

Planning and the Real Origins of Input-Output Analysis

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In an interview with a Polish journal in 1957, Wassily Leontief, Soviet planner of the 1920s and American Nobel Prizewinner in Economics of 1973, answered a question as to the main informative influences on his 'input-output' work with the following remarks:¹

"I have no exact idea. There were many of them. When I was young I studied the theories of the classics of political economy: I count among them Marx, whom I acknowledge as one of the most important economists. The classical works I have in mind, Sismondi and Marx, amongst others — doubtless exercised an influence in the emergence of my input-output analysis."

Despite this admission, there has been a general tendency to denigrate Marx's possible influence and to upgrade Leontief's alleged debts to the work of the neo-classical economist Leon Walras. For example, one writer has remarked: "Notwithstanding its Walrasian parentage, input-output analysis has close kinship to the conceptual system of Keynes' *The General Theory*."² Another, in winding up a discussion about Walras and Leontief's work concluded: "Although the Leontief matrix may be an historical derivative of the Walrasian general equilibrium system, its right to a position as an operational logical derivative may be questioned."³ Yet another, when surveying theoretical models which purport "to explain or predict the effect of any change in final demand upon sector outputs or the allocation of inputs", stated categorically:

"The pioneer of this type of model is Wasily Leontief who made, as early as 1931, an attempt to give quantitative form to the abstract, mathematical theory of general economic equilibrium which had been developed, towards the end of the last century, by Leon Walras and his disciples."⁴

This comment shows a myopic perspective of the origins of general equilibrium analysis and the possible intellectual influences on Leontief.⁵ Leontief himself, however, has done little to clarify the issue, for in 1966 he wrongly claimed that input-output analysis "is an adaption of the neo-classical theory of general equilibrium."⁶ More comments on this error will be made below.

There is no doubt as to the similarities between a simple Leontief model and the 18th century pioneering *Tableau Economique* of Francois Quesnay. By

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delineating the major streams of analysis which flowed from the *Tableau* Leontief's intellectual debts become easier to discern. There have been three main streams involved (a) from the *Tableau* through to Marx's models of reproduction to the Soviet experiments with input-output tables in the economic planning of the USSR in the early 1920s (b) from the *Tableau* to a critique of Physiocratic doctrine by A.N. Isnard, from which Walras gained major but unacknowledged inspiration; and finally (c) a more cluttered, less easily traceable stream involving Ricardo's corn model of growth; the so-called 'transformation problem' of Marx and the attempts to solve it by V.K. Dmitriev and L. von Bortkiewicz at the turn of this century; and P. Sraffa's challenging *Production of Commodities by Means of Commodities* in 1960. The last of these works points to generic relationships between the labour theory of value and the general question of interdependence which add weight to the argument that Leontief owed his key debts to Marx. It also helps to maintain the resurgence of interest in the labour theory of value as an analytical tool shown by M. Morishima and many others after 1970.

By showing the close affinities between aspects of these streams of political economy and input-output analysis, and by examining Leontief's participation in the Soviet economic planning of the 1920s and in the work of a little known group of researchers at Kiel University in the early 1930s, a case is presented below to challenge the alleged primacy of Leontief's debts to Walras.

By introducing his pioneering work on the American economy with the words that it "may best be defined as an attempt to construct . . . a *Tableau Economique* of the United States",⁷ Leontief explicitly recognised the similarities between his own method and that of Quesnay. Other researchers in his field have also made similar, though undetailed, observations.⁸ To show the relationships between the *Tableau* and Leontief's input-output scheme, as well as to clarify certain aspects of Quesnay's analysis, A. Phillips has constructed a simple Leontief model using the same data provided by Quesnay in his 'Analyse' form of the *Tableau*.⁹

It has been doubted whether this comparison between the *Tableau* and Leontief's work is very significant in itself, the grounds being the ease with which the Physiocratic aggregates can be translated into the terms of any other model which deals with the equilibrium of similar aggregates, provided that the necessary alterations are made in the basic theoretical assumptions.¹⁰ However, if one is interested in tracing the linear descendancy of the *Tableau* perspective and particularly its influence on Leontief, then surely the ease with which the original *Tableau* can be transformed into a simple, closed Leontief model is of some significance? Leontief's own acknowledgement to Quesnay must indicate this. Moreover, to doubt the utility of this comparison by quoting the relative ease with which similar comparisons of the *Tableau* can be made with Marx's reproduction schemes and Keynes' system of aggregates is to disguise in effect the importance of the *Tableau* and its influence on economic analysis generally. What the comparison provides is supporting evidence for the theme of this paper — the intellectual debt owed by Leontief to Quesnay and Marx. It is no simple coincidence that the Physiocratic

aggregates can be readily translated into any other model of surplus circulation.

Marx and Leontief

The tracing of similarities between the work of Leontief and Marx is a complex procedure, necessitating a number of different approaches and unfortunately the debate concerning the genealogy of the Leontief model has become embroiled in the exigencies of the 'Cold War'.¹¹ The context of the debate is political from another angle as well, as it involves the de-Stalinisation of Soviet economics and the upgrading within the Soviet Union of mathematical approaches to economic planning.¹²

As early as the 1930s, Leontief paid tribute to the Marxian scheme of the two-sector interrelationship as a fundamental exercise, though he considered it "far from being the 'ultima ratio' of this line of analysis".¹³ The modern work of Leontief contains mathematical symbols, certainly not utilised by Marx, and it is undeniable that Leontief and von Neumann were the first to make extensive use of matrix algebra in examining macro-economic problems. But Marx, even in one of his earliest economic manuscripts showed his awareness of approaches later utilised by these writers.¹⁴ Obviously building on his knowledge of the *Tableau Economique* he constructed the following table which one writer claims "may well be the first (fictive) input-output tabulation in economic science."¹⁵

| | <i>For Labour</i> | <i>Raw Material</i> | <i>Machinery</i> | <i>Surplus Product</i> | |
|-----------------------------|-----------------------|-------------------------|------------------|----------------------------|-------|
| A Raw material manufacturer | 20 | 40 | 20 | 20 | = 100 |
| B Ditto | 20 | 40 | 20 | 20 | = 100 |
| C Machinery manufacturer | 20 | 40 | 20 | 20 | = 100 |
| D Worker's necessities | 20 | 40 | 20 | 20 | = 100 |
| E Surplus producer | 20 | 40 | 20 | 20 | = 100 |

In his discussion of this table Marx takes as his starting point the construction of input coefficients. These coefficients are expressed in the above table in percentage form, and each adds up to 1. Even in this rough manuscript Marx is indicating his interest in quantitative interdependencies as well as his ability to depict these in a matrix form, thus advancing the simpler approach utilised by Quesnay. He uses the above table to discuss analytically complex problems. By differentiating between product flows and value flows, and by discussing the subsequent impact on actual output proportions if the surplus is spent on increased production instead of luxury items, he is carrying out analysis far from unrelated to the much later work of Leontief. The complexity of the algebra used by both writers is the main difference between their discussions of similar analytical problems. The ease which Marx's model of expanded reproduction can be transposed into a four sector input-output table adds to the argument for Leontief's debts to Marx and puts his acknowledgement of Marx's influence on him in a clearer light. Thus:¹⁶

| | <i>Department I</i> | <i>Department II</i> | <i>Labourers</i> | <i>Capitalists</i> |
|---------------|---------------------|----------------------|------------------|--------------------|
| Department I | c_1 | c_2 | — | — |
| Department II | — | — | $v_1 + v_2$ | $s_1 + s_2$ |
| Labourers | v_1 | v_2 | — | — |
| Capitalists | — | — | — | — |

Where s = capitalists' consumption

v = variable capital

c = constant capital;

and the subscripts 1 and 2 denote the two respective departments.

According to the labour theory of value the capitalists do not add to the value of the product or contribute to the actual process of production, thus in the above table their respective row is left empty.

To show the flows for the various annual increases in stocks, the following table can be utilised:

| | <i>Department I</i> | <i>Department II</i> | <i>Labourers</i> | <i>Capitalists</i> |
|---------------|---------------------|----------------------|------------------|--------------------|
| Department I | Δc_1 | Δc_2 | — | — |
| Department II | — | — | — | — |
| Labourers | Δv_1 | Δv_2 | — | — |
| Capitalists | — | — | — | — |

By taking Marx's original numerical values for expanded reproduction the following table can be constructed.¹⁷

| Tableau Economique of Marx | | | | | | | | |
|----------------------------|----------|-------|-------|-------|----------|-----|---|---|
| | <i>A</i> | | | | <i>B</i> | | | |
| 1st year | 4,000 | 1,500 | — | — | 400 | 100 | — | — |
| | — | — | 1,750 | 1,100 | — | — | — | — |
| | 1,000 | 750 | — | — | 100 | 50 | — | — |
| 2nd year | — | — | — | — | — | — | — | — |
| | 4,400 | 1,600 | — | — | 440 | 160 | — | — |
| | — | — | 1,900 | 1,110 | — | — | — | — |
| 3rd year | 1,100 | 800 | — | — | 110 | 80 | — | — |
| | — | — | — | — | — | — | — | — |
| | 4,840 | 1,760 | — | — | 484 | 176 | — | — |
| 4th year | — | — | 2,090 | 1,221 | — | — | — | — |
| | 1,210 | 880 | — | — | 121 | 88 | — | — |
| | — | — | — | — | — | — | — | — |
| 5th year | 5,324 | 1,936 | — | — | 532 | 193 | — | — |
| | — | — | 2,299 | 1,344 | — | — | — | — |
| | 1,331 | 968 | — | — | 133 | 97 | — | — |
| 6th year | — | — | — | — | — | — | — | — |
| | 5,856 | 2,129 | — | — | 586 | 213 | — | — |
| | — | — | 2,529 | 1,477 | — | — | — | — |
| 7th year | 1,464 | 1,065 | — | — | 146 | 107 | — | — |
| | — | — | — | — | — | — | — | — |
| | — | — | — | — | — | — | — | — |

Of particular interest in the above is the fact that every magnitude increases at a uniform rate of 10 per cent per year, and the same increase takes place for all the sectors, surplus, investment and so on. The reason why matrix B flows are exactly one tenth of the corresponding flow elements of matrix A, is because of the assumption of a uniform turnover time of one year. In the first year can also be seen one of Marx's rare slips.

Oscar Lange has provided the best comparative attempt to show the similarities between Marx's schema and that of Leontief.¹⁸ Lange, using the Marxist concepts of aggregate social product, material input (c), the wage fund of workers (v) and surplus product fund (s), has constructed a mathematical interpretation of Marx's reproduction models which he claims enables them to be transformed by expansion, into an input-output model with a large number of interconnections between different branches of the economy. Unlike Marx's models, however, Leontief's tables are designed to study the relationship between a large number of sectors of an economy instead of only between two broad ones — the producers' and consumers' goods sections. Nevertheless, this is a quantitative, rather than a qualitative difference as can be seen in Lange's comparative exercise.

Lange draws the following conclusion from his exposition of the similarities between the two theoretical approaches; namely that "input-output analysis . . . is a method of applying Marx's production formulas in concrete terms".¹⁹ Whereas Marx promulgated the general idea that a balanced exchange of products between departments I and II of the economy was a prerequisite for the smooth continuation of the reproduction process, Leontief's input-output analysis extends this idea to a multi-sectoral approach and as Lange indicated this is merely a quantitative rather than qualitative difference between the respective tableaux of Marx and Leontief. As Lange shows the basic foundation for this claim lies in the presentation by Leontief of the means of production used up in the production process (Marx's gross C) as the sum of the means of production produced by individual sectors of the economy, that is

$$\sum_i \sum_j x_{ij}.$$

To Lange, Leontief's extension of Marx's general balance concept enables the construction of more detailed economic balances, yet the general concept remains initially that of Marx.

Whilst Lange's arguments appear impressive, this is not to say they cannot be criticised; they can be both clarified and supplemented. It must be asked whether Lange is ignoring the value foundations of the Marxian reproduction models and by doing so whether he is in fact negating the claim of 'concrete expression'? A. Zauberman has claimed that Lange, along with the doyen of Soviet post-war mathematical economists V.S. Nemchinov, has failed to graft

"the Marxian dichotomy into the Leontief system".²⁰ Before assessing and evaluating this claim, it is relevant and important to take a brief look at the Lange-Nemchinov debate on this matter.

Nemchinov has described Lange's analysis as "impermissible" on two major grounds; that the reconciliation of the two schemas becomes much more difficult once the hyper-aggregation of the productive system into two sectors is abandoned, that in effect by doing so the basic dichotomy of the system into two sectors becomes lost; and secondly, that Lange's analysis ignores the centrality of Marx's division of the social product into c , v , and s .²¹

Nemchinov argues that Lange's approach by failing to maintain the division of social product into two basic parts when proceeding to a multi-sectoral, multi-variable model, makes it impossible to compare material input in the sphere of consumers' goods production (c_2) with newly created value in the sphere of capital goods production ($v_1 + s_1$). He emphasises this deficiency because of Lenin's strictures on the ratio between c_2 and ($v_1 + s_1$) or the expanded reproduction potential. But this ratio is not central to the kind of comparison Lange is making; it is an attack on Lange for ignoring ideological aspects of the schemes.²²

Nemchinov abandoned an attempt to calculate accurately the value of the surplus product in his contribution to showing the affinities between Leontief and Marx. Zauberman uses this failure as the basis for his claim that: "No less than Lange, did he (Nemchinov) fail in grafting the Marxian dichotomy onto the Leontief. To Zauberman 'the obstacle which he has not and could not overcome is not merely technical — one of difficulty in matrix design — but of substance; it is the nature of the Leontief interdependence of industries that precludes the disentangling of Marxian departments. Indeed it does so because industries of real life evade it'".²³ As supporting evidence Zauberman also quotes from a Soviet comment on early attempts to construct a balance sheet of the economy to the effect that "It is impossible to group production according to Marx's schema in its pure shape".²⁴ But it is not the intention of this paper to "graft the Marxian dichotomy onto Leontief". This is to view the whole issue of the antecedents of Leontief's analysis in too myopic a light. Surely it is unfair to describe Lange's analysis as a simple attempt to 'graft' Marx's schema to that of Leontief? As Lange shows it is not just the dichotomy that is important; in fact he shows convincingly that a balance condition for any specialised pair taken at random from among the Leontief table industries holds for Marx's departments, and *vice-versa*.²⁵ It must be emphasised that Lange's claim is that the general idea behind Marx's reproduction models finds 'concrete expression' in Leontief's model; to him, as noted earlier, Leontief's extension of Marx's general balance concept enables the construction of more detailed economic balances, yet the general concept remains initially that of Marx.

The general affinities between the work of Leontief and Marx can be further illustrated by noting key and interesting similarities between a set of equations enunciated by Leontief, and Marx's model of simple reproduction. In Appendix I to his *The Structure of the American Economy, 1919-1939*,

Leontief provides the following set of equations:

$$(1) \quad -X_1P_1 + x_{12}P_2 = 0$$

$$(2) \quad x_{21}P_1 - X_2P_2 - X_{23}P_3 = 0$$

$$(3) \quad x_{31}P_1 - X_3P_3 = 0$$

Where: X 's = the industries' physical-term outputs,

x 's = inter-industry flow,

P 's = prices, and

subscripts 1, 2, 3, = household, consumer goods and producer goods industries respectively.

Now, as equation (3) above implies that the total output of the producer goods industry is channelled into the consumer goods industry, then X_3P_3 is the equivalent of Marx's, C_2 .

Further, as $x_{31}P_1$ is equal to the total value added in the production of producers goods; and as Leontief's system assumes that the competitive equilibrium profit equals zero, then $x_{31}P_1$ is the equivalent of Marx's $v_1 + s_1$. Thus equation (3) can be rewritten in Marx's terms as $v_1 + s_1 - c_2 = 0$, the balance condition in simple reproduction.

Zauberman points to these affinities but merely uses them to warn of the operational limitations for economic planning of Marx's original two-sector model and this equivalent approach of Leontief.²⁶ For our purposes though, it is another striking example of the similarities between the two schemes; another example of the relative ease with which the two schemes are transferable into one another. It can only add weight to the case for Marx as the most important influence on Leontief.

Value, Price and Input-Output Analysis

The general affinities between Marx's reproduction models and Leontief's tables have been delineated, but it also is important to draw out some of the differences between the ways in which the general approach has been developed and utilised. Of special significance are the quite different theories of value and capital implicit in the two schemas of Marx and Leontief respectively. These require closer examination. It is also necessary to look in some detail at the very nature of the general equilibrium, implicit in both schemas, and stress the different uses of these equilibria by the two schools.

In keeping with the Walrasian concept of a static general equilibrium, input-output analysis is based on an approach to interdependence where, to quote Leontief, "the effect of an event at any point is transmitted to the rest of the economy via the chain of transactions that links the whole system together. A table of ratios for the entire economy gives us, in as much detail as we require, a quantitatively determined picture of the internal structure of the system".²⁷ Leontief aims to reveal the "complex series of transactions in which actual goods and services are exchanged among real people and reduce these to

some kind of order by classifying and aggregating them into groups''.²⁸ With the aid of his matrices he provides a crude disaggregated picture of the general structure of an economy, which rests on the technically determined interdependencies of that economy at a particular state of time.

Input-output analysis takes the final demand as given; it does not pretend to provide guidance as to how this final demand may change over time. Changes over time are discernible however, by comparing the respective input-output tables for the commencement and conclusion of the time period under study. Such an exercise is essentially one of comparative static analysis, and is comparable to the various tableaux Quesnay constructed to illustrate the effects of various policies on the original tableau equilibrium. It could be argued however that just as comparative static analysis does cast some light on processes of change, so would successive input-output tables. As each table indicates the final demand for goods and services and the actual inter-industry relations that lie behind this demand, it is possible, by assuming no change in the technical relations of production, to estimate the impact of a change of aggregate final demand or a change in its composition on each particular sector and in doing so, show the overall implications for all the inter-relations of the particular table. Leontief showed recognition of the limitations of his static input-output tables and in fact implied that the difficulties of obtaining adequate data are a major reason for the non-dynamic nature of his analysis. Moreover a dynamic input-output model "requires accounting for stocks as well as flows of goods, for inventories of goods in process and in finished form, for capital equipment, building and, last but not least, for dwellings and household stocks of durable consumer goods. Dynamic input-output analysis requires more advanced mathematical methods; instead of ordinary linear equations it leads to systems of linear differential equations".²⁹

Dynamic input-output models require more than just these conditions outlined by Leontief. Once Leontief raises the question of stocks of capital, as distinct from just flows, he has entered the 'under-world' of capital theory and the problems of capital measurement. Moreover, the type of equilibrium implicit in input-output analysis must be considered. Disaggregation may produce a more realistic picture of the production process, which shows the various sets of relationships between technologically specific inputs and outputs. Yet in doing this 'capital' is depicted as various specific goods in the various industries. Additionally, this creates the problem of capital valuation, and extreme difficulties with the determination of prices. If it is assumed that prices are set so as to return at least a normal level of profit on capital advanced, and if 'capital' consists of a variety of separate items which can only be expressed in *value* terms, then a massive obstacle occurs. Only if we know the prices of the particular 'capital' items can we then express the inputs in value terms. But it is assumed that prices are determined in terms of a return on the value of 'capital' advanced! These kinds of problems were in fact quite familiar to K. Wicksell, who pointed out that Walrasian general equilibrium faces such difficulties in accommodating the concept of capital.³⁰ Moreover, if 'capital' is composed of a variety of goods then the concept of a marginal

physical product of capital loses all analytical utility, and the insights the model can provide regarding distribution and relative shares accruing to the owners of productive resources, are highly limited. In sum, the very value theory implicit in a theory of growth produces a multiplicity of problems, problems which any attempt to produce dynamic input-output analysis must face.

The original Marxian reproduction models whilst practising disaggregation were less susceptible to these types of problems. This is largely the result of the theory of capital and distribution implicit in them, which enabled Marx to avoid the problems faced by later neo-classical practitioners. Leontief obtained inspiration and guidance from the linear, disaggregated models of Marx, but in ignoring the Marxian approach to capital theory, he was reducing the value of his own analysis as a possible contribution to the theory of economic growth. In Marxian terms this could be even described as a process of mistaking the 'form' for the 'substance', of utilising the analytical skeleton without its highly useful contents. This interesting distinction between the work of Marx and Leontief has not been pointed out by the various commentators on the similarities between their work.

To construct satisfactory dynamic input-output analysis would involve more specific information on the determinants of capital stocks and of further additions to them. On these matters Leontief's input-output analysis provides no real guidance. Devoid of a theory of profit or of capital accumulation, his analysis would either have to return to Marxian (or Ricardian) guidelines on these matters, or further add to the limitations of input-output analysis by utilising neo-classical guidelines. It is little wonder that there has been a resurgence of interest in the labour theory of value on behalf of economists interested in constructing dynamic disaggregated models of production.³¹ Such economists are, in effect, re-forging links between linear models of production and the original value foundations on which the earliest types of these models were constructed. The value aspect of such models is no longer seen as having doubtful analytical import.

In view of the emphasis placed on tracing the affinities between Leontief and Marx it is interesting to clearly distinguish the similarities and differences between Leontief's system and that of Marx, and now Piero Sraffa.³² Certainly a Leontief system depicts production in the same way as Marx and Sraffa do, and it shows a similar approach to the nature of technological interdependence and the importance of intermediate goods. Yet as suggested above, a Leontief system does not raise questions about, for example, a uniform rate of profit on capital or the effects of changes in distribution upon prices. In contrast Marxian analysis and the 'Ricardian' type analysis of Sraffa is primarily concerned with the relations between wages, profits, and prices. It was Marx's combination of the reproduction models with this latter concern that produced an analysis capable of handling many problems that cannot be adequately dealt with in a Leontief system.

Notwithstanding what has been argued above, possible similarities between the price theory of input-output models and the labour theory of value are now

being increasingly recognised and utilised by a growing number of researchers.³³ For example a basic compatibility between the labour theory of value and marginal utility theory was propounded by Leif Johansen.³⁴ He argued that such a combination provides a highly precise meaning to the Marxian thesis that the value of a product is determined by its labour content, provided it possesses use value. His model still left prices determined by the labour theory of value, even though he accepted the marginal utility theory of consumers' behaviour. Such a model is utter anathema to an orthodox Marxist. Moreover it conflicts with those neo-Keynesians who argue that the labour theory of value is redundant and should be jettisoned from even radical economists' tool boxes.

Precise links between classical value theory with its base the labour theory of value, and general theorising about economic interdependence, are indeed complex to discern and analyse. Even Leontief has acknowledged that despite many differences the general theory of interdependence is 'generically related' to the classical theory of value.³⁵ To elucidate this claim and its implications is a far from easy task. The simplest way of associating the labour theory of value with the general problem of interdependence is to note that the concept of a "socially necessary amount of labour embodied in a commodity" becomes meaningless unless it is discussed in the context of the various stages of production that went into its production. The precise way this context is delineated, whether it be in the form of a set of simultaneous input-output relations or not, is not the real issue. What does matter for our purposes is that the very delineation of such stages of production requires consideration of economic interdependencies.³⁶ It has already been argued that Marx's reproduction models constitute a crude form of input-output analysis, but the analogy goes much deeper, and permeates the very crux of the Ricardian and Marxian discussions of value. This can be seen in the so-called transformation problem and its relationship not only to questions of economic interdependence generally, but to the work of Sraffa and to the expanding current dissatisfaction with neo-classical economics.³⁷

Central to the classical theory of distribution and growth is the concept of a surplus product and the question as to why some inputs (or outputs) produce more of themselves in output form, than is used up in the production process. The fact that Marx used this question as his starting point for an analysis of growth and distribution provides a direct, generic link with later work on economic interdependence. Here the contribution of a little known Russian economist is especially important in providing a further crucial link between Marx and Leontief.

In the late 1950s Soviet economists and planners led by V.S. Nemchinov, began to show increased interest in Leontief type methodologies. This was very much a product of the need to overcome inadequacies in the material balances approach to planning and was facilitated by a thawing in the Stalin initiated suspicion of mathematical economics. For obvious political reasons it was helpful if Soviet or Marxian precursors could be found for techniques that the planners wished to experiment with. A related aspect of these claims brought

to light an obscure Russian economist, V.K. Dmitriev, whose main work was carried out at the turn of this century. It was claimed by V.S. Nemchinov that in fact the first system of interdependence in economics, which rests on equations of labour input, was formulated by Dmitriev. Because such equations are absent from the Walrasian system this can be used to support the claims already made regarding Leontief's debts to Soviet and Russian economics.³⁸ However the main relevance of Dmitriev's work, not only includes his precursorial role regarding input-output analysis, but also his contribution to the theory of the transformation problem and hence, in the long-run to the work of Sraffa. Because of Dmitriev's relative obscurity, his contribution will now be examined in some depth.

One of the fascinations of the study of the filiation of ideas is the discovery of the little-known precursor of much later work. It is not unusual for a writer to remain in relative oblivion for decades before his 'rediscovery'. V.K. Dmitriev suffered such oblivion, but the 'rediscovery' of his work has begun a process of wider recognition for his contributions. Dmitriev set out to provide a "synthesis between the labour theory of value and the marginal utility theory" in the words of the sub-title to his *Essais Economiques*. In doing so, he was continuing a dominant stream of thought amongst Russian economists of his era which began with an initial wide acceptance of Marxian economics, which was later tempered by efforts to incorporate aspects of the 'Marginal Revolution' into a kind of Ricardian-Marxist-Austrian synthesis. Dmitriev's major interest was in the Ricardian theory of value and in fact he defends Ricardo against Marx's 'developed' form of a labour theory of value, although he does not use Marx's name in this context.³⁹

By looking in some detail at the work of A. Cournot and L. Walras, Dmitriev attempts to draw some general conclusions and synthesize 'objective' and subjective approaches to the theory of value. But he views value not in terms of abstract, metaphysical value, but largely in terms of the prices of production. This approach naturally brings him into the famous transformation problem and its possible resolution. Starting from Ricardo's labour theory of value, he argues that demand and supply elements do in fact influence production prices, that the technical conditions of production are not the sole determinants of production. The influence of the 'Marginal Revolution' is clearly illustrated here. Nevertheless, he still holds to a theory based on the amount of 'socially-necessary labour' because in the final analysis the conditions of supply and demand determine what quantity of labour is in fact 'socially necessary'. As Zauberman has pointed out⁴⁰ this approach is comparable with a much later use of such an approach to the labour theory of value by J.R. Hicks, who reduced the theory to the proposition that where labour is assumed to be the only scarce factor, prices of production would therefore be proportional to labour costs.⁴¹

Particularly significant about Dmitriev's contribution is the rigorous manner in which he develops these arguments. He presents a system of equations to illustrate his theory of price formation; a system in which the unknowns are of both a 'subjectivist' and 'objectivist' nature, and both the

actual technical conditions of production and the possible determinants of consumption levels are incorporated.⁴² In doing so, Zauberman claims Dmitriev is rejecting Marx's theory of surplus value, and by implication the Marxian theory of profit.⁴³ Zauberman's opinion of the similarities between Dmitriev's and Marx's approach in this area appears to contrast strongly with that of L. von Bortkiewicz, who was himself a trenchant, but constructive critic of Marx. To the latter, Dmitriev "has presented a theoretical model which keeps entirely to the Marxian way of posing the problem. Just like Marx, Dmitriev's model shows as ultimate and exclusive determinants of price the technical conditions of production of commodities, including the technical conditions of production of the commodity labour, the latter finding their expression in a given real wage."⁴⁴

To assess these two claims and to draw out the fundamental contribution Dmitriev made in this area, it is necessary to examine a key passage in his *Essais Economiques*. Quoting Adam Smith, Ricardo and Marx, he draws attention to their examination of the question of measuring the amount of capital used to produce a particular good.⁴⁵ "There's no doubt that capital is actually always produced by capital itself" but it is "impossible to calculate the amount of labour used in a given product from the moment of the creation of the initial capital, by labour alone".⁴⁶ Dmitriev rejects attempts to undertake such an exercise and offers the following equation as a substitute for "some seemingly historical incursions". To follow his argument closely it is useful to quote him at length.⁴⁷

"Let us denote by X the total amount of labour, directly or indirectly expended in order to produce a unit of product A ; for instance let n equal the amount of labour directly expended in its production; let us suppose moreover that different types of 'capital techniques', K_1, K_2, \dots, K_M are utilised in the production process and that the latter constitutes some fractions $\frac{1}{m_1}$ of the capital K_1 , $\frac{1}{m_2}$ of the capital K_2 , $\frac{1}{m}$ of the capital K_M ; suppose finally that X_1, X_2, \dots, X_M signify the amount of labour entering directly or indirectly into the production of the capital K_1, K_2, \dots, K_M respectively; then the sum total of labour entering into the production of a unit of product A will be

$$X = n_A + \frac{1}{m_1} X_1 + \frac{1}{m_2} X_2 + \dots + \frac{1}{m_M} X_M$$

The amounts m_1, m_2, \dots, m_M are representing here, just like some identities that are given by the technical conditions of production the amounts X_1, X_2, \dots, X_M being some unknowns."⁴⁸

The equation is most interesting for our purposes. It is the basis for the claims made about Dmitriev as a precursor of Leontief, but its significance is far more extensive. By developing analogous equations he shows how the amount of labour used up in producing the necessary capital to produce further units of capital can be determined.⁴⁹ Thus he is emphatic that no historical exercises are required; that the sum total of labour expended to produce the capital to produce the capital, etc., can be determined in this simple manner. In offering this solution he provides a mathematical solution to a problem that had plagued most previous expositions of the labour theory of value. He shows that the labour theory of value as an analytical tool need not be rejected on the grounds of the alleged impossibility of tracing labour

inputs back to infinity. Although he was not setting out to explicitly do so, he made an important contribution to the transformation problem and the further development of Ricardian and Marxian economics, of relevance to a wide section of economic problems, and later of course to economic planners. Looking further into his model helps to substantiate such claims.

Discussing Ricardo's theory of value, Dmitriev points out that in Ricardo's discussion of the determinants of the rate of profit, there is the idea that the conditions of production of wage goods, to which in the final analysis expenditures in the production of all goods can be reduced constitute a way of 'directly' determining the rate of profit.⁵⁰ Dmitriev develops this idea into a formal model. Taking a simple two-product case, where one product is input in the production both of itself and of the other,⁵¹ he constructs the following equation:

$$Y_{AB} = \frac{N_A x_a (1 + r)^t A}{N_B x_a (1 + r)^t B}$$

| | | |
|-------------|---|--|
| where Y | = | the price ratio of A to B |
| a units of | | |
| wage good A | = | the real wage per unit of labour time |
| x_a | = | price per unit of A |
| N_a | = | the number of units of labour (e.g. days) required to produce a unit of product A. |
| r | = | the rate of profit |
| t | = | the time over which labour is advanced (or the production period). |

Then if $t_A = t_B$, thus

$$Y_{AB} = \frac{N_A}{N_B}$$

His next step illustrates how r can be directly derived from N and t in the wage good industry once A is known. Most importantly he shows it is not necessary for the wage good A to be determined before r is determined. As a result the price ratio is determinable on the conditions of production of A . He then relaxes key assumptions to show, for example, that several wage goods can be used instead of just one. But the most important result for our purposes, is the fact that he shows that to express cost in terms of labour does not lead into an infinite regress when labour always combines with other inputs; this is easily expressible in terms of simultaneous production via a set of input-output equations.

To summarise the above: if there is any input (or groups of inputs) a which when fed into a productive process turns out more of itself (or themselves) as well as turning out other products, the rate of surplus product or profit will be perfectly determinate and independent of the price of a .⁵² This holds

regardless of whether the input is human labour power or in other forms. The fact that Dmitriev shows the ultimate determinants of price to be the technical conditions of production of labour itself, clearly links him with Marx's discussion of the value-prices transformation.

Dmitriev's work also provides a further link back to Marx. Given the similarities between Dmitriev and Marx, Dmitriev's foreshadowing of a key concept of Leontief's simple input-output model, adds further weight to the claims made about Leontief's intellectual debts to Marx. To develop this theme briefly it is useful once again to examine Dmitriev's key equation:

$$X = n_A + \underbrace{\frac{1}{m_1} X_1}_{\text{}} + \underbrace{\frac{1}{m_2} X_2}_{\text{}} + \dots + \frac{1}{m_M} X_M$$

This equation is the formal basis of the claims made about Dmitriev as a precursor of Leontief. The intentions of these two contributors to the theory of economic interdependence certainly differed. As already noted Dmitriev's key equation was constructed as a tool to discuss the labour theory of value; whilst Leontief set out to construct a theoretical foundation for the development of tools of analysis of inter-industry flows. In the simple static Leontief model inputs of primary factors are reduced to inputs of labour. Labour is seen as the dominant social cost. Leontief later extended this initial starting point of his analysis and it bears remarkable resemblance to Dmitriev's equation. Just because Dmitriev did not develop an operational dynamic model, this does not conflict with this basic similarity of identities. Could an explanation of this remarkable similarity be yet another link between Leontief's work and that of his Marxian Russian confreres? Although the similarities between Leontief and Dmitriev provide strong circumstantial evidence to support such a claim, a more definitive judgement is impossible regarding this matter.

Leontief and the Soviet Planning Experiments with Input-Output Analysis in the 1920s

With the expansion of 'planometrics' in the Soviet Union since the 1950s there has developed a greater interest in the origins of input-output analysis. The most important claim made by Soviet economists is that Leontief gained major inspiration from the early attempts by Soviet planners to construct national balances of the Soviet economy as crude guides to future planning.⁵³ This claim has been supported by Naum Jasny a veteran commentator on Soviet economics, who has hardly been a sympathetic supporter of Soviet economic claims.⁵⁴ A strong attempt at refutation of this claim has come from H. Levine who concludes that: "Neither the Soviets nor Dr Jasny have, in any way, proved that Leontief's ideas were developed on the basis of early Russian experiments with national income balances. Indeed it would be difficult to prove such a claim because to the extent that anything is ever clear in these matters such a claim is clearly untrue".⁵⁵ To this writer, Walras must be considered, if any one writer is to be given major credit, as the mentor of

input-output analysis.⁵⁶ Whilst Levine acknowledges the difficulties in this type of exercise his method of refutation of the alternative claim is far from satisfactory.

The core of Levine's dismissal of the influence of the 1923-24 Soviet Balance of the Economy, is that it was "less than an Input-Output study in that those parts of it which did contain Input-Output information were at best poor, very aggregative forms of just one of the three basic tables which make up an Input-Output study; namely, the first table, the flow table."⁵⁷ Leontief's most important contribution, Levine claims, was the mathematisation of the various flows and their interrelationships, something not done in these early planning attempts. Leontief may have gained more general inspiration from Quesnay, rather than from this first National Balance, but: "All in all, if one were forced to choose the influence which could in some sense be said to have been dominant, the choice from the evidence available, would almost have to be Walras and his mathematical economic analysis".⁵⁸ To cast serious aspersions on this approach involves a brief foray into the development of Soviet planning and the theoretical and ideological influences which operated on the early Soviet planners. It will be shown that the theoretical foundations of early Soviet planning are more complex than is usually claimed, and that in fact there are links back to economists who are removed from Walrasian influences, but very much influenced by links with the *Tableau Economique* and Marx. The debates about the similarities between Input-Output analysis and the first National Balance of 1923-24, and the broader question of the general origins of input-output analysis, then take on a less definitive air, the claims of Levine and others notwithstanding. The starting point will be to examine possible influences on the Soviet 'method of balances'.⁵⁹

All the contributors to the debate over the genesis of input-output analysis appear to be unaware of an important Marxian influence, albeit an indirect one. They ignore the influence that the work of Karl Ballod had on Soviet planning techniques during the first few years of the Soviet Revolution. It is usually claimed by historians that early Soviet planning got its inspiration from the way in which the Imperial German War Machine was organised during World War 1. This argument is used to stress the pragmatism of Lenin and the failure of Marxian economics to provide assistance in the planning sphere. However, Ballod's major work *The State of the Future* and its enthusiastic and wide reception in the Soviet Union provides strong evidence against such claims.⁶⁰ The first edition of this book, published in 1898, contained estimates of input-output ratios for the most efficient industries in each sector of a theoretical economy. Most interestingly, and importantly, it appears that Ballod's work had its theoretical genesis in the reproduction schemas of Marx. Lenin's keen interest in Ballod's work, as evidenced by his detailed study of the book, and his subsequent invitation to Ballod to visit the Soviet Union in 1920, together with the publication of a special Russian edition which was widely circulated in the Soviet Union, adds support to the argument that Ballod's role cannot be ignored.⁶¹ Ballod's efforts to provide a concrete plan of how a centrally planned economy could be run, not only

helped to fill a gap in Marxist literature on the post-revolutionary situation, but also provided yet another link between Marx and Leontief. Although it cannot be substantiated in a direct sense, it would seem foolish to deny that early Soviet planners, including those who constructed the National Balances, at least had some familiarity with Ballod's book and the insights it offered. This becomes even more likely when the dearth of guidance available to these planners is recalled. Moreover, Ballod's adherence to Marxist beliefs made his work even more capable of attracting Bolshevik attention.⁶²

Other Marxian influences on early Soviet planning made themselves felt through the activities of key Menshevik economists, who played important roles in drawing up the early balance sheets and in the Soviet industrialisation debates generally.⁶³ Prominent amongst this group were P.I. Popov, G. Groman and V.A. Barazov, all serious students of Quesnay and Marx.⁶⁴ Contrary to the oft-repeated claims about these early planners as being simple pragmatists, Marxian influences were in fact omnipresent⁶⁵ With Stalin's drastic suppression of economists who were unwilling to be simple sycophants and mere implementors of his strategies, the Soviet work in inter-industry economics was terminated, making it easier for economists in Western countries to deny, or remain ignorant of, these pioneering efforts. The poverty of Soviet economics, under Stalin especially, makes claims about these precursors of Leontief less believable.

It is thus obvious that the question of Leontief's intellectual debts does not stop with arguments about the possible similarities between Soviet planning techniques and Leontief's precise technical contributions. In fact Levine has tended to obscure the broader influences on Leontief, which were given short-run, concrete form during his studies at Leningrad University in 1924. At this stage, given the general interest in Marxian economics in the Soviet Union at this time, it is highly unlikely that Leontief was not exposed to the reproduction models and the *Tableau Economique*. He himself likened the 1923-24 Balance to the *Tableau Economique*,⁶⁶ and given the ways in which Tableau concepts had filtered through Marx's reproduction models down to Soviet economists at this time, it is rather difficult to dismiss the possible influences of the Soviet experiments with inter-industry analysis. These must have had at least a general inspirational effect on Leontief. Even at this early stage Leontief was well aware that this basic approach could be developed. In fact he chided the efforts of those who constructed the 1923-24 Balance Sheet as rather simplistic, and advocated the elaboration of an approach for which he expressed considerable enthusiasm.⁶⁷ Moreover, this reaction indicates a close contact with these early Soviet efforts and a strong interest in the techniques involved. Certainly this first balance sheet was uncomplicated, compared with later efforts, but this was a necessary reflection on the state of the Soviet economy at this time and the low level of development of planning techniques of any kind. But just because the techniques were relatively undeveloped doesn't mean that these early attempts are unrelated to more sophisticated later developments. Leontief's development of this type of approach is not under dispute in these debates; the key question concerns the

basic inspiration for his work.

In the face of available evidence and despite its speculative nature, this writer finds it extremely difficult to seriously accept a claim made in 1960 by Leontief that his work "had no relation" to this earlier Soviet work,⁶⁸ or his claim that input-output analysis "is an adaption of the neo-classical theory of general equilibrium".⁶⁹

Despite the keenness of American commentators to dismiss claims that input-output analysis had a Marxian or Soviet parentage, in 1953 the US government had no doubts on the issue. Official sponsorship of a government input-output research programme was discontinued because of allegations that such analysis was 'socialist' and therefore a threat to private enterprise and the American way of life.⁷⁰

Leontief and the Kiel School

Even further evidence for a direct link between the reproduction models and Leontief's work comes from Leontief's links with a group of researchers at Kiel University during the 1930s.⁷¹ Although this group included Jacob Marschak, Hans Neisser and Wassily Leontief, it was F.A. Burchardt's work which is of most relevance to this study. Burchardt's main concern was with trying to devise a model of the stationary flow that could be made amenable to dynamic transformation. Neither Lausanne nor Cambridge with their emphasis on price variables to the exclusion of the physical and technical aspects of the industrial market, offered a fruitful starting point. An early dissertation on Schumpeter's model of market equilibrium led him to call into question Schumpeter's assertion that dynamics, understood as a theory of development or growth was necessarily less 'exact' than, and unrelated to, the theory of stationary equilibrium.⁷² He then turned his attention to the earlier attempts at constructing a *Tableau Economique*, of Quesnay and Marx, and the criticisms Böhm-Bawerk had made of these.

Burchardt revived interest in Marx's reproduction models by pointing to the deficiencies in Böhm-Bawerk's theory of capital which could be eliminated by adopting the approach of Marx to this problem. His attack on the 'linear' model of the structure of production challenged the orthodox 'Austrian' school whose conception of capital involved the aggregate of intermediate products and rested heavily on the concept of 'period of production'.⁷³ To Böhm-Bawerk the productive process consisted of 'intermediate products' steadily moving down the strictly one way road of the process towards their final goal of consumption.⁷⁴ Burchardt disputed such a picture and demonstrated that the reproduction and expansion of the stock of fixed capital goods in a state of full resource-utilisation cannot be explained by simply tracing the technical process of production back to some original combination of labour and natural resources, as the Böhm-Bawerk model claimed.⁷⁵ Such fixed capital, though itself an output, he argued, can only be maintained and increased with the help of a circular process in which fixed capital goods also act as inputs. Nevertheless, the 'Austrian model' can be used to describe the

structure of working capital; if on the highest stage a stock of fixed capital goods is added to the original inputs of labour and natural resources, the downward flow to the final stage of finished output properly describes the structure of working capital. Burchardt indicated, however, that to account for the presence and change of fixed capital the linear model needs to be supplemented by the Marxian schema of reproduction, which depicts the self-reproduction of characteristic equipment goods. The relevance of this conciliation of the two approaches to the debate concerning the origins of input-output analysis is this: a colleague of Burchardt, working in close association with him, developed an analysis of inter-industrial relationships, which by using Burchardt's work on fixed capital foreshadowed by many years Leontief's description of the 'whirlpools' of intersectoral flows.⁷⁶ Though couched in arithmetical terms only, this work, in Lowe's opinion contains "the first attempt at input-output analysis, applied not only to stationary equilibrium but also to the intersectoral shifts required for capital formation".⁷⁷

This evidence throws further light on to the formative influences on Leontief's development of input-output analysis. As mentioned Leontief was a member at this crucial time of the Kiel group.

That Leontief did not remain uninfluenced by the work of his colleagues can be seen in article he wrote in the 1930s on Marxian economics in which he discussed both Marx and Böhm-Bawerk in relation to their contribution to linear analysis.⁷⁸ One could also assume that his under-graduate training in Marxian economics in the country of his birth would make him more than competent to understand and extend the line of enquiry initiated by Burchardt.⁷⁹

All in all, the question of Marx's influence on Leontief, if complicated is clear enough, especially taking into account that the development of the input-output analysis does go direct from Marx to Soviet economics to Leontief, rather than (as economists so often assume) to Leontief's neo-classical predecessor, Leon Walras.

With developments in Marxian economics (some stimulated by the "Cambridge controversies" in capital theory), and the growing disquiet with neo-classical orthodoxy, the subject of this article assumes a wider significance. If Leontief's debts to the Quesnay-Marx tradition are, as I have argued, much greater than has been commonly accepted, then the analytical utility and standing of this tradition is concomitantly reinforced. This must be borne in mind by all those who wish to understand the link between Marxian growth theory and problems of economic planning. Without it, the planning debate will not only lack historical perspective but firm theoretical underpinning.

Footnotes

1. Quoted by A. Zauberman in his review of O. Lange, *Balance of Outlays and Output of Production* (Moscow, 1958), in *Soviet Studies*, Vol. 10 (April 1959), p.392.
2. Cameron, B., 'Input-Output Analysis', *Economic Record*, Vol. 30 (May 1954), p.33.

3. Kuenne, R.W., 'Walras, Leontief and the Interdependence of Economic Activities', *Quarterly Journal of Economics*, Vol. 68, No. 2 (August 1954), p.353.
4. Simkin, C.G.F., *Economics at Large: An Advanced Textbook on Macro-Economics* (London, 1968), p.110.
5. Another example of this emphasis is found in Shackle, G.L.S., *The Years of High Theory: Invention and Tradition in Economic Thought 1926-1939* (Cambridge, 1967). In a chapter entitled 'Leontief's Tableau Economique', Shackle notes the similarity in intent between Quesnay and Leontief, 'but does not examine the intellectual influences on Leontief. In fact, it is strongly implied that Walras was the main mentor ('Leontief's basic attitude throughout has been a conviction of the theoretical truth and practical relevance of Walras's conception', p.280). Marx's possible influence is not even noted.
6. Leontief, W., *Input-Output Economics* (New York, 1966), p.134.
7. Leontief, W.W., *The Structure of the American Economy, 1919-1939* (2nd edition), New York, 1951), p.9.
8. For example, Duane Evans, W. and Hoffenberg, M., *The Interindustry Relations Study for 1947* (US Bureau of Labour Statistics, 1951); Charnes, A., Cooper, W.W. and Henderson, A., *An Introduction to Linear Programming* (New York, 1953), p.2.
9. Phillips, A., 'The Tableau Economique as a Simple Leontief Model', *Quarterly Journal of Economics*, Vol. 69 (1955), pp.137-44. The originality of Phillips' exercise has been challenged, with the claim that a similar exercise was carried out as early as 1946. See Samuels, W.J., 'The Tableau Economique as a Simple Leontief Model: A Precursor to Phillips', *Indian Economic Journal*, Vol. 7, No. 1, pp.112-17.
10. Meek, R. L. 'The Interpretation of the Tableau Economique', *Economica*, Vol. 27, No. 108 (November 1960), p.346.
11. For example, see Leontief, W., 'The Decline and Rise of Soviet Economic Science', *Foreign Affairs* (October 1960), pp.261-72.
12. Major articles on the revolution in Soviet planning include: Zauberman, A., 'New Winds in Soviet Planning', *Soviet Studies* (July 1960), pp.1-12; Levine, H.S., 'Soviet Economic Planning and Input-Output Analysis', *American Economic Review, Papers and Proceedings*, Vol. 52, No. 2 (May 1962), pp.127-37; Benard, J., 'Quesnay, Leontief et la Théorie Economique en USSR', *Cahiers du Monde Russe et Soviétique*, Vol. 1, No. 2 (January-March 1960), pp.251-59.
13. Leontief, W., 'The Significance of Marxian Economics for Present-Day Economic Theory', *American Economic Review Supplement*, Vol. 28 (March 1938).
14. Marx, K., *Grundrisse der Kritik der Politischen Ökonomie 1857-58* (reprinted Berlin, 1953), pp.345 ff. This has been translated by Martin Nicolaus and published by Penguin Books in 1973. See page 441 of this translation for Marx's 'input-output table' discussed above.
15. Brody, A., *Proportions, Prices and Planning* (Budapest, 1970), p.33.
16. This procedure follows that used in Brody, A., *op.cit.*, pp.46-49.
17. Using Marx's values in *Capital*, Vol. II (Moscow edition 1967), pp.514-17.
18. Lange, O., 'Some Observations on Input-Output Analysis' in Nemchinov, V.S. (ed.), *The Use of Mathematics in Economics* (Edinburgh, 1964), pp.191-216. The comparison originally appeared in *Sankhya* (February 1957), and is also discussed in Lange's, *Introduction to Econometrics* (London, 1962).
19. *Ibid.*, p.200.
20. Zauberman, A., *Aspects of Planometrics* (London, 1967), p.36.
21. Nemchinov, V.S., *op.cit.*, pp.28-32.
22. This again raises the validity of the 'law' of the priority of Department I. For an interesting debate on this law and its relationship to the expanded reproduction schema see Dobb, M., 'Comparative Rates of Growth in Industry', *Soviet Studies*, Vol. 7 (July 1955), pp.52-57, and the rejoinders and criticisms which followed in subsequent issues.
23. Zauberman, *Aspects of Planometrics*, *op.cit.*, p.36.
24. *Ibid.*
25. Quoted by Zauberman, *op.cit.*, p.35.
26. Zauberman, *Ibid.*, pp.37-38.
27. Leontief, W., *Input-Output Economics* (New York, 1966), p.24.

28. *Ibid.*, p.15.
29. *Ibid.*, p.28.
30. See Wicksell, K., *Value, Capital and Rent* (London, 1954 edn.), p.169.
31. The work of M. Morishima is the best example of this trend.
32. Sraffa, P., *Production of Commodities by Means of Commodities: Prelude to a Critique of Economic Theory* (Cambridge, 1960).
33. For example, Cameron, B., 'The Labour Theory of Value in Leontief Models', *Economic Journal*, Vol. 62 (1952); Morishima, M. and Seton, F., 'Aggregation in Leontief Matrices and the Labour Theory of Value', *Econometrica*, Vol. 29 (1961); and, Johansen, L., 'A Note on Aggregation in Leontief Matrices and the Labour Theory of Value', *Econometrica*, *ibid.*
34. Johansen, L., 'Labour Theory of Value and Marginal Utilities', *Economics of Planning*, Vol. 3 (1963).
35. Leontief, W., 'Input-Output Analysis and the General Equilibrium Theory' in Barna, T. (ed.), *The Structural Interdependence of the Economy* (New York-Milan, 1956), p.42. See also the editor's comments. pp.22 ff.
36. For an example of Marx's discussion of quite complex interdependencies see his careful examination of possible influences on the rate of profit of changes in the rate of turnover, in *Capital*, Vol. III (Moscow, 1962 ed.), pp.70-76.
37. The literature on the transformation problem, already voluminous, continues to expand rapidly. It would be precocious to discuss it in this context.
38. It appears that Dmitriev and Leontief even worked at the same institution, albeit at different periods, originally the St Petersburg (later Leningrad) Polytechnic. This can be deduced from a footnote in Nove, A. and Zauberman, A., 'A Resurrected Russian Economist of 1900', *Soviet Studies*, Vol. 13 (1961), p.101, n. 20. The other secondary sources of information on Dmitriev are: Zauberman, A., 'A Few Remarks on a Discovery in Soviet Economics', *Oxford Institute of Statistics Bulletin*, Vol. 24 (1962), pp.437-45, and the introduction by the same author to the French translation of Dmitriev's major treatise: Dmitriev, V.K., *Essais Economiques — Esquisse de synthèse organique de la théorie de la valeur-travail et de la théorie de l'utilité marginale* (Paris, 1968). This was originally published in Moscow in 1904, and received its first complete English translation under the aegis of D.M. Nuti at Cambridge and was published by Cambridge University Press 1974.
39. Dmitriev, V.K., *Essais Economiques*, *op.cit.* See his first essay on Ricardo (pp.17-94). He does however refer in footnotes to Vol. I of *Capital* (p.26) and Vol. II (p.253).
40. Zauberman, A., 'A Few Remarks on a Discovery in Soviet Economics', *op.cit.*, p.444.
41. See Hicks, J.R., 'Linear Theory', *Economic Journal*, Vol. 70 (1960), p.683.
42. Zauberman, A. in his introduction to *Essais Economiques*, *op.cit.*, p.12.
43. Henri Denis in a postscript to the 1968 French edition of *Essais Economiques*, pp. 261-69 discusses these divergences from Marx, and shows considerable annoyance with Dmitriev.
44. Bortkiewicz, L. von., 'Value and Price in the Marxian System,' reprinted in *International Economic Papers*, no. 2 (1952). p.22.
45. Dmitriev, V.K., *op.cit.*, p.26.
46. *Ibid.*, p.27.
47. *Ibid.*
48. *Ibid.* The fact that this book was translated from Russian into French probably accounts for the difficulty involved in obtaining a fluent English translation from the French. Moreover the mathematical symbolism utilised by Dmitriev tends to be unnecessarily confusing in places.
49. *Ibid.*, pp.28-34.
50. *Ibid.*, p.47.
51. Clearly the 'production of commodities by means of commodities'.
52. *Ibid.*, pp.51-52.
53. For a general outline of these early attempts see: Ryabushkin, T., 'From the history of the balance-sheet of the USSR national economy', *Bulletin de l'Institut International de Statistique*, Vol. 38, No. 2 (Brussels, 1960).

54. See Jasny, N., 'The Russian Economic Balance and Input-Output Analysis: A Historical Comment', *Soviet Studies* (July 1962), pp.75-80 and his *Soviet Economists of the Twenties: Names to be Remembered* (Cambridge, 1972).
55. Levine, H.S., 'The Russian Economic Balance and Input-Output Analysis: A Reply', *Soviet Studies* (January 1964), p.356.
56. *Ibid.*, p.355.
57. *Ibid.*, p.354.
58. *Ibid.*, p.355.
59. It will become clear in what follows, that Levine's attempt to suggest a qualitative difference exists between the first National Balance and earlier industry balances, dating from the period of War Communism, is in fact difficult to substantiate. Supporting evidence for this position can be found in: Schlesinger, R., 'A Note on the Content of Early Soviet Planning', *Soviet Studies*, Vol. 16 (1964-65), pp.22-24; and Benard, J., 'Quesnay, Leontief et la Théorie Economique', *Cahiers du Monde Russe et Soviétique*, Vol. I, No. 2 (Paris, January-March 1960).
60. Ballod, K., *The State of the Future* (1898).
61. Specific examples of Ballod's possible influence can be seen in his emphasis on electrification programmes which were strongly supported by Lenin and put into effect in 1921 with the setting up of a national co-ordinating body for this purpose (GOELRO), and his emphasis on the need for Five Year Plans. It is not being claimed that Ballod's work was the exclusive source of such ideas, just that the available evidence suggests his possible influence should not be overlooked.
62. The major sources of information for these claims were: Smolinski, L., 'Planning Without Theory, 1917-1967', *Survey*, Vol. 64 (July 1967), pp.117-20; Wiles, P., 'The Political and Social Prerequisites for a Soviet-Type Economy', *Economica*, Vol. 34, No. 33 (February 1967), pp.12-13; and Trembl, V., 'Interaction of Economic Thought and Economic Policy in the Soviet Union', *History of Political Economy*, Vol. I, No. 1 (Spring 1969), p.189. All these references only give a passing mention to Ballod and do not relate his work to the debate over the origins of input-output analysis. The Trembl article contains only a footnote reference to Ballod. One leading economic history of the USSR does not even mention Ballod: Nove, A., *An Economic History of the USSR*, Middlesex, 1969.
63. For an outline of these debates see: Erlich, A. *The Soviet Industrialisation Debates 1924-1928* (Cambridge, Mass., 1960).
64. For biographies of these two economists and other major Menshevik economists see Jasny, N., *Soviet Economists of the Twenties*, *op.cit.* Even Levine (*op.cit.*) acknowledges the Marxian influence on Groman, but only in a footnote (p.355).
65. In utilising Marx's reproduction models economists such as P. I. Popov ignored the value element implicit in the original models. To Popov the balances he experimented with studied "the relationships of production and circulation in material terms" and even if money "must play the role of yardstick, a means of reducing the assorted fruits of social production to a common denominator . . . the balance studies the relationships in the production and consumption of products as a material process" (P. I. Popov, 'Introduction to the Balance of the National Economy', reprinted in Spulber, N., *Foundations of Soviet Strategy for Economic Growth* (Bloomington, 1964), p.18).
66. Leontief, V. (later W.W. Leontief), 'Balans Narodnoy Khozyaistva SSR', *Voprosy ekonomiki* 1925, No. 12. Reprinted in Spulber, N., *Foundations of Soviet Strategy for Economic Growth*, *ibid.*
67. *Ibid.*
68. In correspondence with B. Seligman, Leontief claimed that his work had no relation to Soviet sources. See Seligman, B., *Main Currents in Modern Economics* (New York, 1962), p.835, note 246. Given this claim, it is perhaps ironical to find Leontief assisting countries such as Cuba in more modern attempts to draw up National Balances for planning purposes. A suggestion (Brody, A., *Proportions, Prices and Planning*, Budapest, 1970), p.54 that Leontief was influenced by G.A. Feldman, a precursor of the later dynamic work of Harrod and Domar need not be considered here, as this refers to the *dynamic* work of Leontief.

69. Leontief, W., *Input-Output Economics* (New York, 1966), p.134. His interest in Soviet economics did not end once he left the Soviet Union. In 1934 he wrote an interesting critique of Soviet planning; 'Wahrung, Lebenshaltung und Finanzen in Sweiten Fünfjahresplan der Sowjetunion', *Ost-Europe Market* (April-May 1934).
70. See *Business Week* (New York) (29th August, 1953), p.26.
71. Major works covering this school's contribution include: Burchardt, F.A., 'Die Schemata des Stationären Kreislaufs bei Böhm-Bawerk und Marx', *Weltwirtschaftliches Archiv*, Vol. 34 (1931), pp.525-64 and Vol.35, pp.116-76; Gaitskell, H.T.N., 'Notes on the Period of Production', *Zeitschrift für Nationalökonomie*, Vol. 9 (1938), pp.215-44; Nurkse, R., 'The Schematic Representation of the Structure of Production', *Review of Economic Studies*, Vol. 2 (1935), pp.232-44.
72. Burchardt was thus providing a serious critique of J.A. Schumpeter's *Theory of Economic Development*.
73. A discussion of the 'period of production' is contained in Marcus-Fleming, J., 'The Period of Production and Derived Concepts', *Review of Economic Studies*, Vol. 3, No. 1 (October 1935), pp.1-17.
74. Böhm-Bawerk depicted the process in graphical form, as a system of concentric circles in which all groups according to their distance from the consumer were located. He used it to illustrate both the structure of production maintained by a given 'time distribution' of original factors and the general synchronised productive process. One of many brief discussions of Böhm-Bawerk's capital theory is that contained in Seligman, B.B., *Main Currents in Modern Economics* (New York, 1962), pp.294-310.
75. Nurkse, *op.cit.*, pp.232-34.
76. Kaehler, A., *Die Theorie der Arbeiterfreisetzung durch die Maschine* (The Theory of Technological Unemployment) (1933), Ch. IV.
77. Lowe, A., 'F.A. Burchardt', *Oxford University Institute of Statistics Bulletin*, May 1959, p.64. It is worthy of note that Burchardt's analysis did not require an acceptance or usage of the labour theory of value. This provides further evidence for the claims of Joan Robinson which have been accepted in the writing of this paper. Yet this is not to say that the labour theory of value is incompatible with Leontief's approach. See Morishima, M., and Seton, F., 'Aggregation in Leontief Matrices and the Labour Theory of Value', *Econometrica*, Vol. 29, No. 2 (April 1961), pp.203-20. This article examines the relationships between the price of commodities and their total labour content when the productive contributions of all other factors are imputed to labour. For Lowe's major utilisation of Burchardt's insights see his 'A Structural Model of Production', *Social Research*, Vol 19, No. 2 (June 1952), pp.135-76.
78. Leontief, W., 'The Significance of Marxian Economics for Present Day Economic Theory', *American Economic Review, Supplement*, March, 1938.
79. Further work by Burchardt on what are now called 'the sectoral conditions for once-over growth' spread further than Leontief, and especially to Adolph Lowe's work on real capital formation. An interesting connection to remember here is that Lowe, while a key member of the Kiel school, received the young Leontief as a temporary assistant after the latter left the USSR. It is important to note in this context that Lowe's analysis of the 'structural model of production' contained many elements in the spirit of Marxian economics.

If Leontief was interested in aspects of the work initiated by Burchardt, Lowe took the same view. He wrote that "so far from being exhausted, the stimulation derived from Burchardt's theoretical thinking is likely to gather momentum in the years to come?" (Lowe, 'F.A. Burchardt', *op.cit.*, p.64). Unfortunately for the development of this line of analysis, Burchardt went to Oxford in 1935 and devoted almost all his attention not to growth theory but to applied economic research until his death in 1959.