonnegative, indecomposable and primitive matrix A^* with the maximum eigenvalue $\alpha(A^*) = 1$ converges to a stable limit matrix $\Omega > 0$.[8]

By definition, the matrix $A \geq 0$ of the basic system is indecomposable. Since $A = A_C + l\phi b$, with $l > o$ and $b \geq o$, it is guaranteed that $a_{ii} > 0$ for at least one i, which is a sufficient condition for primitivity of A. Therefore we divide A by its maximum eigenvalue $\alpha(A) < 1$ in order to obtain $A^* = \frac{1}{\alpha}A$ with the maximum eigenvalue $\alpha(A^*) = 1$. A^*, as a scalar multiple of A, is nonnegative, indecomposable and primitive as well.

Obviously $[A^*]^{t+1} = [A^*]^t A^*$. For $t \rightarrow \infty$ we get $\lim [A^*]^{t+1}$

$= \lim [A^*]^t A^*$. Denote $\lim [A^*]^t = \Omega$, with $\Omega > 0$. Similarly $\lim [A^*]^{t+1} = \Omega$, such that $\Omega A^* = \lim [A^*]^t A^* = \lim [A^*]^{t+1} = \Omega$. Conversely, $A^*\Omega = \Omega$ implying $\omega^{i\cdot} = \omega^{i\cdot} A^*$ resp. $\omega^{\cdot j} = A^*\omega^{\cdot j}$ for every row resp. every columnn vector of Ω ($i,j, \in \{1,\dots,m\}$). Therefore all rows and columns of Ω are eigenvectors corresponding to $\alpha(A^*) = 1$ which are identical up to a scalar multiple; the limit matrix Ω is the vector product of the eigenvectors p^* and x^* corresponding to the maximum eigenvalue $\alpha(A^*) = 1$ resp. $\alpha(A) < 1$:

$$\lim [A^*]^t = \Omega = p^* x^*.$$

The row vectors of Ω with the structure x^*

are all of one and the same composition and therefore do not differ qualitatively any more but only quantitatively, i.e. merely in their dimensions. <u>They are only different quantities of one and the same capital.</u> (111)

This structure of inputs x^* Charasoff calls the <u>original capital</u>.

Thus, for a given technique A, the series of production of an arbitrary product q converges to the structure of the original capital

$$q\,\Omega = q\,p^* x^* = \xi x^*, \quad \xi \in \mathbb{R}.$$

3.2. The original capital and the system of prices

The ratios of the row vectors in $\Omega = p^* x^*$ correspond to the ratios of exchange p^*. Ω, therefore, transforms an arbitrary system of prices p into the vector of prices p^*

$$\Omega\, p = p^* x^* p = p^* \pi, \quad \pi \in \mathbb{R}.$$

p^* is the vector of prices of production of the basic system:

$p^* A(1+R) = p^*$,

for $A^* = \frac{1}{\alpha} A = (1+R)A$.[9]

The prices of the products are proportional to the values, or dimensions, of the expended [sectoral] *original capitals, and the capitalist rate of profit is determined by the rate of growth of the original capital.* (117)

As to the problem of transforming labour values into prices of production Charasoff's theory of the original capital provides a clear cut solution: As A runs to Ω Marxian cost prices $A\lambda$ finally are transformed into prices of production

$$\Omega\, \lambda = p^* \bar{\pi}, \quad \bar{\pi} \in \mathbb{R}.$$

We see that our solution of the problem of pricing can be given a form where the notion of labour can be avoided al-

most completely and where the prices of products are defined immediately by capitalist original cost...This could have been expected too, because prices are formed on a capitalist market where practically nothing is understood about the labour theory. Nevertheless, the law of value regulates prices, for capitalists, saving original capital, save labour embodied in the original capital as well. They don't know it, yet they do it. (111 - 112)

Charasoff does not leave off with the result that *the internal dimensions of the original capital* in the matrix Ω yield the vector of prices of production p^*. He also argues that x^*, *the composition of the original capital*, has a significant meaning for the physical structure of the economy. If the composition of the original capital itself is taken as the dimensions of the activites of the basic system, this yields a vector of outputs of composition x^*.[10]

The original capital is nothing but the basic production the branches of which are taken in quite specific dimensions. That is, the criterion for these dimensions is that the gross produce of the basic production, if it should represent an original capital, must be of the same type as its total capital. (126)

If A is activated by x^* we obtain Charasoff's analogue of Sraffa's standard system:

$x^*A(1+R) = x^*$

$$x^*A + s^* = x^*$$

$$s^* = Rx^*A$$

From this one concludes that that original type, to which all capitals of lower order converge as their common limit, has the property of accruing in the process of production without any qualitative modification and that its rate of growth must serve as the general rate of profit. (124)

With the original capital we have the idea of a value creating growing capital represented in its purest form, and, in fact, its rate of growth P^* emerges as the general capitalist rate of profit R. (111 - 112)

Since the maximum eigenvalues of the quantity system and the price system are the same, Charasoff is right in concluding that the maximum rate of growth of a capitalist economy equals its rate of profit, for

the height of the rate of profit of the basic production, and, therefore, of the money rate of profit too, does not give the actual but merely the virtual rate of growth of capital. (179)

If the actual rate of growth should equal the rate of profit some peculiar conditions must be satisfied:

Imagine a capitalist society where all surplus labour is *directed exclusively to accumulation, or, to the production of new capitals, the process of accumulation being uniform throughout, such that all firms grow at the same pace and all entrepreneurs receive profits proportional to their capitals which they use without any deduction for the expansion of their firms. Under these conditions the social capital will be of the original type and the rate of profit, obviously, will have that meaning which it always has in our analysis - it gives the rate of growth of the original capital in the annual process of production.* (126 - 127)

Charasoff here describes the system $x^*Ap^* = \alpha x^*p^*$ which Morishima (1973) calls the "Marx-von Neumann model". In this model Marx's formula for the rate of profit holds.

The more the capitalist restricts his luxury consumption the more his revenue contracts, luxury capital vanishes, and the more exaxtly the average rate of profit, as it should be calculated according to Marx, gives the actual height of money profit. At the same time it turns more and more into the rate of accumulation, the rate of growth of capital. (177) [11]

3.3. The Fundamental Marxian Theorem

Charasoff emphazises that profit depends on surplus labour and refers to the physiocratic idea *that profit must be based on physical surplus.* (83) *If no surplus labour is performed in the basic production, no profit can arise* _at all_, *because it lacks foundation - the capital* $s_B p^*$. (87)

If labour time is reduced to necessary labour time, the surplus product vanishes and the material basis is withdrawn from profit. This statement Charasoff denotes as

the fundamental law according to which profit stems from surplus labour. (97)

The equation of the quantity system of the basic production is

$x_B[A_B \;\; O] + [s_B, O] = [x_B, O]$,

and that of *secondary production*

$x_N[A_{NB} \;\; A_N] + [O, s_N] = [x_N A_N, x_N]$.

In physical terms the Marxian condition of equilibrium for simple reproduction is

$x_N A_{NB} = s_B$.

The capital of the secondary production equals the _surplus_, *that of total production equals the* _produce_ *of the basic production.* (96)

Charasoff now argues as follows:

If the working day is reduced to labour time necessary for the reproduction of labouring power, that is $T = T^{V} = b\lambda$, then exclusively the processes of the basic production can be activated in certain, fixed proportions.

If $\Phi = \overline{\Phi} = \frac{1}{b\lambda_B}$, then A_B turns into $\overline{A}_B$, the maximum eigenvalue of which is $\overline{\alpha}(\overline{A}_B) = 1$. Though Charasoff calculates $\overline{A}_B$ and its corresponding eigenvectors only by example, it is clear that this might be done by his concept of original capital as well:

The limit matrix $\overline{\Omega} = \overline{px}$ corresponding to $\overline{A}_B$ encompasses the correspondig vectors of outputs and prices, unique up to a scalar factor:[12]

$$(4) \qquad \overline{x}_B = \overline{x}_B\overline{A}_B = \overline{x}\ A_{CB} + \overline{x}\ 1_B\overline{\Phi}\ b,$$

$\overline{x}1_B\overline{\Phi} = x1\Phi = \frac{L}{T} = N$ is the number of workers, with L as the total amount of labour performed.

The output $\overline{x}_B$ merely replaces the used up means of production and the consumption goods of the workers; thus the system is transformed into a subsistence economy. By $\Phi = \overline{\Phi}$ follows $s_B = x_N A_{NB} = o$ and, finally, because of the specific structure of A_N we get $x_N = o$.

The system of prices dual to (4) is

(5) $\bar{p}_B = \bar{A}_B\bar{p}_B = \bar{A}_{CB}\bar{p}_B + l_B^{\phi}\, b\bar{p}_B.$

Both departments will lack profit. All that will be produced will have to serve for the reproduction of constant capital and labour power. (98)

In this system prices $\bar{p}_B$ are proportional to labour values λ_B.

Charasoff always proves the equivalence of surplus value and profit with respect to the difference of necessary labour and surplus labour. He critizises Conrad Schmidt[13] who

plainly reveals his main interest in the problem of distribution thereby ignoring completely the problem of production: the length of the working day is assumed as given and the wage as variable - and not the opposite. (XXVI)

Moreover, there always would be a functional relation between the height of the rate of profit and the wage of labour, a relation which is expressed by the fact that - the technique and the length of the working day assumed as constant - profit turns out to be the higher the lower the wage is and vice versa. But, strangely enough, Schmidt regards these arguments of his as a proof that one can study the social relationships of today without the law of value quite well too, instead, quite the reverse, of recognizing in this a confirmation of the law of value. Obviously, this is be-

cause he ranks the problem of distribution above the problem of production and because he is unable to see the factory behind the market. (XXVI)

4. The reproduction capital and the reproduction basis

Starting with the series of production Charasoff develops in chapter 12 the concepts of the reproduction capital and the reproduction basis. Both represent sub-systems the first of which is related to total basic production, while the second one exclusively refers to means of production.

4.1. The reproduction capital

The series of production can conceptually be regarded as a sequence in time of capitals of increasingly higher order which in the final period once yield a definite product as a surplus. If this product is to be consumed periodically, the sum of the components of the series of production can be regarded as just that capital which society simultaneously must apply:

The idea of a reproduction capital can most simply be made clear if one thinks of a society producing only one definite article X as net product or as a fruit of its surplus labour. To be able to produce this article at all, society

must operate the different industries, yet all of them only as an aid in the main production of commodity X. Whatever else is produced society itself consumes for the sake of production (and as necessary consumption of the working class), only the article X can be withdrawn from production, can enter into luxury consumption, or can be exported.
(141)[14]

This is in full analogy with Sraffa's definition of a sub-system: "Consider a system of industries (each producing a different commodity) which is in a self-replacing state... Such a system can be subdivided into as many parts as there are commodities in its net product, in such a way that each part forms a smaller self-replacing system the net product of which consists of only one kind of commodity."[15]

Continuous reproduction of the surplus product requires that the capitals of higher order which constitute the production series act simultaneously - so to speak *in spatial coexistence* (140). The reproduction capital of any product, therefore, is the sum of the infinite sequence of capitals of higher order:

If some product X is annually consumed beyond the wages of labour then the capital $X' + X'' + \ldots + X^{*} + \ldots$ *necessarily must be available to society. Then this capital reproduces itself with an increment X, and this is why we may call it the* <u>*reproduction capital*</u> *of product X....This, again, is* *nothing but the basic production taken in such proportions that the entire surplus takes on the pure shape of product X*. (140 -141)

Let us assume that exclusively the quantity s_i of commodity i is produced as society's surplus then the corresponding sub-system is
$x^i = x^iA + s^i$, where $s^i = s_ie^i$, and e^i is the i-th unit vector in $\mathbb{R}^m$.

The reproduction capital of this product therefore is the infinite series
$s^iA + s^iA^2 + s^iA^3 + \ldots$

As $\sum_{t=1}^{\infty} A^t = [I - A]^{-1}A$ and $x^i = s^i[I - A]^{-1}$ for the reproduction capital we get
$s^i[I - A]^{-1}A = x^iA$.

Since Charasoff did not know this modern form of solution, he approximates the Leontief-inverse by assuming that the original capital is reached at a finite stage in the production series:

Consider, if X^* is of sufficient high order and approximately of the original type, that the reproduction capital of X^* is of the original type again, that is, it must equal X^*/P^*, where P^* is the rate of growth of the original capital: since X^*/P^* annualy yields an interest X^*. Therefore the *reproduction capital of a product X is not $X' + X'' + \ldots + X^*$, as would have been obtained, if the addition would have been broken off arbitrarily, but $X' + X'' + \ldots + X^* + X^*/P^*$.* (144)

4.2. The reproduction basis

Because $A = A_C + 1\phi b$ and therefore workers' consumption goods enter into the reproduction capital, the latter *depends on the level of wages and on the length of the working day as well.* (146)

If we are merely interested in the means of production which enter directly and indirectly into a given product, then we have to exclude workers' consumption goods from the reproduction capital.

Unless wages are considered as expenditure but as part of social income one has to attribute special theoretical importance to <u>that</u> reproduction capital which was calculated with the assumption that wages are null. Let us denote such a reproduction capital the <u>reproduction basis</u>. The reproduction basis is calculated by the same method as the reproduction capital except that wages, thoroughly, are assumed as non-existent or null; in other words, only physical aids of the process of production are conceived as means of production, but not the necessary consumption of the working popu-

lation. (146 - 147)

Thus, the reproduction basis for y_i units of commodity i is

$$y^iA_c + y^iA_c^2 + y^iA_c^3 + \ldots, \quad \text{where } y^i = y_ie^i$$
$$= y^i[I - A_c]^{-1}A_c$$
$$= q^c(y^i)A_c$$

$[I - A_c]^{-1}A_c$ is the matrix of vertically integrated coefficients of means of production:

$q^c(y^i) = q^c(y^i)A_c + y^i = y^i[I - A_c]^{-1}$ is a sub-system for y^i in Sraffa's sense.

Like Sraffa Charasoff applies the reproduction basis for the calculation of the value of an arbitrary commodity i:

If R is the reproduction basis of a particular product X and a is the quantity of human labour which annually has to be expended on this reproduction basis, then the annual reproduction of the product X costs society exactly the quantity a of labour. With this, eventually, the concept of labour cost of production and, consequently, of value of commodities is defined. (147)

This is exactly Sraffa's sub-system method of calculating labour values:

$$y_i\lambda_i = y^i[I - A_c]^{-1}\mathbf{1}$$
$$= q^c(y^i)\mathbf{1}$$

"Thus in the sub-system we see at glance, as an aggregate, the same quantity of labour that we obtain as the sum of a series of terms when we trace back the success of stages of the production of the commodity."[16]

For the value of an arbitrary product y we get with this method

$$(6) \qquad y\lambda = q^{c}(y)l.$$

In his first book Charasoff already sketches the possibility of determining labour values by reduction to dated quantities of <u>surplus labour</u>.[17]

The value equation $\lambda = A_{c}\lambda + l$ can be reformulated as follows:

$$\lambda = A_{c}\lambda + \frac{1}{1+\mu}l + \frac{\mu}{1+\mu}l$$
$$= A_{c}\lambda + \frac{1}{(1+\mu)b\lambda}lb\lambda + \frac{\mu}{1+\mu}l$$

By equation (2) follows $\frac{1}{(1+\mu)b\lambda} = \phi$. Thus

$$\lambda = [A_{c} + \phi lb]\lambda + \frac{\mu}{1+\mu}l = \frac{\mu}{1+\mu}[I - A]^{-1}l$$
$$= \frac{\mu}{1+\mu}[l + Al + A^{2}l + \dots]$$

or, for an arbitrary product y we have

$$(7) \qquad y\lambda = \frac{\mu}{1+\mu}q(y)l$$

with $y[I - A]^{-1} = q(y)$.

4.2.1. The reproduction basis of the socially necessary product

Special importance Charasoff ascribes to *the reproduction basis of the socially necessary product* Nb. Charasoff develops this concept by the following argument:

Given the technical conditions of production, the needs of the working class and the number of workers N, then, if L^v, the labour which has to be performed on this basis, is distributed equally on the workers, each worker has to work $\frac{L^v}{N} = T^v$ hours. If the length of the working day is exactly $T = T^v = b\lambda$ hours, then all workers must be employed on the reproduction basis of the necessary product. Consequently there can be neither surplus labour nor profit. If, however, the working day is prolonged beyond T^v such that each worker works $T = T^v + T^m$ hours, then the necessary labour L^v can be performed by merely $N^v = \frac{L^v}{T}$ instead of N workers. Thus, $N - N^v$ workers are set free to work on the reproduction basis of the surplus product.

Replacing L^v by NT^v yields $N^v = N\,\frac{T^v}{T}$ for the number of workers engaged in the reproduction of the necessary product.

and $N^m = N \frac{T^m}{T}$ for the workers producing the surplus product. The ratio of N^v and N^m is equal to the ratio $\frac{T^v}{T^m}$, that is the ratio of necessary labour to surplus labour.

The workers of the first category perform the socially necessary labour and, therefore, may be called necessary labourers - in contrast to the surplus labourers which are immediately engaged in the production of the surplus product. (149)[18]

The core of this idea of assigning necessary and surplus labour to certain groups of workers is already found in Marx:

"Developed further, the total agricultural labour, both necessary and surplus labour, of a segment of society must suffice to produce the necessary subsistence for the whole of society, that is, for non-agricultural labourers too. ... Although the labour of the direct producers of means of subsistence breaks up into necessary and surplus labour as far as they themselves are concerned, it represents from the social standpoint only the necessary labour required to produce the means of subsistence."[19]

The reproduction basis of the necessary product Nb is

$Nb[I - A_c]^{-1}A_c = q^c(Nb)A_c$,

its value

$Nb[I - A_c]^{-1}1 = Nb\lambda$, where $b\lambda = T^v$.

We see at glance that *the system of necessary production* is identical to the subsistence economy of equation (4) with the exception that in the former surplus labour exists:

$$q^c(Nb)A_c + Nb = q^c(Nb) \iff \bar{x}A_c + Nb = \bar{x} \iff \bar{x}\,\bar{A} = \bar{x} \quad \text{or}$$

$$\bar{x}[A_c + 1\bar{\Phi}b] = \bar{x}.$$

With $\Phi = \dfrac{1}{(1+\mu)b\lambda}$ and $\bar{\Phi} = \dfrac{1}{b\lambda}$ we get $\bar{\Phi} = \Phi(1+\mu)$.

Therefore

$$\text{(8)} \qquad \bar{x}[A_c + 1\Phi(1+\mu)b] = \bar{x} \quad \text{or}$$

$$\bar{x}\,A_c + (N^v + N^m)b = \bar{x}.$$

Thus, in the system of necessary production a surplus product is obtained which simply is a scalar multiple of the vector of real wages b. The rate of surplus value, therefore, can be derived as a ratio of physical quantities.

If the working day is reduced with wages remaining constant, a definite part of the surplus labourers has to migrate to the category of the necessary ones until there are no more surplus labourers and the working day is restricted to necessary labour time alone. Here the connection between surplus labour and profit is a transparent one and it is easy, too, to demonstrate the further connection between the rate of surplus value and the rate of profit. (149)

Therefore Charasoff formulates the following

Proposition

The organic composition of necessary capital, measured in money units, is equal to the organic compostion of original capital measured in labour units. Let us denote it by Z, then the general rate of profit is $\mu/(Z + 1)$. (150)

$$\text{(i)} \qquad \frac{x^* A_C \bar{p}}{x^* 1\phi b\bar{p}} = Z = \frac{\bar{x} A_C p^*}{\bar{x} 1\phi b p^*}$$

$$\text{(ii)} \qquad R = \frac{\mu}{Z + 1}$$

<u>Proof:</u>

The quantity system of original capital is

$x^*[A_C + 1\phi b](1+R) = x^*$.

multiplied by $\bar{p}$

$x^*[A_C + 1\phi b]\bar{p}(1+R) = x^*\bar{p}$.

Similarly, the value system activated by x^* is

$x^*[A_C + (1+\mu)1\phi b]\bar{p} = x^*\bar{p}$.

Thereby for the rate of profit follows

$$(9) \qquad R = \frac{\mu x^* 1\phi b\bar{p}}{x^* A_C\bar{p} + x^* 1\phi b\bar{p}} = \frac{\mu}{\dfrac{x^* A_C \bar{p}}{x^* 1\phi b\bar{p}} + 1} = \frac{\mu}{Z(x^*) + 1}$$

As is seen from equation (8), the system of necessary production is

$\bar{x} = \bar{x}[A_C + (1+\mu)1\phi b]$,

multiplied by p^*

$\bar{x}p^* = \bar{x}[A_C + (1+\mu)1\phi b]p^*$

On the other hand the system of prices of production, activated by $\overline{x}$, is

$$\overline{x}p^* = \overline{x}[A_C + 1\phi b]p^*(1+R).$$

Thus for the rate of profit follows

$$(10) \qquad R = \frac{\mu\overline{x}1\phi bp^*}{\overline{x}A_C p^* + \overline{x}1\phi bp^*} = \frac{\mu}{\dfrac{\overline{x}A_C p^*}{\overline{x}1\phi bp^*} + 1} = \frac{\mu}{Z(\overline{x}) + 1}.$$

Setting (9) equal to (10) yields

(i) $\qquad Z = Z(x^*) = Z(\overline{x})$ and

(ii) $\qquad R = \dfrac{\mu}{Z + 1}.$ $\qquad \|$

We see that, formally, Charasoff has all the elements for a complete theory of "dual duality":[20]

The system of necessary production (8)

$$\overline{x} = \overline{x}[A_C + (1+\mu)1\phi b]$$

is dual to the system of labour values

$$\overline{p} = [A_C + (1+\mu)1\phi b]\overline{p},$$

while the original capital

$$x^* = x^*[A_C + 1\phi b](1+R)$$

is dual to the system of prices of production

$$p^* = [A_C + 1\phi b]p^*(1+R).$$

From the point of view of modern debates on Marxian economic theory it is straightforward that the original capital x^* becomes identical with the system of necessary production $\overline{x}$, if the vector of real wages b is an eigenvector of A_C, as well as p^* becomes identical with $\overline{p}$, if 1 is an eigenvector of A_C, i. e., if the organic composition of capital is uniform. In either case Marx' formula for the general rate of profit holds.[21]

4.2.2. The relation between the reproduction basis and the reproduction capital of a given product

To analyze the relation between the reproduction basis $q^c(y)A_c = y[I - A_c]^{-1}A_c$ and the reproduction capital $q(y)A = y[I - A]^{-1}A$ of a product y, Charasoff argues:

Let $y\lambda$ be the value of a given product y and $\frac{y\lambda}{T} = y\lambda\Phi$ the number of workers, which produce y. Then $y\lambda\Phi b\lambda$ is the wage of these workers. In order to yield this wage, an additional number of workers have to be employed on the reproduction basis, such that their surplus labour is exactly $y\lambda\Phi b\lambda$. Their number is $\frac{y\lambda\Phi b\lambda}{T^m}$ thereby producing the necessary product of value $\frac{y\lambda}{T^m} b\lambda$ and, thus, for $\frac{y\lambda}{T^m}$ workers. Then

The reproduction capital of our product originates from its reproduction basis, if the reproduction basis of the necessary product for $\frac{y\lambda}{T^m}$ *workers is added to it and if, moreover, this necessary product itself is added to this capital.* (152) i.e.:

$$q(y)A = y[I - A_c]^{-1}A_c + \frac{y\lambda}{T^m}b[I - A_c]^{-1}A_c + \frac{y\lambda}{T^m}b$$

Charasoff leaves the proof as an exercise to the reader:

As

$$q(y) = q(y)A_c + q(y)1\phi b + y = [q(y)1\phi b + y][I - A_c]^{-1}$$

and

$$q(y)A = q(y)A_c + q(y)1\phi b$$

it follows, that

$$q(y)A = [q(y)1\phi b + y][I - A_c]^{-1}A_c + q(y)1\phi b.$$

Therefore, between $q(y)A$ and $q^c(y)A_c$ the following relation is established:

$$q(y)A = q^c(y)A_c + q(y)1\phi b[I - A_c]^{-1}A_c + q(y)1\phi b$$

It only has to be shown that $\frac{y\lambda}{T^m} = q(y)1\phi$. By equations (6) and (7) one has

$$\frac{q(y)1}{q^c(y)1} = \frac{1+\mu}{\mu} = \frac{T}{T^m} = \frac{q(y)1}{y\lambda}.$$

Thus $$\frac{y\lambda}{T^m} = \frac{q(y)1}{T} = q(y)1\phi, \text{ completing the proof.}$$

Obviously $\lim_{\mu\to 0} q(y) = \infty$, since

the greater T^m the smaller the reproduction capital of a peculiar product and vice versa: given the social capital, the more products are produced beyond the wages of labour. If, however, $T^m = 0$, then the reproduction capital of any *item is infinitly great and no product can be produced beyond the necessary one. This, again, confirms that profit, also in its physical shape, stems from surplus labour.* (152)

5. Competition

The concept of original capital serves Charasoff for solving the dual problem of equalization of profit rates on the one hand and of adjusting output to demand on the other hand.

5.1. The equalization of profit rates

Charasoff assumes that the sectoral distribution of capitals is adapted exactly to social needs but market prices still deviate from prices of production thus yielding different profit rates for different capitals. He describes the formation of a general rate of profit by a procedure known today as Markov-process:[22]

$$p^t = [A^*]^t p^0.$$

For $t \to \infty$ follows $p^* = \Omega\, p^0 = \frac{1}{\alpha} A\, \Omega\, p^0$ and thus
$p^* = (1+R)A\, p^*$.

We see that the initial price p^0 *of a commodity X at first is transformed into a magnitude proportional to the initial* *price* p' *of capital X'; further into a magnitude proportional to the initial price* p'' *of the capital of second order X'', and so forth in infinitum....In this manner the analysis of capitalist competition leads us to the theorem of original capital anew. If such an original capital would not exist, a general rate of profit would be impossible, and that is why everyone who speaks of regular prices of production or of a general rate of profit unconsciously recognizes the fact that such an original capital exists.* (137 - 138)

Marx's algorithm of transformation merely performs the first step of iteration for $p^0 = \lambda$:

$$p^1 = (1 + \frac{\mu x l \phi b \lambda}{x A \lambda}) A \lambda$$

Here Marx breaks off his conversion of values into prices and this is the first imperfection of his theory of prices which one does not grow tired of blaming him for, instead of improving it by a dialectical development of the basic idea. A second imperfection is that Marx absolutely wanted to start with labour values of commodities. This, however, is really inessential for the theory of prices as such. The initial prices are allowed to be arbitrary. Identifying them with values may be an inevitable logical necessity, if one recognizes the law of economizing on human labour as the major premise of all human economy. For the theory of capitalist competition this is of no concern. (138)

5.2. Supply and demand

Charasoff asks, how a reallocation of social capital to the different sectors is possible, if the physical composition of the sectoral capitals is not uniform, because this implies that by contraction and expansion of sectoral outputs inputs are set free and demanded in incompatible proportions.

The original capital allows for a simple solution of this problem: By definition, in $\Omega = p^* x^*$ sectoral inputs are of uniform physical composition. Charasoff argues that by successively expanding the series of production input demands of all sectors finally tend to the structure x^*. Therefore on a sufficiently low stage of production it is possible

to turn a certain portion of original capital which earlier formed ,say, the fourth stage in the production of product A into the fourth stage of production of product B. (133)

This idea is dual to the Markov-process above:

$$x^t = x^0 [A^*]^t \text{ and for } t \to \infty$$

$$x^* = x^0 \ \Omega = (1+R) x^* A$$

Since any initial vector $x \in \mathbb{R}^m$ in the limit is transformed into the original capital x^*, so, conversely, any activation

$x' \in \mathbb{R}^m$ of the <u>disaggregated</u> original capital $\Omega = p^* x^*$ can be derived from the original capital x^*:

$$x\ \Omega = x^* = x'\ \Omega$$

for the normalization $x\ p^* = x'\ p^* = 1$.

In other words, Charasoffs idea is that the sectoral portions of the original capital are regrouped corresponding to the shift of demand for the surplus product $s = x[I - A]$ to the demand for $s' = x'[I - A]$.

$$\hat{x}\ \Omega = \begin{bmatrix} x_1 p_1^* x^* \\ \vdots \\ x_m p_m^* x^* \end{bmatrix} \longrightarrow \begin{bmatrix} x_1' p_1^* x^* \\ \vdots \\ x_m' p_m^* x^* \end{bmatrix} = \hat{x}'\Omega,$$

where $\hat{x}$ and $\hat{x}'$ are diagonal matrices formed by the vectors x and x'.

It is as if a field had been used for the production of rye which previously had served for the production of wheat. (134)

6. Technical change and the rate of profit

The correct determination of prices of production leads Charasoff, like before him v. Bortkiewicz, to a critique of Marx's law of the falling profit rate. Marx maintains that the fall of the profit rate inevitably results from the rising ratio of dead to living labour.[23] To this Charasoff remarks:

It is certainly possible that the ratio a/c of living labour to dead labour falls and that simultaneously the rate of profit m/c+v rises; in the long run such an inverse movement of both magnitudes, yet, is impossible. (155)

This Okishio (1961) points out too, stressing that on this condition a rising rate of surplus value ultimately cannot compensate for the rising organic composition of capital. Both Marx's value rate of profit m/(c+v) as well as the correct rate of profit R, cannot exceed this tendentially falling least upper bound.

But Charasoff agrees with Okishio who demonstrates that R must rise even if the above condition is satisfied, because capitalists introduce technical innovations only if they are cost-reducing. Moreover, Charasoff recognizes the specific role of the basic system in this context:

The regular system of prices of production **p** *taken as a basis, the general profit rate equals R and the cost price of a particular commodity equals k, whereas its selling price equals k(1+R). Now a new method of production is introduced, where the cost price turns out to be lower, namely equal to K, while the commodity, for the time being, is sold off at the old selling price k(1+R) and its seller realizes a profit rate* R_j *situated above the regular one. Since under the*

old system of prices of production the profit rate does not turn out to be equal throughout but has become equal to R_j for a new capitalist, while it has remained the same for all the others, competition with its equalizing tendency will have to come in, and for all commodities new prices of production will be brought about, through which, finally, the profit rate will become equal, everywhere, say, = R'.
(189 - 190)

If, therefore, the initial system of prices of production is $Ap(1 + R) = p$ and if a new technique A' is introduced, such that

$a^i = a'^i$, $i \neq j$, $i,j \in \{1,\dots,n\}$,

then the innovating sector j realizes an extra-profit because

$a'^i p(1 + R) = p_i$ and

$a'^j p(1 + R) < p_j \iff a'^j p(1 + R_j) = p_j$, $R_j > R$.

Equalization of the profit rates yields the new system of prices p' and a new rate of profit R'.

$$A'p'(1 + R') = p'$$

The question now is whether the magnitude R' will be lower or greater than R.
Yet, put this way, the question is answered at once, since the general rate of profit is always the average of the partial rates of profits calculated on the basis of an arbitra-

ry system of prices. Thus the new average rate of profit will be situated between R_j, the rate of profit of our capitalist, and R, the rate of all the other capitalists, calculated on the basis of the old regular system. It never can fall below the previous rate of profit R, but will always exceed it, if it refers to the basic system. (190)

Mathematically, Charasoff's argument is as follows:[24]

$$A'p(1 + R) \lesssim p \quad \Rightarrow$$

$$\min_i \frac{a'^i p}{p_i} < \alpha' < \max_i \frac{a'^i p}{p_i},$$

where α' is the maximum eigenvalue of A'.

Thus

$$a'^j p(1 + R_j) > a'^i p'(1 + R') > a^i p(1 + R).$$

Footnotes:

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[1] Charasoff (1910) was reviewed by the Austrian Marxist Bauer (1911). Bucharin (1925) and Grossmann (1929) mention him casually. Moszkowska (1929). pp. 31 - 32. recognizes that his transformation of values into prices is correct without considering it in detail.

[2] We received the article of Egidi and Gilibert only after our argument had been worked out. According to Porta (1986). p. 453. an italian edition of *Das System des Marxismus* is prepared by G. Gilibert.

[3] In this paper all quotations from Charasoff's book are in italics. Moreover, for the sake of consistency, in quotations we sometimes replace Charasoff's symbols by ours.

[4] Charasoff's critique of Marx's formula is analogue to v. Bortkiewicz's. See Charasoff (1910), ch. 6.

[5] See Charasoff (1910), p. 96.

[6] If we refer exclusively to the basic system, the subscript B is omitted.

[7] The following deduction, of course, is not Charasoff's, who, however, points out that the original capital is obtained after a finite number of steps $t(\Delta)$ with an arbitrary small deviation: $q[A^t + \Delta] = q\,\Omega$, $\Delta \in \mathbb{R}^{mxm}$. Moreover, our normalization is not the only possible one. The objection might be raised that the presumption of the eigenvalue is tantamount to the presumption of the profit rate which Charasoff gets as the result of his analysis. The point, however, is to give a simple and precise view of Charasoff's idea. For an alternative normalization see Egidi/Gilibert (1984), p. 48.

[8] See Nikaido (1968), Theorem 8.1., p. 110.

[9] The prices of secondary products are irrelevant for the determination of R. As soon as the prices of the basic products and the profit rate are known, the prices of the secondary products $A_N p^*(1+R) = p^*$ are determined as well. In this generalized form Charasoff (1910), pp. 90 - 92, formulates the "theory of the corn profit rate" which, according to Sraffa (1951), pp. xxx - xxxiii, is due to Ricardo - an idea also found in Dmitriev (1904) and v. Bortkiewicz (1907).

[10] The original capital, of course, consists of basic products alone, since - as Charasoff presupposes - A_N is nilpotent and, therefore, all secondary products are eliminated

after a finite number of steps. Under a certain condition, however, the expansion of the series of production also eliminates those non-basics which enter into their own reproduction: Even if A_N is not nilpotent such non-basics vanish as t tends to infinity, provided that the maximum eigenvalue of the basic system A_B is greater than that of A_N, as Egidi (1975) shows in appendix 5. For the economic meaning of this condition see Sraffa (1960), appendix B.

[11] In chapter 17 Charasoff develops the idea of synchronized labour costs. He maintains, that for a population growing at the rate j socially necessary labour amounts to $V + jK$, where $K = C + V$ is the value of society's total capital. *Let $jK = M$, then j proves to be equal to $M:K$, from which it follows, that exactly so much surplus labour has to be performed for the sake of reproduction of the population that the average social rate of profit turns out to be equal to the rate of reproduction j.* (218) Cf. v. Weizsäcker/Samuelson (1971).

[12] For a subsistence economy Seidelmann (1965) develops from the matrix $\bar{A}$ the corresponding limit matrix $\bar{\Omega}$ in order to obtain the vector of labour values $\bar{p} = \lambda$. Seidelmann is mentioned in the foreword of Behr and Kohlmey, the editors of the german edition of Sraffa (1960).

[13] Conrad Schmidt is perhaps known to the reader from Engels' Preface to Marx (1894), where Schmidt is mentioned in

the context with the transformation problem.

[14] Charasoff gives two examples for a reproduction capital: 1. The efforts of american steel trusts to integrate vertically the entire production process of steel products. 2. Let Russia produce all means of production for her export, consisting of corn alone, exactly in those proportions as is required by this sole final product. Anyway he interpretates mercantilism as the effort of forming a social capital which can be decomposed into several distict reproduction capitals.

[15] Sraffa (1960), appendix A, our emphasis.

[16] Sraffa (1960), appendix A.

[17] Charasoff (1909), pp. 67 - 69; see also Charasoff (1910), pp. 153 - 154.

[18] In Morishima (1973), pp. 49 - 50, a similar argument is to be found.

[19] Marx (1894), p. 635.

[20] For "dual duality" in Marx see Morishima (1973), p. 4.

[21] Samuelson (1971), who wants to show that Marx's formula for the rate of profit is valid for a peculiar structure of

coefficients, independent of the assumption of uniform organic composition of capital, in his famous system of equal internal compositions of capital presupposes that the material inputs in every industry, as well as the vector of real wages, are of the same composition as gross output. This implies that the augmented matrix **A** is an Ω-matrix.
As is shown in Morishima (1973), p. 78, this assumption is an extreme case of the necessary condition of linearly dependent industries. A weaker assumption, sufficient to bring about Samuelsons result and dual to that of uniform organic composition, is a vector of real wages **b** as an eigenvector of A_C in the system of necessary production.

[22] For the application of Markov-processes in Marxian theory see Morishima/Catephores (1979), ch. 6.

[23] Marx (1894), p. 213.

[24] Cf. Roemer (1981), p. 98.

References:

Bauer, O. 1911. Bücherschau. *Der Kampf*. February

Bortkiewicz, L. v. 1907. Zur Berichtigung der grundlegenden theoretischen Konstruktion von Marx im 3. Band des Kapital. *Jahrbücher für Nationalökonomie und Statistik*

Bucharin, N. 1919. *Die Politische Ökonomie des Rentners*. Wien,Berlin, 1925, Verlag für Literatur und Politik

Charasoff, G. 1909. *Karl Marx über die menschliche und kapitalistische Wirtschaft. Eine neue Darstellung seiner Lehre*, Berlin, Bondy

Charasoff, G. 1910. *Das System des Marxismus. Darstellung und Kritik*, Berlin, Bondy

Dmitriev, V. K. 1904. *Economic Essays on Value, Competition and Utility*, ed. by Nuti, M. D., Cambridge, CUP, 1974

Egidi, M. 1975. Stabilita ed instabilita negii schemi sraffiani. *Economia Internazionale*. nn. 1-2

Egidi, M., Gilibert, G. 1984. La teoria oggettiva dei prezzi, *Economia Politica*, April

Grossmann, H. 1929. *Das Akkumulations- und Zusammenbruchsgesetz des kapitalistischen Systems*. Leipzig, Hirschfeld

Marx, K. 1894. *Capital*, Volume III, London, Lawrence & Wishart, 1977

Morishima, M. 1973. *Marx's Economics. A Dual Theory of Value and Growth*, Cambridge, CUP

Morishima, M., Catephores, G. 1979. *Value, Exploitation and Growth*, London, McGraw-Hill

Moszkowska, N. 1929. *Das Marxsche System. Ein Beitrag zu dessen Ausbau*, Berlin, Engelmann

Nikaido, H. 1968. *Convex Structures and Economic Theory*, New York, Academic Press

Okishio, N. 1961. Technical changes and the rate of profit, *Kobe University Economic Review*

Porta, P. L. 1986. Understanding the significance of Piero Sraffa's Standard commodity: A note on the Marxian notion of surplus, *History of Political Economy*, Autumn

Roemer, J. E. 1981. *Analytical Foundations of Marxian Economic Theory*, Cambridge, CUP

Samuelson, P. A. 1971. Understanding the Marxian notion of exploitation: a summary of the so-called transformation problem between Marxian values and competitive prices. *Journal of Economic Literature*, June

Seidelmann P. 1965. Eine wahrscheinlichkeitstheoretische Fassung des Wertgesetzes in seiner Verallgemeinerung als Gesetz der Verteilung der gesellschaftlichen Arbeit. *Wirtschaftswissenschaft*. January

Sraffa P. 1951. "Introduction" in: Ricardo, D. *Works and Corresondence, Vol. 1*, Cambridge, CUP

Sraffa P. 1960. *Production of Commodities by Means of Commodities*, Cambridge, CUP. German edition: *Warenproduktion mittels Waren*. Frankfurt, Suhrkamp, 1976

v. Weizsäcker, C. C., Samuelson, P. A. 1971. A new labor theory of value for rational planning through the use of the bourgeois profit rate, *Proceedings of the National Academy of Science*