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HIGHER EDUCATION AS A FILTER

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This note sketches a model of the economic role of higher education rather different from the current human capital orthodoxy. It is designed to formalize views expressed by some sociologists (e.g., Berg, 1970) that the diploma serves primarily as an (imperfect) measure of performance ability rather than as evidence of acquired skills. I think the model is capable of illuminating certain aspects of the economic returns to higher education and gives an interpretation alternative to the conventional one.

The model certainly abstracts from aspects which have been much considered. I am not apologetic for this abstraction, but I am for the fact that the model is still so primitive in form and in particular for the fact that it seems so difficult to test. I hope to work further on it and to encourage others to do the same.

The conventional view among economists is that education adds to an individual's productivity and therefore increases the market value of his labor. From the viewpoint of formal theory, it does not matter how the student's productivity is increased, but implicitly it is assumed that the student receives cognitive skills through his education. Educators on the other hand, have long felt that the activity of education is a process of socialization; the latent content of the process, the acquisition of skills such as the carrying out of assigned tasks, getting along with others, regularity, punctuality, and the like, being at least as important as the manifest objectives of conveying information. This last doctrine is currently revived by radical economists, though

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with a negative rather than a positive valuation. But from the viewpoint of economic theory, the socialization hypothesis is just as much a human capital theory as the cognitive skill acquisition hypothesis. Both hypotheses imply that education supplies skills that lead to higher productivity.

I would like to present a very different view. Higher education, in this model, contributes in no way to superior economic performance; it increases neither cognition nor socialization. Instead, higher education serves as a screening device, in that it sorts out individuals of differing abilities, thereby conveying information to the purchasers of labor. (Perhaps I should make clear that I personally do not believe that higher education performs only a screening purpose. Clearly professional schools impart real skills valued in the market and so do undergraduate courses in the sciences. The case is considerably less clear with regard to the bulk of liberal arts courses. But in any case I think it better to make a dramatic and one-sided presentation of the screening model in order to develop it than to produce a premature synthesis. It should also be understood that I am speaking only about the contribution of higher education to production; the consumption aspects are real and important, but they are irrelevant to the points being made here.)

The screening or *filter* theory of higher education, as I shall call it, is distinct from the productivity-adding human capital theory but is not in total contradiction to it. From the viewpoint of an employer, an individual certified to be more valuable is more valuable, to an extent which depends upon the nature of the production function. Therefore, the filtering role of education is a productivity-adding role from the private viewpoint; but as we shall see, the social productivity of higher education is more problematic.

The filter theory of education is part of a larger view about the nature of the economic system and its equilibrium. It is based on the assumption that economic agents have highly imperfect information. In particular, the purchaser of a worker's services has a very poor idea of his productivity. In this model, I assume instead that the buyer has very good statistical information but nothing more. That is, I assume that there are certain pieces of information about the worker, specifically whether or not he has a college diploma, which the employer can acquire costlessly. He knows, from general information or previous experience, the statistical distribution of productivities given the information he has, but has no way of distinguishing the productivities of individuals about whom he has the same information.

It will probably be argued that this description is valid enough at the time of hiring but that after a period of time the employer will know his workers and their productivities on an individual basis. No doubt there is something to this viewpoint but not as much as may be thought. After all, what is needed for allocative efficiency is the marginal productivity of each individual. But in a complex production process, the employer has simply no way of determining that. All he can do is act like an ideal econometrician, relating his output to the numbers of different kinds of workers (and other inputs, from which I am abstracting in this paper). Here two workers are of the same kind if the employer's information about them is the same.

The general point that information in the real world is much more limited than that assumed in our usual equilibrium models has a long history among critics of the mainstream of economic thought. In recent years it has been especially stressed by Herbert A. Simon and his followers. The particular emphasis on lack of information concerning the productivity of workers has been argued by me in the context of racial discrimination in employment (Arrow, 1972a, b) and, in a more general way, by A. Michael Spence in a recent Harvard dissertation (Spence, 1972). The hypothesis that the actors in an uncertain world have a correct perception of the probability distribution of that uncertainty is a fairly standard one. In particular, this can be applied to lack of information about endogenous economic variables, such as prices or productivities; it becomes a condition of equilibrium that the distribution, when believed, helps generate such behavior as to maintain the distribution.

1. The basic model

We shall assume that each individual has three characteristics, his record before entering college, the probability of his getting through higher education, and his productivity. These have a joint distribution and presumably are positively correlated. The producers know about an individual only whether or not he is graduated from college.

The colleges serve really as a double filter, once in selecting entrants and once in passing or failing students. In admitting students, the colleges aim to maximize the expected number of graduates. Let,

y = record before college,
z = productivity,
$f(y, z)$ = joint density of the two variables.

For applicants with a record y, the college is only interested in the conditional probability of their graduating. Hence it can be assumed without loss of generality that y is the probability of graduating conditional on the pre-college record, for the conditional probability of success is the only aspect of the pre-college record relevant to admission and to the model as a whole. If the capacity of the college is limited, then choice of admission procedures to maximize the expected number of graduates implies choice of a cut-off number, y_0 such that an applicant is admitted if and only if,

$$y \geqq y_0. \tag{1}$$

Let,

N_e = proportion of population admitted to college,
N_g = proportion graduating.

Since y has been transformed to be a probability, it varies from 0 to 1. The variable z is only constrained to be a non-negative variable and therefore may range from 0 to $+\infty$. From the definitions let,

$$g(y) = \int_0^{+\infty} f(y, z)\,dz,$$

be the marginal density of y. From the definitions,

$$N_e = \int_{y_0}^{1} \int_0^{+\infty} f(y, z)\,dz\,dy = \int_{y_0}^{1} g(y)\,dy = P(y \geqq y_0), \tag{2}$$

$$N_g = \int_{y_0}^{1} \int_0^{+\infty} y\, f(y, z)\,dz\,dy = \int_{y_0}^{1} y\, g(y)\,dy$$

$$= E(y \mid y \geqq y_0)\, P(y \geqq y_0) = \bar{y}_e P(y \geqq y_0), \tag{3}$$

where

$$\bar{y}_e = \mathrm{E}(y \mid y \geqq y_0) \tag{4}$$

is the probability of graduation of a random college entrant.

A detailed interpretation of productivity has not yet been given; in the sequel, two alternative interpretations will be used. However, under either interpretation, we will regard total output of the appropriate commodity to be the sum of the productivities of individuals. Then the average productivity of all individuals is,

$$\bar{z} = \int_0^1 \int_0^{+\infty} z\, \mathrm{f}(y, z)\, \mathrm{d}z\, \mathrm{d}y = \mathrm{E}(z), \tag{5}$$

and the total product of college graduates (per unit of total labor force) is,

$$Z_g = \int_{y_0}^1 \int_0^{+\infty} zy\, \mathrm{f}(y, z)\, \mathrm{d}z\, \mathrm{d}y = \int_{y_0}^1 y\, \mathrm{E}(z \mid y)\, \mathrm{g}(y)\, \mathrm{d}y$$

$$= \mathrm{E}(zy \mid y \geqq y_0)\, \mathrm{P}(y \geqq y_0). \tag{6}$$

Under what conditions does college filtering convey any information? From (6) and (3), the expected productivity of a college graduate is,

$$\bar{z}_g = Z_g / N_g = \mathrm{E}(zy \mid y \geqq y_0) / \mathrm{E}(y \mid y \geqq y_0). \tag{7}$$

College graduation has some (positive) information content if the productivity of a randomly chosen college graduate exceeds that of a randomly chosen member of the population, i.e., if

$$\bar{z}_g > \mathrm{E}(z). \tag{8}$$

The existence of the admission procedure suggests the following additional question; is it the admission or the college itself that performs the screening function? This is, after all, an important policy question, since admission procedures are much cheaper; for a first approximation we may suppose them free. Then admission procedures convey information if,

$$\mathrm{E}(z \mid y \geqq y_0) > \mathrm{E}(z), \tag{9}$$

and college itself has additional informational content over simple admission, if,

$$\bar{z}_g > E(z \mid y \geqq y_0). \tag{10}$$

Since,

$$E(z) = E(z \mid y \geqq y_0) P(y \geqq y_0) + E(z \mid y < y_0) P(y < y_0),$$

$$E(z \mid y \geqq y_0) - E(z)$$
$$= P(y < y_0)[E(z \mid y \geqq y_0) - E(z \mid y < y_0)], \tag{11}$$

i.e., if the expected productivity of those admitted is greater than that of those rejected, then the admission procedure has predictive value.

$$\bar{z}_g - E(z \mid y \geqq y_0) = \frac{E(zy \mid y \geqq y_0) - E(z \mid y \geqq y_0) E(y \mid y \geqq y_0)}{E(y \mid y \geqq y_0)}$$
$$= \frac{\sigma_{yz \mid y \geqq y_0}}{E(y \mid y \geqq y_0)}, \tag{12}$$

where use is made of (7), and $\sigma_{yz \mid y \geqq y_0}$ means the conditional covariance of y and z, given admission to college. Thus college education conveys information about productivity beyond admission if there is a positive correlation between productivity and probability of college success among those admitted.

It is easy to see that both (9) and (10) hold if we make the following, *Positive screening assumption*: $E(z \mid y)$ is an increasing function of y. Under this assumption, it is obviously true that,

$$E(z \mid y \geqq y_0) > E(z \mid y_0) > E(z \mid y < y_0),$$

so that, from (11), (9) holds.

Also, under any condition on the range of y, the covariance of y and z is the same as that between y and $E(z \mid y)$, by the definitions. But the covariance between any random variable, over any range, and an increasing function of it is certainly positive, so that, from (12), (10) is certainly true.

The productivity advantage of college graduates over the average member of the population can be found by adding (11) and (12).

2. The social value of college screening: the one-factor case

But even if college does have a positive informational value, it by no means follows that it is socially worthwhile. The filter model thus leads to a very different conclusion from the human capital model; for, as we will now see, there can easily be a divergence between social and private demands for information.

Consider the simplest model of production; all individuals are perfect substitutes in production with ratios given by their productivities. Then there is no social value to information about productivity. The total output of society will be $\mathrm{E}(z) = \bar{z}$ (normalized on labor force); the more productive individuals will produce more whether or not anyone knows who they are. (I am abstracting from incentive questions here.) There will, however, be a private value to a college diploma for those most likely to get it, if we assume a competitive world. For then, the wage of an individual will be the expected value of his product conditional on the information available to the employer. Let us assume that the individual has no better information about his prospects of going through college than the college has. Suppose further that anyone can go to college if he pays its cost, c. (Since education is also a consumer's good, the cost, c, is to be interpreted here empirically as the cost over and above its consumption value.) Suppose no one is going to college initially. Then a few individuals go to college; if they have selected themselves properly, then the expected value of their productivity, conditional on graduation, is greater than the overall average, $\bar{z}$. Clearly, if it is sufficiently greater and if the probability of passing is high enough, then it pays to incur the costs. But these costs are simply a social waste.

In fact, a detailed examination shows that, under certain informational assumptions, everybody would gain by prohibiting college (the following argument is really a special case of Spence's). Let us study the equilibrium. Some go to college, and some don't. The employers know the expected productivities of college graduates and of others (I assume here that employers do not distinguish between college failures and those who do not enter; such a distinction could easily be introduced if deemed realistic). Further assume that potential entrants know the overall probability of graduation among those who enter but not the probability conditional on their own record. The colleges do know these conditional probabilities. As before, there is a critical level, y_0, such that individuals are admitted to college if and only if $y \geq y_0$ (the critical level here is determined by demand for college entrance, not, as before, by capacity restrictions). The employers then pay to college graduates $\bar{z}_g = \bar{z}_g(y_0)$, as defined in (7). Let $\bar{z}_n(y_0)$ be the expected productivity of the non-graduates. Then,

$$\bar{z} = \bar{z}_g(y_0)N_g + \bar{z}_n(y_0)(1 - N_g); \tag{13}$$

N_g, as defined in (3), is, of course, also a function of y_0.

If an individual goes to college, he graduates with probability $\bar{y}_e$, and fails with probability $1 - \bar{y}_e$. He incurs a cost of c in any case, so that his expected return from college is,

$$\bar{z}_g(\bar{y}_e) + \bar{z}_n(1 - \bar{y}_e) - c.$$

If he does not go to college, his return is $\bar{z}_n$ with certainty. In the absence of risk-aversion, equilibrium requires that the two returns be equal; for if the expected return to going to college were the greater, individuals with records slightly less than y_0 would find it profitable to go to college.

$$\bar{z}_g\bar{y}_e + \bar{z}_n(1 - \bar{y}_e) - c = \bar{z}_n,$$

or, after simplification,

$$\bar{y}_e(\bar{z}_g - \bar{z}_n) = c, \tag{14}$$

which immediately shows that $\bar{z}_g > \bar{z}_n$. But then, from (13), $\bar{z}_n < \bar{z}$. Therefore, the income of a non-graduate is lower than it was before; but since the expected income of college entrants equals that of non-graduates, the college entrants do not benefit either, at least not ex ante.

Hence, we have the remarkable possibility that if college is a filter, its abolition may help everyone. Not only is there no efficiency gain, but college has also created an inequality in ex post income where none existed before.

We are accustomed in theory to argue that information may be under-produced because its social value is greater than its private. But the opposite possibility has also been shown by Hirshleifer (1971) in a recent and very important article. Information not used in production may merely convey a competitive advantage.

Of course, our conclusion depended on free entry to college. If college entrance is limited in some manner, then the equality in (14) becomes an inequality, and the college entrants may gain on average. The non-entrants certainly lose in any case, and further it would have paid them to bribe the entrants not to enter. The effects on equality are even worse in this case.

William Brainard has pointed out to me that the strong result found so far depends on the assumption that the potential entrant does not know the probability of graduation conditional on his own record but only the probability conditional on the fact of entrance. If the stronger informational condition holds, the expected return from college for an individual whose pre-college record is y is,

$$\bar{z}_{\mathrm{g}} y + \bar{z}_{\mathrm{n}}(1 - y) - c = (\bar{z}_{\mathrm{g}} - \bar{z}_{\mathrm{n}})y + \bar{z}_{\mathrm{n}} - c, \tag{15}$$

and therefore equilibrium is obtained by setting this return equal to $\bar{z}_{\mathrm{n}}$ for the *marginal* man, for whom $y = y_0$. Therefore, (14) is replaced by

$$y_0(\bar{z}_{\mathrm{g}} - \bar{z}_{\mathrm{n}}) = c. \tag{16}$$

It remains true that $\bar{z}_{\mathrm{g}} > \bar{z}_{\mathrm{n}}$ and therefore $\bar{z} > \bar{z}_{\mathrm{n}}$, so that the non-graduates are worse off than they would be in the absence of college education. However, individuals with sufficiently good records, i.e., sufficiently high values of y, may have expected returns (15) which are at least equal to $\bar{z}$, the expected return in the absence of college filtering. If y_1 is the smallest such value of y,

$$(\bar{z}_{\mathrm{g}} - \bar{z}_{\mathrm{n}})y_1 + \bar{z}_{\mathrm{n}} = \bar{z} + c.$$

If (13) is subtracted from this equation, we see that,

$$(\bar{z}_{\mathrm{g}} - \bar{z}_{\mathrm{n}})(y_1 - N_{\mathrm{g}}) = c,$$

and dividing through by (16) yields,

$$y_1 = N_{\mathrm{g}} + y_0.$$

If there are no values of y above y_1, then it remains true that everybody gains by changing from competitive equilibrium to the abolition of college. However, in any case, the fundamental point remains unaltered; there is a net gain in social output by abolishing college, and everybody could be made better off by doing so and redistributing income suitably.

(This statement may appear hard to reconcile with our usual expectation that college education is insufficient. Of course, I am abstracting from credit rationing, which has been the most powerful force working in the opposite direction. My guess is that we are moving into a period where public subsidies to higher education plus improved credit facilities are making effective credit restriction on higher education a thing of the past.)

3. The social value of screening: two-factor model

To understand the social role of education as a filter better, one must consider more complicated production functions, in which there are complementary kinds of labor. Then education has a positive value in sorting out types of workers. For simplicity, suppose there are two kinds of labor. Everyone is capable of supplying one unit of type 1 labor, and this fact is known to all. However, there is also needed type 2 labor, of which different individuals can supply different amounts. In this model, z will be interpreted as the supply of type 2 labor, measured in efficiency-units. To emphasize the complementarity of the different types of labor, we will assume that production requires fixed proportions of the two types of labor (remembering always that type 2 labor is measured in efficiency-units). By proper choice of units, we can require without loss of generality that one unit of each type of labor is needed to produce one unit of product.

In this model, filtering is no longer useless. Suppose there are two classes of people, say A and B with expected productivities $\bar{z}_A$ and $\bar{z}_B$, respectively, $\bar{z}_A > \bar{z}_B$ (remember that "productivity" here means supply of efficiency-units of type 2 labor). Then clearly it can never be optimal to have simultaneously individuals of class A performing type 1 labor and individuals of class B performing type 2 labor. For suppose this happened. Efficiency also requires that the sum of the z's of those in type 2 labor equal the number in type 1 labor. Then remove some class A individuals from type 1 labor and replace them with an equal number of class B individuals from type 2 labor, making all selections at random. The number of type 1 laborers is unchanged, the expected number of efficiency-units of type 2 labor is increased. Output does not yet increase, since the number of type 1 laborers is a bottleneck. But then we can transfer individuals from type 2 to type 1 labor; by increasing the number of type 1 laborers, total output is increased, provided not so many are transferred that the supply of type 2 labor is reduced below that of type 1 labor.

Thus total output is increased by successful filtering, provided, of course, that the cost of the filter is not too high. We can easily calculate the gain in output due to filtering at zero cost. Let N_A and N_B be the proportions of individuals of the two classes, $N_A + N_B = 1$. Then,

$$\bar{z} = N_A \bar{z}_A + N_B \bar{z}_B .$$

If the filter is not used, a fraction N_1 of the entire population is assigned to type 1 labor and the rest to type 2 labor. Since the assignment is random, the total supply of type 2 labor in efficiency units is $(1 - N_1)\bar{z}$. Efficiency requires that,

$$(1 - N_1)\bar{z} = N_1 ,$$

and the output is the common value of the two sides, so that,

$$N_1 = \bar{z}/(1 + \bar{z}) = \text{output without filtering.} \tag{17}$$

Now suppose for the moment that the filter is used naively, that is, all individuals of class A are assigned to type 2 jobs and only such. The supply of type 2 labor is then $N_A \bar{z}_A$, that of type 1 labor, N_B. Of course, these two need not be equal; if they are not, the allocation is inefficient. Suppose first that $N_B < N_A \bar{z}_A$. Then clearly some class A labor will have to be assigned to type 1 jobs. That is, $N_1 > N_B$. Clearly, efficiency requires,

$$(1 - N_1)\bar{z}_A = N_1 ,$$

so that,

$$N_1 = \bar{z}_A/(1 + \bar{z}_A) = \begin{array}{l}\text{output with filtering and excess of}\\ \text{class A labor.}\end{array} \tag{18}$$

The increase in output due to filtering is, then,

$$\frac{\bar{z}_A}{1 + \bar{z}_A} - \frac{\bar{z}}{1 + \bar{z}} = \frac{\bar{z}_A - \bar{z}}{\bar{z}(1 + \bar{z}_A)} \; \frac{\bar{z}}{1 + \bar{z}} ,$$

the first factor showing the proportionate increase in output.

If there is a deficiency of class A labor for the type 2 jobs, $N_{\mathrm{B}} > N_{\mathrm{A}}\bar{z}_{\mathrm{A}}$, optimal allocation requires that all the class A labor be assigned to those jobs plus enough of the class B labor so that the supplies of the two types of labor are equal.

$$N_{\mathrm{A}}\bar{z}_{\mathrm{A}} + (N_{\mathrm{B}} - N_1)\bar{z}_{\mathrm{B}} = N_1,$$

so that

$$N_1 = \bar{z}/(1+\bar{z}_{\mathrm{B}}) = \text{output with filtering and a deficiency of class A labor} \tag{19}$$

and the increase in output is,

$$\frac{\bar{z}}{1+\bar{z}_{\mathrm{B}}} - \frac{\bar{z}}{1+\bar{z}} = \frac{\bar{z}-\bar{z}_{\mathrm{B}}}{1+\bar{z}_{\mathrm{B}}}\,\frac{\bar{z}}{1+\bar{z}}.$$

Now let us identify these general classes A and B with graduates and non-graduates respectively. Then it has been shown that college education pays if it is free (in which case everyone goes to college and the screening is solely through passing or failing), and, by continuity, some college education pays if c is sufficiently small. Suppose at the optimum $N_{\mathrm{g}}\bar{z}_{\mathrm{g}} > 1 - N_{\mathrm{g}}$. Then the output is given by (18) but from this must be subtracted the cost of education. If c is measured in terms of output, the cost is $c\mathrm{P}(y \geqq y_0)$, where y_0 is the cut-off record for admission. Hence, the net output of society for a given y_0 is,

$$[\bar{z}_{\mathrm{g}}/(1+\bar{z}_{\mathrm{g}})] - c\mathrm{P}(y \geqq y_0).$$

But an increase in y_0 can be shown to increase $\bar{z}_{\mathrm{g}}$ and therefore will increase $\bar{z}_{\mathrm{g}}/(1+\bar{z}_{\mathrm{g}})$ and it will also decrease $\mathrm{P}(y \geqq y_0)$; hence such an allocation cannot be optimum, a contradiction.

To see that an increase in y_0 will increase $\bar{z}_{\mathrm{g}}$, first differentiate the definition (7) logarithmically with respect to y_0.

$$\frac{1}{\bar{z}_{\mathrm{g}}}\frac{\mathrm{d}\bar{z}_{\mathrm{g}}}{\mathrm{d}y_0} = \frac{1}{Z_{\mathrm{g}}}\frac{\mathrm{d}Z_{\mathrm{g}}}{\mathrm{d}y_0} - \frac{1}{N_{\mathrm{g}}}\frac{\mathrm{d}N_{\mathrm{g}}}{\mathrm{d}y_0}$$

$$= \frac{-y_0\,\mathrm{E}(z\,|\,y_0)\,\mathrm{g}(y_0)}{Z_{\mathrm{g}}} + \frac{y_0\,\mathrm{g}(y_0)}{N_{\mathrm{g}}} \qquad \text{(from (3) and (6))}$$

$$= [y_0\,\mathrm{g}(y_0)/Z_{\mathrm{g}}]\,[\bar{z}_{\mathrm{g}} - \mathrm{E}(z\,|\,y_0)],$$

which is positive if $\bar{z}_g > E(z \mid y_0)$; but since every graduate has a record at least equal to y_0, the last inequality follows from the positive screening assumption.

It has therefore been shown that the optimal amount of college education will be such that,

$$N_g \bar{z}_g \leqq 1 - N_g. \tag{20}$$

It is interesting to note that the cost c does not enter into this condition, so it is valid even if $c = 0$. Even if education is free, it is socially optimal to restrict it so as to improve its screening function.

Since (20) holds, the net output of society is given by (19), and the optimal amount of higher education is obtained by choosing y_0 to maximize

$$[\bar{z}/(1 + \bar{z}_n)] - cP(y \geqq y_0) = F(y_0, c)$$

$$= H(y_0) - cP(y \geqq y_0), \tag{21}$$

subject to (20).

The full statement of the derivative conditions implied by this maximization can be easily written down, but they do not appear simple enough for useful interpretation. However, some implications are useful to draw, in particular conditions under which the constraint (20) is binding. When this holds, the filter is working most smoothly, in that every graduate goes into type 2 jobs and every non-graduate into type 1 jobs. In this case, we will say that the filter is *complete*.

First note that the function,

$$G(y_0) = N_g \bar{z}_g + N_g = Z_g + N_g$$

$$= \int_0^{+\infty} \int_{y_0}^{1} y(1 + z)\, f(y, z)\, dy\, dz,$$

from (3) and (6), is clearly a strictly decreasing function of y_0 in any region of positive density. Hence, the equation,

$$G(y_0^*) = 1, \tag{22}$$

has a unique solution, and the inequality (20) can be written, $G(y_0) \leqq 1$, or

$$y_0 \geqq y_0^*. \tag{23}$$

The variables $\bar{z}_g$ and $\bar{z}_n$ are functions of y_0; their values when $y_0 = y_0^*$ will be denoted by $\bar{z}_g^*$ and $\bar{z}_n^*$, respectively. The starred magnitudes characterize the complete filter.

Next we note that the value of y_0 which maximizes (21) subject to (20) or, equivalently, (23) must be a monotone increasing function of c. Actually, this conclusion must be stated more precisely, since nothing has been said which implies that the maximum must be unique. Suppose $c_1 < c_2$. Then we show that every maximal value of y_0 for $c = c_2$ exceeds every maximal value for $c = c_1$, except that if y_0^* is the unique maximal value of y_0 for $c = c_1$, then it can happen that y_0^* is also a maximal value for $c = c_2$.

To see this, let y_0^1 be any maximal value for $c = c_1$ and y_0^2 for $c = c_2$. From (21),

$$F(y_0, c_1) + (c_1 - c_2)P(v \geqq y_0) = F(y_0, c_2).$$

By definition of a maximum,

$$F(y_0^1, c_1) \geqq F(y_0^2, c_1).$$

Add,

$$\begin{aligned}(c_1 - c_2)&P(y \geqq y_0^1) \\ &= (c_1 - c_2)P(y \geqq y_0^2) + (c_1 - c_2)[P(y \geqq y_0^1) - P(y \geqq y_0^2)]\end{aligned}$$

to this inequality.

$$\begin{aligned}F(y_0^1, c_2) &\geqq F(y_0^2, c_2) + (c_1 - c_2)[P(y \geqq y_0^1) - P(y \geqq y_0^2)] \\ &\geqq F(y_0^1, c_2) + (c_1 - c_2)[P(y \geqq y_0^1) - P(y \geqq y_0^2)],\end{aligned}$$

from the fact that y_0^2 maximizes $F(y_0, c_2)$ in the range (23). Hence,

$$(c_1 - c_2)[P(y \geqq y_0^1) - P(y \geqq y_0^2)] \leqq 0,$$

or, since $c_1 < c_2$,

$$\mathrm{P}(y \geqq y_0^1) \geqq \mathrm{P}(y \geqq y_0^2),$$

which is possible only if $y_0^2 \geqq y_0^1$. That is, if the cost of education rises, any optimal cut-off level for entrance must be at least as high as it was before the raise.

But when can the equality, $y_0^1 = y_0^2$, hold? This means that y_0^1 is optimal for $c = c_1$ and for $c = c_2$. Suppose $y_0^1 > y_0^*$. Then the constraint (23) is not effective, so that $\partial \mathrm{F}/\partial y_0 = 0$ at $y = y_0^1$ for both values of c. But

$$\mathrm{dP}(y \geqq y_0)/\mathrm{d}y_0 = -\mathrm{g}(y_0), \text{ and,}$$

from (21),

$$\frac{\partial \mathrm{F}}{\partial y_0} = \mathrm{H}'(y_0) + c\mathrm{g}(y_0),$$

so that if y_0^1 is optimal for two different levels of c and $y_0^1 > y_0^*$, we would have,

$$\mathrm{H}'(y_0^1) + c_1 \mathrm{g}(y_0^1) = 0,$$

$$\mathrm{H}'(y_0^1) + c_2 \mathrm{g}(y_0^1) = 0,$$

which is impossible if $\mathrm{g}(y_0) > 0$, which we can assume. Hence, the equality $y_0^1 = y_0^2$ can hold only if $y_0^1 = y_0^*$.

To complete the argument, suppose that at $c = c_1$, y_0^* is optimal, but there is also another optimal value, say $y_0 = y_0'$. But then, as already shown, $y_0^2 \geqq y_0' > y_0^*$ for any value of y_0 optimal for $c = c_2$. Hence, there can be a common optimal value for $c = c_1$ and $c = c_2$ only if y_0^* is the unique optimal cut-off point for the lower cost.

We can then conclude that the complete filter is optimal, if ever, only for an interval of c-values starting at $c = 0$. If, for any c, y_0^* is not the unique optimal value, then it is not optimal for any larger values of c.

It is, of course, clear that y_0^* cannot be optimal for all c. For consider what happens when y_0 is raised to its upper limit, 1; this means approaching the no-filter situation, in which there is no higher education. In that case, $\bar{z}_n \to \bar{z}$, while $\mathrm{P}(y \geqq y_0)$ approaches zero; hence, the net output tends to $\bar{z}/(1 + \bar{z})$, as already seen in (17). On the other hand, if the complete filter were used for all values of c, it can be seen from (21) that the net output would eventually fall below this level (and indeed would eventually become negative and therefore infeasible). Clearly, the complete filter cannot be optimal if it is inferior to no filter. Hence the c-interval in which the complete filter is optimal, if it exists, is bounded above.

It remains to see if there is any interval of costs of education for which the complete filter is optimal. Actually, this interval can be shown to exist only under an additional, though natural, assumption. Note that $F(y_0, 0) = H(y_0)$. If it is true that $H'(y_0) < 0$ for all y_0, then the only optimal cut-off for free education would clearly be to make y_0 as small as possible, i.e., to let $y_0 = y_0^*$. What happens for c slightly greater than 0? I shall show that there must be an interval of c-values in which y_0^* is the unique optimum.

For suppose not; then we can find a sequence $\{c^\nu\}$ of c-values, arbitrarily small, such that for $c = c^\nu$, there is an optimal value $y_0 = y_0^\nu > y_0^*$. By definition of an optimum,

$$F(y_0^\nu, c^\nu) \geqq F(y_0^*, c^\nu), \text{ each } \nu.$$

Either the sequence $\{y_0^\nu\}$ has a limit point $y_0^{**} > y_0^*$ or else $y_0^\nu \to y_0^*$. In the first case, by continuity, we would have $F(y_0^{**}, 0) \geqq F(y_0^*, 0)$, in contradiction to the fact that y_0^* is the unique optimum when $c = 0$. Hence,

$$\frac{F(y_0^\nu, c^\nu) - F(y_0^*, c^\nu)}{y_0^\nu - y_0^*} \geqq 0, \quad y_0^\nu \to y_0^*. \tag{24}$$

From (21),

$$\frac{F(y_0^\nu, c^\nu) - F(y_0^*, c^\nu)}{y_0^\nu - y_0^*} = \frac{H(y_0^\nu) - H(y_0^*)}{y_0^\nu - y_0^*} - c^\nu \frac{P(y \geqq y_0^\nu) - P(y \geqq y_0^*)}{y_0^\nu - y_0^*}.$$

From the definition of a derivative and other remarks made above, we know that, as ν approaches $+\infty$,

$$\frac{H(y_0^\nu) - H(y_0^*)}{y_0^\nu - y_0^*} \to H'(y_0^*), \; c^\nu \to 0,$$

$$\frac{P(y \geqq y_0^\nu) - P(y \geqq y_0^*)}{y_0^\nu - y_0^*} \to - g(y_0^*);$$

hence,

$$\frac{F(y_0^\nu, c^\nu) - F(y_0^*, c^\nu)}{y_0^\nu - y_0^*} \to H'(y_0^*),$$

and, from (24), $H'(y_0^*) \geqq 0$, in contradiction to the assumption that $H'(y_0) < 0$ for all y_0.

Thus, the condition $H'(y_0) < 0$ implies the existence of a cost $c = c_1 > 0$ such that the complete filter is optimal for $0 \leqq c \leqq c_1$ and not for higher values of c. It remains only to restate the condition, $H'(y_0) < 0$. Clearly, from (21), this holds if and only if,

$$\frac{d\bar{z}_n}{dy_0} > 0.$$

As already seen in (13), $N_g\bar{z}_g + (1 - N_g)\bar{z}_n = \bar{z}$, and this for all y_0. From (7) and (6),

$$N_g\bar{z}_g = Z_g,$$

and,

$$\frac{dZ_g}{dy_0} = - y_0 \int_0^{+\infty} z f(y_0, z)\, dz.$$

If we differentiate (13) with respect to y_0, we find, after some transposition, that,

$$(1 - N_g)\left(\frac{\mathrm{d}\bar{z}_n}{\mathrm{d}y_0}\right) = \bar{z}_n\left(\frac{\mathrm{d}N_g}{\mathrm{d}y_0}\right) - \left(\frac{\mathrm{d}z_g}{\mathrm{d}y_0}\right)$$

$$= y_0 \int_0^{+\infty} z\mathrm{f}(y_0, z)\,\mathrm{d}z - y_0\bar{z}_n \int_0^{+\infty} \mathrm{f}(y_0, z)\,\mathrm{d}z$$

$$= y_0 \int_0^{+\infty} \mathrm{f}(y_0, z)\,\mathrm{d}z\,[\mathrm{E}(z|y_0) - \bar{z}_n]\,,$$

with the aid of (3). Hence, $\mathrm{H}'(y_0) < 0$ if and only if,

$$\mathrm{E}(z|y_0) > \bar{z}_n\,. \tag{25}$$

This asserts that, for any given cut-off admission criterion, the average productivity of those marginally admitted exceeds the average productivity of non-graduates. Notice that this assumption is somewhat strong, for the non-graduates include those with pre-admission records predicting a probability of graduation greater than y_0. Indeed, (25) can hardly hold for $y_0 = 0$; for in that case, $\bar{z}_n$ is the average productivity of all those who failed when everyone is permitted to go to college, while $\mathrm{E}(z|y_0)$ is the average productivity of the subgroup whose failure was perfectly predictable. By the same token, we would certainly expect (25) to hold when $y_0 = 1$. In this case, there is no filter at all, i.e., no higher education, so that $\bar{z}_n = \bar{z}$; we would certainly expect that the expected productivity of those who, on the basis of their pre-college records, would be certain to pass if admitted would be higher than the average in the population. We can therefore assume that (25) holds for y_0 sufficiently high; in particular, we assume that it holds for $y_0 \geqq y_0^*$, which is all that is needed.

We can thus conclude as follows: If (25) holds for cut-off points y_0 at least equal to that for the complete filter, then there is a cost, $c_1 > 0$, such that the complete filter is uniquely optimal for education costs $0 \leqq c \leqq c_1$ and not for higher cost levels. If (25) does not hold, then the same may be true, or else it may be that the complete filter is never optimal. In any case, for cost levels for which the complete filter is not optimal, the cut-off point increases with educational costs, and the number of graduates is less than the number of type 2 jobs. For sufficiently high costs, the abolition of higher education is optimal.

4. Competitive equilibrium with screening: the two-factor model

As in the one-factor model, it is important to ask to what extent the competitive market achieves an optimal or satisfactory level of education. It remains true that there is a divergence between private and social benefits in filtering, but, as has been shown, it is no longer true that the socially optimal level of college education is zero.

The following will now be shown: If the complete filter is optimal, then it is achieved by the competitive market in which college education is supplied to everyone willing to pay for its cost. The complete filter remains the competitive allocation for higher cost levels, even up to levels such that everyone is worse off than they would be under no filtering. For still higher cost levels, the competitive equilibrium is no longer the complete filter but rather one in which the number of graduates is less than the number of type 2 jobs; it remains true that, under the same informational assumptions made as earlier, everyone is worse off under the equilibrium allocation than they would be under no filtering.

In the two-factor model, let w_1 be the price per unit of type 1 labor, w_2 the price per efficiency-unit of type 2 labor. Obviously, there will be at least one graduate working at type 2 labor; since the wages per man of graduates must be the same in all uses, a graduate must earn $w_2 \bar{z}_g$ per man. Similarly, a non-graduate must earn w_1 per man. At equilibrium, the expected wage of an entrant, less cost of education, must equal the wage of a non-graduate.

$$\bar{y}_e(w_2 \bar{z}_g) + (1 - \bar{y}_e)w_1 - c = w_1,$$

or, analogously to (14),

$$\bar{y}_e(w_2 \bar{z}_g - w_1) = c. \tag{26}$$

Since one unit of type 1 labor and one efficiency-unit of type 2 labor together produce one unit of product, exhaustion of the product implies,

$$w_1 + w_2 = 1. \tag{27}$$

From (26), it follows immediately, that $w_2 \bar{z}_g > w_1$ if $c > 0$. This statement in turn implies that no graduate is working at a type 1 job, for, since all graduates are indifferent in the market place, all can earn

$w_2\bar{z}_g$ and therefore none will work for w_1 at a type 1 job. The total supply of type 2 labor by all graduates therefore does not exceed the number of units of type 2 labor used in the economy at equilibrium.

$$N_g\bar{z}_g \leqq 1 - N_g, \tag{28}$$

a condition which also holds at the optimal allocation, according to (20). If the equality held, then no non-graduate would prefer to work in a type 2 job. Since his income in such a job would be $w_2\bar{z}_n$, we must have, in this case, $w_1 \geqq w_2\bar{z}_n$. On the other hand, if the inequality holds in (28), some non-graduates are working in type 2 jobs, so that $w_2\bar{z}_n = w_1$. Thus, one of the two following situations must hold at equilibrium:

$$N_g\bar{z}_g = 1 - N_g \quad \text{and } w_1 \geqq w_2\bar{z}_n; \tag{29}$$

$$N_g\bar{z}_g < 1 - N_g \quad \text{and } w_1 = w_2\bar{z}_n. \tag{30}$$

As will now be shown, which of these holds will depend upon the parameters of the problem; in particular, given other parameters, on the education cost c.

As we know from (22–23), when (29) holds, $y_0 = y_0^*$. Let $\bar{y}_e^*$ be the corresponding value of $\bar{y}_e$, the probability of graduation conditional on admission. If (29) holds, then, solve for w_1 and w_2 from (26) and (27):

$$w_1 = \frac{[\bar{z}_g^* - (c/\bar{y}_e^*)]}{(1 + \bar{z}_g^*)}, \tag{31}$$

$$w_2 = \frac{[1 + (c/\bar{y}_e^*)]}{(1 + \bar{z}_g^*)}. \tag{32}$$

From (29), these are the equilibrium wages, and the complete filter is the equilibrium allocation, provided that $w_1 \geqq w_2\bar{z}_n^*$. From (31–32), this condition can be written,

$$c \leqq \frac{\bar{y}_e^*(\bar{z}_g^* - \bar{z}_n^*)}{(1 + \bar{z}_n^*)} = c_2. \tag{33}$$

Thus, for c in this range, the complete filter is competitive equilibrium.

Recall that, as in the one-factor model, the equilibrium condition for choosing entrance to college implies that the ex ante expected income (net of educational costs) is the same for all, and therefore equal to w_1. We can then compare w_1 with the expected output in the absence of a filter, $\bar{z}/(1 + \bar{z})$ from (17). Since w_1 is linear in c, from (31), we need make the comparisons only for c approaching 0 and $c = c_2$, as far as equilibria satisfying (29) are concerned.

First note that from (26) and either (29) or (30),

$$w_2 \bar{z}_g > w_1 \geqq w_2 \bar{z}_n,$$

so that,

$$\bar{z}_g > \bar{z} > \bar{z}_n, \tag{34}$$

in any equilibrium. Then, for c approaching 0,

$$w_1 \rightarrow \frac{\bar{z}_g^*}{(1 + \bar{z}_g^*)} > \frac{\bar{z}}{(1 + \bar{z})};$$

as might be expected, when c is small, the competitive equilibrium is better than no filter. On the other hand, when $c = c_2$, we find, on substitution from (33) into (31) and some simplification, that,

$$w_1 = \frac{\bar{z}_n^*}{(1 + \bar{z}_n^*)} < \frac{\bar{z}}{(1 + \bar{z})}.$$

Therefore, there is a cost level, c_3, $0 < c_3 < c_2$, such that the complete filter is better than no filter for $c < c_3$ and worse for $c_3 < c \leqq c_2$. A fortiori, the complete filter, though a competitive equilibrium, is not optimal for $c \geqq c_3$. Hence, if there is any range of costs for which the complete filter is optimal, the upper limit of that range, c_1, must be less than c_3.

Now consider equilibria which are not complete filters, those for which (30) holds. The argument here is simple: from (27) and the condition in (30) that $w_1 = w_2 \bar{z}_n$, it follows immediately that,

$$w_1 = \frac{\bar{z}_n}{(1 + \bar{z}_n)} < \frac{\bar{z}}{(1 + \bar{z})}, \tag{35}$$

from (34), so that again the equilibrium filter is worse for everyone than the absence of college education.

(One technical remark is needed here. If $c > c_2$, the only possible competitive equilibrium must satisfy (30). However, it is possible that for some values of $c \leqq c_2$ there may be more than one equilibrium; one will be the complete filter, but there are others which satisfy (30). Note that if we substitute (35) and the corresponding value of w_2, $1/(1 + \bar{z}_n)$, into (26), we have,

$$c = \frac{\bar{y}_e(\bar{z}_g - \bar{z}_n)}{(1 + \bar{z}_n)} = A(y_0),$$

say, and therefore there is an equilibrium satisfying (30) for any c in the range of $A(y_0)$, with $y_0 \geqq y_0^*$. From (33), $A(y_0^*) = c_2$; hence, if $A(y_0)$ were an increasing function of y_0, there could not be any equilibria satisfying (30) for c-values for which the complete filter is also an equilibrium. I have not studied whether this monotonicity condition is reasonable; if not, then it is possible that the minimum value of $A(y_0)$ for $y_0 \geqq y_0^*$, say $\bar{c}$, may be less than c_2. In that case, for any c, $\bar{c} < c < c_2$, the equation $A(y_0) = c$ may have one or more solutions, so that there will be several competitive equilibria, one a complete filter and the others not.)

5. Concluding remarks

There are perhaps two final remarks that should be made, though I must be cursory in both. One is the comparison between this model and a human capital model. It has long been clear that measures of return to human capital may well be biased upwards because ability differences are confounded with differences in the inputs of schooling. Attempts have been made to correct the measures of return to schooling (see e.g., Griliches and Mason, 1972, and Hause, 1972) by introducing a variable designed to measure ability. But unfortunately these ability measures are wrong in principle. Typically, they are measures of intelligence; but "ability" in the relevant sense means the ability to produce goods, and there is simply no empirical reason to expect more than a mild correlation between productive ability and intelligence as measured on tests. Intelligence tests are designed to predict scholastic success, and this is a function they perform well. But there is considerable evidence in direct studies of productivity (e.g., by the U.S. Navy) that ability to pass tests is weakly related to ability to perform specific productive tasks. It is only the latter ability that is relevant here.

Unfortunately, this argument raises another difficulty; the model of this paper depends upon an unmeasured and unmeasurable variable, "ability". There may be no way of ever achieving a direct measurement; after all, a premise of the model is that employers cannot measure ability directly, and there is no reason to suppose that the economist is going to do better. It remains to be seen if the theory can be made to yield interesting and testable implications in the absence of direct measurements of ability.

Indeed, if we revert to the one-factor model, the filter model has some implications for macroeconomic observations. It says that an increase in the resources devoted to college education will have no positive effect on output in the non-educational sector, if all other variables are controlled for. This is indeed a strong inference, but its usefulness in making intertemporal or international comparisons is limited by the need to hold the statistical distribution of ability constant. If "ability" is influenced by cultural factors, then it will certainly vary internationally and may also be thought to vary over time.

There is also one particularly needed elaboration of the model (which is not to say that it doesn't cry out for elaboration in many other directions). This is the relation between college filtering and on-the-job filtering. Once an employee has been hired, the employer can gradually draw on more directly obtained information to determine his productivity. However, this filtering may be costly. To the extent that the employer does filter and does so accurately, the value of the college filter is reduced. The employer pays the average product of a group with given educational achievement only during the period before his own filter has become effective. Conversely, however, an increase in the college population will mean (and has meant) a depreciation in the quality of non-college students (this is *not* necessarily the same as a decrease in the quality of college students). It may be that, with the increased supply of college-filtered students and with a decrease in the quality of non-college students, the alternative filters become less worthwhile and eventually cease to be profitable. This means that the improvement in the equality of income due to increased college education may therefore be offset by the decrease in alternative filters leading to qualification for type 2 jobs. In particular, it means that the criteria used to select for type 2 jobs become narrower in scope, and it can easily be true that both efficiency and equity suffer.

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