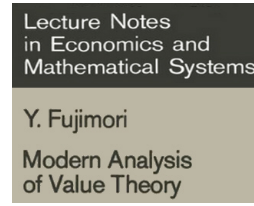

CHAPTER IV

MARX-VON NEUMANN'S THEORY OF VALUE

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Introduction.

The theory of value based on the system of value equations comes up against a crucial difficulty in a von Neumann economy, unless the additional conditions as discussed in the preceding chapter are fulfilled. To presuppose such conditions will, needless to say, circumscribe the validity of Marx's theory of value,

If the value equation is not solvable, the rate of surplus value cannot be evaluated. There still remains the possibility, however, that the rate of surplus labour reflects the volume of surplus. How then is the rate of surplus labour defined in general terms?

Although Morishima put forward a criticism of Marx's value theory with Steedman, he went a step further to pave the way for the generalisation of Marx's value theory: he reconstructed Marx's value theory in terms of the inequality á la von Neumann, and discussed the relevance and significance of the fundamental Marxian theorem. The discussion of the fundamental Marxian theorem and related topics á la von Neumann, originally adovocated by Morishima (5) and Morishima-Catephores, is called Marx-von Neumann's theory of value in this volume.

The purpose of this chapter is to place Marx-von Neumann's theory of value in the development of value theory.

The same framework and the notation as employed in Chapter III will be used and similar assumptions will be made. The basic assumptions are:

(B.1) $A \geqq 0$, $B \geqq 0$, $L \geqq 0_n$, $F \geq 0^m$.

(B.2) $x \geqq 0^n$ and $Dx \geq 0^m \Longrightarrow Lx > o$.

(B.3) The productiveness condition holds: $\exists\, x \geqq 0^n$: $Dx \geqq 0^m$, (W,Pd.C).

Note that the set of techniques A , B and L in this chapter is that of technically possible techniques. Also note that the productiveness condition can be weakened here. The remaining assumptions and conditions will be introduced as the discussion proceeds.

§ 1. The theory of optimum value.

1. In order to evaluate the amount of surplus in general terms, let us begin with the following definiton:

DEFINITION 1.(Minimum necessary labour) The smallest amount of labour necessary for the production of the wage goods distributed to the workers is called minimum necessary labour.

Let

x^a n x 1 : actual intensity vector,

and for a given x^a, consider the subsequent linear programming problem: (LP.A)

(1) $$\text{Min}\,\{ Lz \mid Bz \geqq Az + FLx^a,\ z \geqq 0^n \} .$$

It is easy to see that the minimum magnitude of this gives the amount of minimum necessary labour. It is also seen that the problem represents the maximisation of the productivity of labour. In what follows, let a minimiser be called:

z^o n x 1 : necessary intensity vector.

Since minimum necessary labour represents how much the actually employed workers exert their labour so as to produce the wage goods they receive, it is natural to state:

DEFINITION 2.(Surplus labour) Surplus labour is the actual expenditure subtracted by minimum necessary labour:

$$\text{surplus labour} = Lx^a - Lz^o.$$

This is a natural extension of the definition of surplus labour previously made. Then, the rate of surplus labour can be defined by

(2) $$\eta = \frac{Lx^a}{Lz^o} - 1 .$$

This is formally the same as the traditional definition.

PROPOSITION 1. The rate of surplus labour is uniquely determined.

Proof.

(LP.A) is feasible in view of (B.1), and $Lz > o$ from (B.2). Namely, the objective function is bounded from below. Hence (LP.A) has an optimum solution, and $Lz^o = \min Lz$ is unique. (Cf. Lemmas 15-17.)

Q.E.D.

Note that the proportion of z^o does not depend on x^a.

2. Consider the dual problem of (LP.A). It is stated by: (LP.B)

(3) $$\text{Max}\ \{\ \Lambda FLx^a \mid \Lambda B \leqq \Lambda A + L\ ,\ \Lambda \geqq 0_m.\}$$

As mentioned by Definition III-3, an optimum solution of (3) defines the optimum value with respect to FLx^a.

DEFINITION 3.(M_4-value) An optimum value with respect to FLx^a is called M_4-value, or simply optimum value.[1)]

The optimum value is the shadow price of products with respect to the minimisation of labour necessary for their production. Let

Λ^o $1 \times m$: M_4-value vector,

and, as mentioned by Proposition III-8, a nonnegative M_4-value exists in a productive von Neumann economy. Note that the proportion of Λ^o does not depend on x^a.

In view of the duality theorem of linear programming, one has

(4) $$Lz^o = \Lambda^o FLx^a.$$

The maximum value of (3), i.e., $\Lambda^o FLx^a$, represents the volume of wages in terms of M_4-value, and hence is regarded as paid labour. Thus, the rate of unpaid labour may be defined by

(5) $$\mu' = \frac{Lx^a - \Lambda^o FLx^a}{\Lambda^o FLx^a}\ .$$

Next, net products are represented by

$$y^a = (B - A)x^a\ ,$$

so that surplus value in terms of M_4-value is given by

$$\text{surplus value} = \Lambda^o(B-A)x^a - \Lambda^o FLx^a\ .$$

Therefore, the rate of surplus value can be defined by

(6) $$\mu = \frac{\Lambda^o(B-A)x^a - \Lambda^o FLx^a}{\Lambda^o FLx^a} = \mu(\Lambda^o)\ .$$

The relationship among these three rates of surplus is stated by the following proposition:

PROPOSITION 2. $\eta = \mu' \geqq \mu$.

Proof.

From (4), $\eta = \mu'$ is trivial; whereas, from (3), one has

$$\Lambda^o(B - A)x^a \leqq Lx^a\ ,$$

and hence,

$$\Lambda^o(B-A)x^a - \Lambda^o FLx^a \leqq Lx^a - Lz^o,$$

by dint of (4). Thus, $\eta \geqq \mu$. Q.E.D.

Accordingly, it is seen that the rate of surplus value, which is not uniquely determined, is an incomplete measure of exploitation.

§ 2. The fundamental Marxian theorem, generalised.

1. As opposed to the Leontief economy case in which the quantitative as well as qualitative aspect of equilibrium is made clear by Frobenius' theorem, only a little has been known about equilibrium in the von Neumann economy case. The theory of von Neumann equilibrium concerns chiefly the qualitative aspect of equilibrium.

Given the technical matrices and the wage goods bundle, the von Neumann equilibrium can be defined by the solutions of the subsequent problems:

(7) $$\mathrm{Min}\ \{ \pi^w \mid p^w B \leqq (1+\pi^w) p^w M\ ,\ p^w \geqq 0_m \}\ ,$$

(8) $$\mathrm{Max}\ \{ g^c \mid B x^c \geqq (1+g^c) M x^c\ ,\ x^c \geqq 0^n \}\ .$$

Let, as usual,

π^w		:	warranted rate of profit,
p^w	$1 \times m$	:	von Neumann price vector,
g^c		:	von Neumann growth rate,
x^c	$n \times 1$	:	von Neumann proportion,

and the von Neumann equilibrium is described by the quadruplet, $(\pi^w,\ p^w,\ g^c,\ x^c)$.

Now, let the subsequent assumptions be made:

(B.4) $$B\,1^n > 0^m,\quad 1_m M > 0_n\ .$$

The first assumption here means that each type of good is produced by at least one process. (As for the second, see p.59.)

The following proposition is well-known:

PROPOSITION 3.(i) There exists a von Neumann equilibirum, with

$$\pi^w \leqq g^c\ .$$

(ii) $\pi \geqq \pi^w$, if π is the profit rate satisfying

$$pB^a = (1+\pi) pM^a,\ p \geq 0_m\ ,$$

where B^a and M^a are respectively output and augmented input matrices of the engaging processes.

(As for the proof, refer to Klein, pp.358-67.)

Let us note that the theory of von Neumann equilibrium concerns mainly the qualitative aspects of equilibrium, in the sense that it discusses the existence of equilibrium and establishes the inequality concerning the von Neumann growth rate and the warranted profit rate.

Nevertheless, von Neumann equilibrium does not exclude the case in which the economy is contracting. Since von Neumann equilibrium concerns the possibility of growth and profitability, such a case in which the von Neumann growth rate, for instance, is nonpositive is economically meaningless. Then, one has to ask under which conditions von Neumann equilibrium makes sense -- $g^c > o$ and/or $\pi^w > o$. It was Morishima that discussed the relationship between von Neumann equilibrium and the rate of surplus labour on the basis of the above problem. He thus shed light on the most important problem of modern growth theory.

Let P.Pf.C'. of Chapter III(p.59) be extended as:

(B.5) $$\min \{ \pi^M \mid p^M B \leqq (1+\pi^M) p^M A \, , \, p^M \geqq 0_m \, . \} > o \, .$$

This means that if wages are not paid, the warranted rate of profit can be positive.

Then, Morishima demonstrated the following propositions:

PROPOSITION 4. $\eta > o$ implies $\pi^w > o$.

In fact, from (1), (2) and (7), it follows that

$$\eta p^w F L z^o \leqq \pi^w p M z^o ,$$

and the conclusion is soon obtained.

PROPOSITION 5. $g^c > o$ implies $\mu' > o$.

In fact, premultiply (8) by Λ^o , and postmultiply (3) by x^c. It soon follows that

$$g^c \Lambda^o M x^c \leqq \mu' \Lambda^o F L x^c \, .$$

Hence, the conclusion follows.

THEOREM I. (Fundamental Marxian theorem, generalised) $\pi^w > o$, $g^c > o$ and $\eta > o$ are all equivalent.

(As for the detail of the proof of these three propositions, see Morishima(6), pp.619-21, or Morishima-Catephores, pp.51-3.)

2. Some additional discussion will be made here on an extension of Morishima-Seton's equality.

The organic composition of capital in terms of M_4-value is expressed as

(9) $$\xi(\Lambda^o, x^a) = \frac{\Lambda^o A x^a}{\Lambda^o F L x^a} \, .$$

Although the magnitude of the denominator is given by the maximum value of (LP.B), and hence is unique, that of the numerator is not unique. Hence, note that $\xi(\Lambda^o, x^a)$ itself cannot be unique.

Define the maximum equilibrium growth rate as

(10) $$g^M(x^a) = \max\{g \mid Bx^a \geq (1+g)Mx^a.\}$$

Then, one can show:

THEOREM II. (Fundamental Marxian inequalities)[3)]

(i) $$g^M \leqq \frac{\mu}{\xi(\Lambda^o, x^a)+1} .$$

(ii) $$\frac{\mu^*}{\xi^*(x^c)+1} \leqq \pi^w \leqq g^c \leqq \frac{\mu}{\xi(\Lambda^o(x^c), x^c)+1} ,$$

where $\xi^*(x^c) = \xi(p^w, x^c)$ and $\mu^* = \mu(p^w)$.

Proof.

(i) Premultiply (10) by $\Lambda^o(x^a)$, and, in view of (6) and (9), the inequality soon follows.

(ii) The third inequality can be proved by applying (i) to the $x^a = x^c$ case.

The first inequality can be derived in the same manner as in (i): postmultiply (7) by x^c, and rearrange the result.

Needless to say, the second comes from Proposition 3. Q.E.D.

The above (ii) gives an extension of Morishima-Seton's equality. The inequalities shown here hold with equality in the Leontief economy case.

3. How is the von Neumann equilibrium influenced by changes in the rate of surplus value?

Let the wage goods bundle F be rewritten again as

(11) $$F = cf ,$$

where

f m x 1 : standard wage goods vector,

c : the number of units of the standard wage goods bundle.

The rate of surplus labour can be rewritten as

(2') $$\eta = \frac{Lx^a}{\Lambda^o cfLx^a} - 1 .$$

It is easy to see that M_4-value, Λ^o, does not depend on c, and that if x^a is not influenced by c, $c\Lambda^o fLx^a$ is an increasing function of c. Hence, η is a continuous function of c, and

(12) $$\frac{d\eta}{dc} > o \ .$$

Now, take two different magnitudes η_1 and η_2 correponding to different c_1 and c_1, and write $M_i = A + c_i fL$, g_i^c, x_i^c and π_i^w, where $i = 1$ and 2, corresponding to these two. Then, one has:

PROPOSITION 6. $\eta_1 > \eta_2$ implies $g_1^c \geqq g_2^c$ and $\pi_1^w \geqq \pi_2^w$.

Proof.

From (12), $\eta_1 > \eta_2$ implies $M_1 \leq M_2$. Then, one has

$$Bx_2^c \geqq (1+g_2^c)M_2 x_2^c \geq (1+g_2^c)M_1 x_2^c \ .$$

Now, by the definition of g_1^c, one obtains $g_1^c \geqq g_2^c$. Likewise,

$$\pi_1^w \geqq \pi_2^w \ . \qquad \text{Q.E.D.}$$

This proposition means that the interval (π^w, g^c) tends to shift upward if η is increased. The Marxian interpretation of this is that decreases in real wages are based on harder exploitation in the capitalist economy.

§ 3. Concluding remarks.

1. It has been shown so far that the fundamental Marxian theorem can be extended to the von Neumann economy case, and that the concept of optimum value is significant for the fundamental Marxian inequalities.

What was advocated by Morishima as the most relevant extension, however, was not the optimum value as discussed above, but "true" value. (Morishima-Catephores, p.37.)

DEFINITION 4. (True value) The minimum necessary labour for the production of a composite commodity Y is called its true value:

(14) $$\lambda_Y^o = \min\{ Lz \mid Bz \geqq Az + Y \ , \ z \geqq 0^n . \}$$

Needless to say, the true value of good i is given by $\lambda^o_{e^i}$.

Therefore, it is easily seen that the true value of good i is its M_2-value.

A remarkable property of the true value is that it is not additive: the following inequality holds,

$$\lambda^o_Y \leqq (\lambda^o_{e1}, \ldots, \lambda^o_{em})Y \ .$$

Various critical comments have been made with respect to the nonlinearity of optimum value and true value. It must be noted, however, that the linearity of the valuation of goods is on no account necessary in so far as the fundamental Marxian theorem is concerned.[4)]

It must be observed, moreover, that the true value is no longer related to Marx-von Neumann's theory of value: the propositions in §§1-2 are not dependent on the concept of true value. The true value satisfies the three conditions of value stipulated by Morishima, but it has not yet been shown whether or not the true value is effective as a weight of aggregation.

An important role is played by the concepts of minimum necessary labour and optimum value. The concept of minimum necessary labour alone suffices to show the fundamental Marxian theorem, but the notion of M_4-value is indispensable in establishing the growth constraint and the fundamental inequalities.

Hence, the true value as introduced by Morishima may not be the "true" extension of M_2-value. M_2-value should be generalised in such a way that value is the shadow price of the composite commodity representing wage goods.[5)]

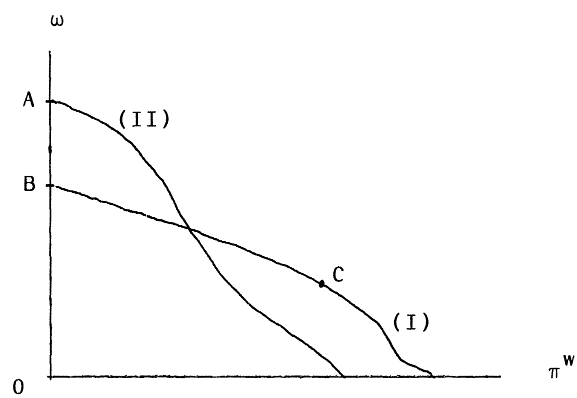
2. The theory of optimum value comprises the theory of value developed in the Leontief or narrow plain economy case.

The discussion made in the following context of "M_2-value ---> the rate of surplus labour ---> the fundamental Marxian theorem and the fundamental inequalities" is completely reconstructed in Marx-von Neumann's theory of value. Moreover, it is seen that the generalised fundamental Marxian theorem now manifests the fundamental duality once discarded in Chapter III: the positivity of the rate of surplus labour is necessary and sufficient for the fundamental duality to hold.

It is also easy to see that if $\pi^w > o$ and $p^w \geq 0_m$, then $\Lambda^o \geq 0_m$. This can be construed as the value basis of price, albeit in a weak sense.

3. It is worth mentioning that the rate of surplus labour is measured in a set of techniques that minimises the expenditure of labour, and that this set is usually different from the set which determines the ongoing profit rate. This can be illustrated by the following figure.

FIGURE IV-1 .



Write $I \subset \{1,..,n\}$, and $I(x) = \{ i \in \{1,...,n\} \mid x_i > o \}$, and $I(x)$ is the index set of the engaging processes for a given level of operation x .

Suppose that a particular x^a is given, which fixes the set of the engaging processes $I(x^a)$, called technique(I), and that the von Neumann equilibrium corresponding to a given F (=cf) is at point C.

The minimisation of labour in (LP.A), however, fixes a set of processes $I(z^o)$, called technique(II). Therefore, the rate of surplus labour is measured at point A on the basis of technique(II), and not at point B corresponding to technique (I).

Nevertheless, it is not necessary to evaluate the rate of surplus labour at point B, because the fundamental Marxian theorem states that the warranted profit rate at point C is positive if and only if the rate of surplus labour at point A is positive. Moreover, since $I(z^o)$ does not depend on x^a, point A represents the state in which the minimisation of labour in the given technically possible processes is attained. This is a remarkable feature of the fundamental Marxian theorem established here, because the set of the technically possible processes does not depend on the market.

This fact is important when the durability of fixed capital with changing efficiency is considered. The economic durability of fixed capital depends on the profit rate if its efficiency is changing, and is supposed to be shorter than its physical durability. The above remark means, however, that in establishing the fundamental Marxian theorem it is not necessary to take into account the dependence of the economic durability of fixed capital on the profit rate.

4. Thus, it can be concluded that Marx-von Neumann's theory of value succeeds the qualitative aspect of the traditional value theory, and as such, is an extension of value theory.

5. Let us observe here again that the von Neumann equilibrium with a positive von Neumann growth rate and a positive warranted profit rate concerns the possibility of growth and profitability of an economy, and that it is not the actual equilibrium. Nevertheless, such an economically meaningful von Neumann equilibrium represents a growing economy with positive profit in the sense that its existence is equivalent to the possibility of growth -- whether or not balanced -- and the profitability of the economy.

It is important to note that the theory of von Neumann equilibrium becomes a truly economically meaningful theory if based on the fundamental Marxian theorem.