Goodwin's Class Struggle Model

A central feature of most of Kalecki's work was income distribution and different behavior captured by the different motivations of different classes of income recipients (mainly, workers and capitalists). Richard Goodwin (1967) drew upon this Marxian-Keynesian tradition to formulate a non-linear model of the cycle on the basis of "class struggle" via the Lotka-Volterra equations employed in biological "predator-prey" scenarios. More succinctly, Goodwin's model attempted to demonstrate the cyclical relationship between employment and wage share in a working economy. Goodwin's model is in fact not as controversial as it may sound: "class struggle" and "predator-prey" can invoke strident images of revolution and reaction, but nothing more radical than a standard Phillips Curve and a Kaleckian profit mechanism is at work.

The basic features of Goodwin's (1967) model can be stated simply: high employment generates wage inflation which can increase the wage share of workers in output; but this will, in turn, reduce the profits of capitalists and thus, in Kaleckian fashion, reduce future investment and output. That reduction in output will in turn reduce labor demand and employment and consequently lead to lower wage inflation or even deflation and thus reduce the wage share of workers. But as workers wage share declines, then profits increase and, with them, investment. This will lead to greater employment and thus improve the bargaining power of workers and consequently wages in Phillips Curve fashion and thus greater wage share in output - and the rest of the cycle then repeats itself. For good measure, Goodwin adds exogenous growth components - namely, labor supply growth and productivity growth.

Goodwin's essential setup is to begin with two classes of income recipients, wage-earning workers and profit-earning capitalists and output Y is divided between them so that wL represents the wage bill (where w is wage and L the amount of labor employed) and P = Y - wL are total profits. Thus, wL/Y is the wage share and P/Y is the profit share so that wL/Y + P/Y = 1. Letting l = Y/L, then the wage share can be rewritten w/l while the profit share is 1-w/l . Following Kalecki, we can assume capitalists save all their income while workers consume all of theirs. Thus, total savings are S = P = (1-w/l)Y. All savings are invested, so:

> $dK/dt = S = (1-w/l)Y.$

and thus the growth rate of the capital stock is:

> $g_K = (dK/dt)/K = (1-w/l)(Y/K)$

or, letting v = K/Y, the capital-output ratio, then:

> $g_K = (1-w/l)/v$

As l = Y/L, then employment L = Y/l . Let labor productivity grow at the rate q so $l = l_0e^{qt}$. Thus, as Y/L = l, then $g_Y - g_L = (dK/t)/dK - (dL/dt)/L = q$ so:

> $g_L = g_K - q$

or, substituting in for g_K:

> $g_L = (1-w/l)/v - q$

Let N be the supply of workers, growing at natural rate n, so $N = N_0e^{nt}$, thus $g_N = n$. Thus, the employment rate m = L/N so, in growth terms:

> $g_m = (dm/dt)/m = g_L - g_N$

or :

> $g_m = (1-w/l)/v - q - n$

reorganizing (and letting u = w/l):

> $g_m = (1 - u)/v - (q + n)$

is employment rate growth. As u = w/l , then:

> $g_u = g_w - gl$

reflects the growth of the labor share. Goodwin assumes a Phillips curve relationship dominating the growth of wages, with g_w = ï½¦ (m) where ï½¦$_m$ > 0, so as employment rate increases growth of wages declines. If linear (or approximating it via a linear function), then we can write the Phillips Curve relations as $g_w = -a + b\,m$ (where a , b > 0). Recalling that $g_l = q$, then plugging these into our g_u equation:

> $g_u = [-a + b\,m] - q$

Thus we can establish two differential equations from g_m and g_u, which reflect growth in labor employment and wage share respectively. Rewriting:

> $dm/dt = [1/v - (q + n) - u/v]m$
>
> $du/dt = [-(a + q) + bm]u$

which are "Lotka-Volterra" equations. Lotka-Volterra equations have vortex dynamics of the type illustrated in Figure 1.

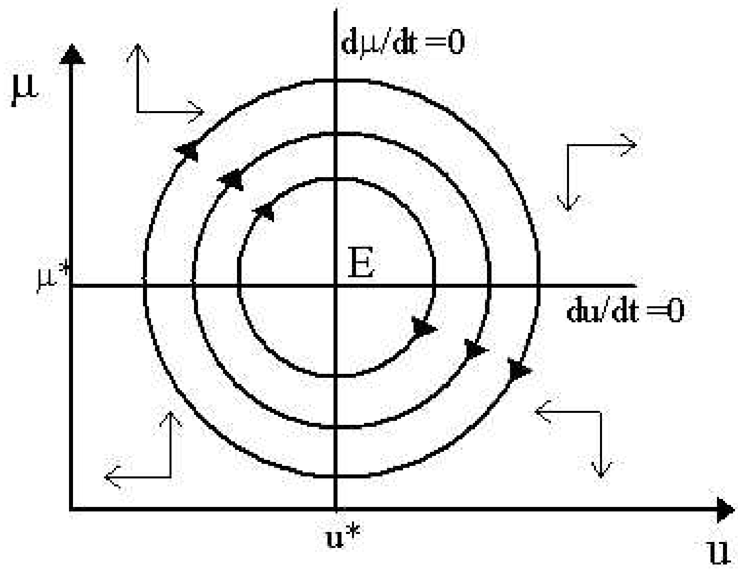

Fig. 1 - Vortex Dynamics in Wage Share and Employment

In Figure 1, *every* trajectory in (u, m) space is a closed orbit around the equilibrium E = (u*, m *). Thus, as is obvious, we have cyclical dynamics of u andm . To understand the logic, look only at the differential equations for employment and the wage share. If employment goes to zero, then obviously the du/dt = -(a +q)u, i.e. the wage share goes to zero exponentionally at rate (a +q). If, on the other hand, wage share goes to zero, then dm /dt = [1/v - (q + n)]m , which implies that employment is increased infinitely at the exponential rate [1/v - (q + n)] (as there are no costs). This is akin to "predator-prey" where employment is the prey and wage share is the predator: if the prey (employment) disappears, then the predator (wage share) dies out; if the predator disappears then the prey grows unbounded.

The clockwise direction of the closed orbits in Figure 1 and the equilibrium values (u*,m *) and isokines are easily ascertained from the differential equations. If dm /dt = 0, then 1/v - (q + n) - u/v = 0 so, dm /dt = 0 where u = 1 - v(q + n) = u* (which is positive becaue 1 > v(q +n)) Similarly, if du/dt = 0, then - (a + q) + b m = 0, or du/dt = 0 where m = (a + q)/b = m *. Thus, the isokines dm /dt = 0 and du/dt = 0 are vertical and horizontal lines respectively which intersect at the equilibrium values (u*, v*). The off-isokine dynamics are easy to evaluate. Note that evaluated at equilibrium m *, d(dm /dt)/du = -(a +q)/b v < 0, so to the right of isokine dm /dt = 0, m declines whereas to the left of it, m rises. Similarly, evaluated at equilibrium u*, d(du/dt)/dm = b (1-v(q +n)) > 0, so above the du/dt = 0 isokine, u rises, whereas below it, u falls.

> dm/dt = [1/v - (q + n) - u/v]m
>
> du/dt = [- (a + q) + b m]u

It is obvious that *which* trajectory dominates will depend on initial conditions as well as the structure of the equations. To get at the resulting dynamic, let us set up the differential equations as a ratio to eliminate dt:

> dm/du = [dm/dt]/[du/dt] = [1/v - (q + n) - u/v]m/[-(a + q) + bm]u

so:

> $$dm\,[-(a + q) + bm]u = du[1/v - (q + n) - u/v]m$$

or factoring out m from the left and u from the right:

> $$dm[-(a + q)/m + b]mu = du[1/vu - (q + n)/u - 1/v]mu$$

so dividing through by mu:

> $$dm[-(a + q)/m + b] = du[1/vu - (q + n)/u - 1/v]$$

integrating both sides:

> ãƒ»/font> [-(a + q)/m + b] dm = ãƒ»/font>[1/vu - (q + n)/u - 1/v] du

which yields:

> $$-(a + q) \ln m + bm = [1/v - (q + n)] \ln u - u/v + c$$

where c is a combined constant of integration. Introducing dummy variable z, then the left side can be written:

> $$z = F(m) = -(a + q) \ln m + bm$$

and for the right hand side:

> $$z = G(u, c) = [1/v - (q + n)] \ln u - u/v + c$$

The curves F(m) and G(u, c) are illustrated in Figure 2. For the first equation dz/dm = Fï½¢ = -(a + q)/m + b and F(m) is convex to the origin. Note that at the bottom of F(z), F' = 0 implies m = -(a +q)/b at the extremum of F(z) - which is, incidentally, exactly the equilibrium value m*. Similarly, for the second equation: dz/du = Gï½¢ = [1/v - (q + n)]/u - 1/v and G(u, c) is concave to the origin; at the extremum, Gï½¢ = 0 so u = 1 - v(q +n), which is exactly the equilibrium value u*.

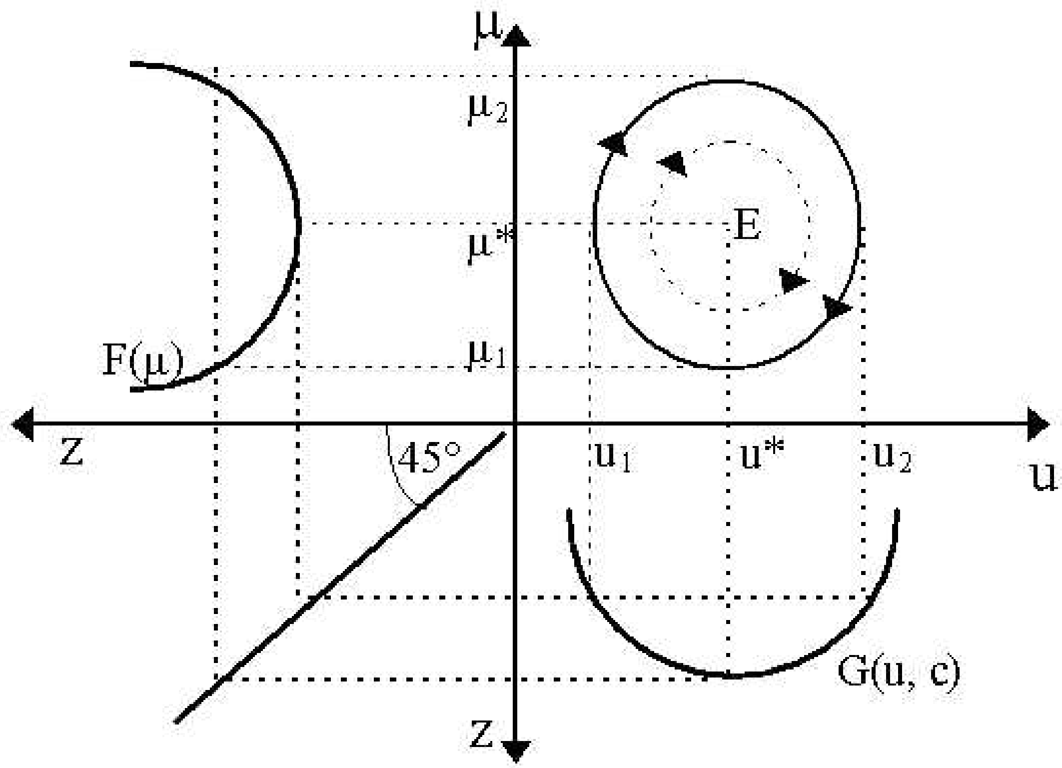

Fig. 2 - Goodwin's Class Struggle Dynamics

We see immediately that the extrema of F(m) and G(u, c) form the boundaries of the values u and m will take and thus the appropriate orbit. For instance, suppose we began at m_1 in Figure 2. This corresponds, via F(m), to a particular z which, bouncing off the 45ï½° line, yields u* which is the extrema of G(u, c). Thus, m *cannot* fall below m_1 because then the corresponding z will be above the maximum G(u, c). Similarly, m cannot be higher than m_2. Conversely, if we take the lower bound of the admissible values of u (i.e. u_1), this corresponds to a particular value z via G(u, c) which, bouncing off the 45ï½° line, is the extrema of the F(m) function. Any u below u_1 would imply a z below the extrema of F(m), which is not feasible. Thus, u cannot be lower than u_1 nor, by a similar argument, can it be above u_2.

The boundaries m_1, m_2, u_1 and u_2 all define a very clear orbit in the positive orthant - which is one out of the many possible orbits, but it will be the one that is chosen. As the boundaries are determined by the shape and position of F(m) and G(u, c) respectively, then obviously *which* orbit is chosen depends not only on the parameters in those functions (a , b , v, q , n) as well as the coefficient of integration, c. If, for instance, c changes so that G(u, c) declines (i.e. is closer to the horizontal axis), it is easy to trace that the smaller, inner orbit traced in Figure 2 will rule the dynamics. In short, given the orbit chosen by the parameters, employment and wage share will fluctuate cyclically in a deterministic manner - without the extraneous paraphenelia of exogenous shocks, ceilings, floors, etc. Thus, Goodwin's (1967) "class struggle" cycle is "endogenous" to the system. Many economists, mostly (but not exclusively) of Keynesian or Marxian persuasion, have made much use of Goodwin's structure and endogenous cyclical dynamics.